The Effect of Spatial Averaging of Earthquake Ground Motion on the Response of Structures and Equipment

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Abstract

A stochastic representation of earthquake ground motion is advocated as a supplement and an alternative to conventional response spectra for purposes of seismic analysis and design of structures and equipment. In the model the strong motion duration captures the essential transient character of earthquake ground motion, while the spectral density function represents its "equivalent stationary" frequency content. This representation provides a starting point for modeling the space-time variation of earthquake ground motion and offers an attractive format for bringing recent geophysical information about source parameters and ground motion frequency content to bear on earthquake engineering practice.

1. Introduction

To date, it has been common in earthquake engineering to focus attention on the time history of ground acceleration (and its response spectrum) at a given location in space. This is a logical consequence of the fact that much of our knowledge about earthquake ground motion comes from recorded accelerograms and that engineers have traditionally focused on "point" facilities for which it seemed reasonable to ignore "local" spatial variation of ground motion. Empirical strong-motion data from closely spaced arrays of seismographs is now gradually becoming available, and engineers are increasingly directing their attention to the effects of earthquakes on spatially distributed systems. It is evident that the "response spectra" format of motion representation is poorly suited to account (for engineering purposes) for critical new information about spatial variability of earthquake ground motion.

In earlier work [1,2], the writer has sought to substantiate a proposal for the use of a direct stochastic representation of earthquake ground motion in terms of the ground motion spectral density function, $G(\omega)$, and the duration of strong shakings, s. While essentially equivalent to response spectra in reference to single-degree linear systems, the stochastic representation leads to improved predictions of the response of linear multi-degree systems and equipment, and the behavior of a variety of nonlinear systems, including those sensitive to low cycle fatigue and liquefaction. It also provides a tractable format for dealing with the effect of spatial variation of ground motion, accounting for the influence of local geology, and relating ground motion frequency content (and duration) to basic earthquake source parameters and source-to-site distance. Only the first of these factors (spatial variation) is discussed in this brief summary paper.

2. Spectral Density Function: Effect of Local Averaging

The spectral density function $G(\omega)$ expresses how the ground acceleration intensity at a given point in space is distributed over frequency. For actual records, $G(\omega)$ is directly connected to the "Arias intensity" I_o (the integral over time of the squared accelerations in the record) and, through the relationship $I_o = \sigma^2 s$, to the r.m.s. ground acceleration σ. Recall from basic theory of stationary random processes that the principal property of $G(\omega)$ is that its integral over positive frequencies equals σ^2. (Attention is restricted here to a single component of ground motion.) Scaling $G(\omega)$ with respect to σ^2 yields the unit-area spectral density function $g(\omega)$.

Of course, earthquake ground motion varies in space as well as with time. The distances may range from tens of centimeters to several kilometers to cover the dimensions of the base of strong-motion instruments as well as foundations of buildings or components of lifeline systems. Recent research [3] based on data recorded by the SMART-1 seismograph array indicates that analytical models of homogeneous random field theory can be used to represent the space-time character of ground motion in a (locally) homogeneous random medium, say, a particular type of bedrock or a layer of alluvial soil, during the strong phase of an earthquake. In a wide alluvial basin, waves tend to propagate in all directions and the resulting random field of ground motions (say, a specified component of horizontal motion) may exhibit an isotropic spatial correlation function (or a more general "ellipsoidal" random field characterized by a correlation function with ellipsoidal iso-correlation contours). In addition, it is appropriate (especially in the near field) to introduce a deterministic phase lag to account for partially predictable wave front propagation.

Random field theory [4] permits calculating the "admittance functions" which multiplied by the "point" spectral density function $g(\omega)$ yield the spectral density function $G_D(\omega)$ of the local spatial average of a component of the ground motion over a region D (characterizing, for example, the dimensions of a rigid foundation slab). The principal effect of the admittance function is to suppress the high frequency content of the "point" s.d.f. $G(\omega)$. It is also possible to generate cross-spectral density functions of seismic inputs at two different support points, or of two local spatial averages of the random field of ground motions over different regions in space.

3. Stochastic Prediction of System Response

The simplest seismic analysis procedures are based directly on response spectra The proposed stochastic representation (i.e. sudden exposure, for "s" seconds, to stationary excitation with given s.d.f.) permits response predictions with equal ease and superior reliability compared to procedures based directly on the response spectrum. Whenever system behavior is highly sensitive to ground motion duration, the stochastic representation has the clear advantage of accounting explicitly for duration.

Random vibration methodology provides approximate closed-form predictions for the seismic response of multi-degree linear systems. In fact, the form of the expression for the multi-degree system response variance motivates improved rules for modal combination that are of immediate benefit in the conventional response spectrum approach. (It suffices to replace the modal standard deviations by the response spectra ordinates). The writer first suggested such a "stochastic modal superposition" (SMS) procedure that fully accounts for cross-correlation between modal responses [2]. An alternate procedure (referred to as "complete quadratic combination", or CQC) was suggested by Der-Kiureghian [5]; however, its derivation assumes white noise excitation, ignores the transient nature of the seismic response, and neglects secondary effects attributable to differences in the peak factors of multi-and single-degree responses. If the response is stationary and the input white noise, the two procedures are identical, as they are simply based on two different ways of expanding the multi-degree response variance. Similar procedures are available to predict floor response spectrum directly.

The main point noted in this paper is that the methodology to predict multi-degree system response or floor response (when the excitation s.d.f. is prescribed) also enables evaluation of the effect of local spatial averaging on system response; it suffices to replace the input "point" s.d.f. $G(\omega)$ by the s.d.f. of the local spatial average, e.g., $G_D(\omega)$. Similarly, random field models of the earthquake ground motion provide the input for random vibration analysis of structures with multiple supports and spatially extended foundations. More details are provided in the full version of this paper to be published in Nuclear Engineering and Design.

Conclusions

It is argued in this paper that the "$G(\omega)$-s" stochastic model of earthquake ground motion is superior to the conventional representation based on the response spectrum. The principal features of the proposed model are: (i) $G(\omega)$ provides direct information about the frequency content of (the strong shaking phase of) ground motions and the model accounts explicitly for motion duration; (ii) it leads to improved predictions of structural response for multi-degree and secondary linear systems and yields predictions of cumulative damage measures; (iii) it permits extending ground motion models to account for local (spatial) variation; (iv) it is compatible with conventional representation of earthquake ground motion such as the response spectrum and provides a convenient starting point for generating synthetic accelerograms.

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