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THE VIBRATION CHARACTERISTICS OF A FLAT PLATE-TYPE FLUID-STRUCTURAL INTERACTION MODEL

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Abstract

The in-fluid natural frequencies of a system of stacked rectangular plates are found to be a function of plate aspect ratio, edge boundary condition, the finite depth of the surrounding fluid layers, and the number of plates in the system. The in-fluid natural frequencies decreases as the depth of a fluid medium decreases. Also as more plates are stacked in parallel, the in-fluid natural frequencies are further reduced due to the plate-plate interactions.

1. Introduction

The analysis of fluid-structure interaction problem is of great importance in many engineering fields such as nuclear, mechanical, marine and aeronautical. In the case of the structures in contact with a finite fluid medium, the fluid reactive loadings can significantly affect the structure responses.

This paper investigates for the in-fluid natural frequencies of a system of thin flat rectangular plates stacked in parallel and separated by channels of a dense fluid (Fig. 1). Each plate is assumed elastically restrained (ER) along two opposite edges and structurally free (F) along the other two opposite edges. Such fluid-structural systems are found in flat plate type nuclear reactor core designs. The principal problem in the design of such reactor cores is the potentiality of hydrodynamic instabilities due to fluid flow during reactor operations. The dynamic response of a plate with ER-F-ER-F edges can not be solved exactly because of inseparable eigenfunctions, and hence may be approximated by superimposing distributed rotational stiffnesses (K_S) along the simply-supported (SS) edges (Fig. 2). Additionally, the response of a structure in contact with a fluid can not be solved exactly due to the intermodal coupling between the fluid and the structure.

The dynamic response of a plate with ER-F-ER-F edges is described approximately by solutions of two plates with SS-F-SS-F edges - one plate is loaded by the external lateral force with homogeneous boundary conditions and another plate is excited by the line

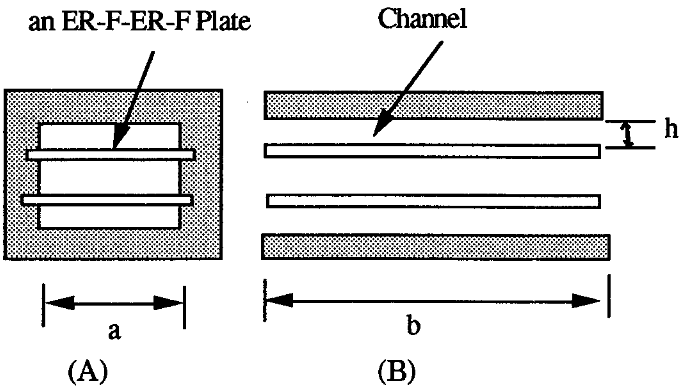

Fig. 1 Two-Plate Model: (A) Front View
(B) Cross-sectional View

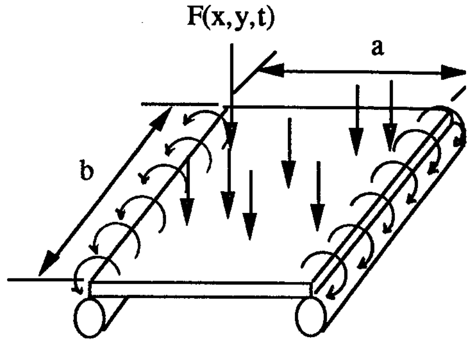

Fig. 2 A Plate with ER-F-ER-F edges

moments distributed along two opposite simply-supported edges. Hence the dynamic response of a plate with ER-F-ER-F edges is expressed as infinite series form based on the eigenfunction of the plate with SS-F-SS-F edges. For the coupled equation of the plate and the fluid, the thin plate theory and the homogeneous wave equation are employed. By using Fourier transformation method and complex plane, the fluid reactive loadings are estimated. Actually the fluid reactive loadings are analyzed in the low frequency range. Finally the in-fluid natural frequencies of a plate with ER-F-ER-F edges are estimated for the fluid-structural systems having one to five plates. The edge boundary conditions are evaluated by varying the rotational stiffness. The plate material assumes an elastic modulus, $E = 6.9 \times 10^7$ KPa (1×10^7 psi). The plate width (a) is kept constant, a = 7.62 cm (3.0 inch). The fluid is water at room temperature.

2. Equation of Coupled Motion

The equation governing the response of a fluid-loaded ER-F-ER-F plate is written in series form of normal modes as:

$$N_{mn}B_{mn}W_{mn} = F_{mn} \tag{1}$$

where N_{mn} is the orthogonality property, $B_{mn} = M_p(\omega^2_{mn} - \omega^2)$, M_p is the plate mass, ω_{mn} is the in-vacuo natural frequency, ω is the exciting frequency and W_{mn} is the amplitude. The normalized forcing function is given by:

$$F_{mn} = \int_0^a \int_0^b \psi_{mn}(x,y)f(x,y)\,dxdy \tag{2}$$

are expressed as an eigenfunction(ψ_{mn}) of an SS-F-SS-F plate and the lateral forcing function,f(x,y). The lateral forcing function is treated separately into two components - the forces induced by the fully-developed turbulent flows, which have the motion-independent and motion-dependent forces, and the acoustic pressure.

The plate vibration induced by turbulent flows may result in the displacement of the fluid layers at the surface of the plate, which is assumed to vibrate in the shapes of the in-vacuo modes of a plate. When the fluid on the flexible surfaces of a plate vibrates in the same mode of a plate, the fluid reaction introduces the additional mass effect to the total plate mass, which is called 'fluid loading'. The fluid loadings are associated with the in-vacuo mode shapes of a plate and have the greatest contributions in the region around the modal wavenumbers of the plate.

3. Estimation of The In-fluid Natural Frequencies

It is well known that the dynamic behaviors of a plate exposed to a high density fluid such as water are changed due to the fluid loading effect. For the case of a rectangular plate vibrating in water, numerous attempts have been made to investigate the fluid loading effects in the past decades. Davies, Pope and Leibowitz estimated the fluid loading effects for a plate having all edges simply-supported by using the modal wavenumber and the Kronecker delta functions. Lomas and Hayek calculated the constants of the proportionality for the fluid loading effect of a plate with all clamped edges. Giman computed approximately the constants of the proportionality for the fluid loading effect of a plate with SS-F-SS-F edges. The fluid loading effect can be described as the intermodal coupling coefficient:

$$J_{mnqr} = J_x + iJ_r \tag{3}$$

where J_x is the reactive component and J_r is the resistive component and the indice, m,n,q,and r represent the mode numbers. The reactive component contributes largely to the additional mass effect in the low frequency region. But the resistive component is negligible in the low frequency region due to its small efficiency.

For a single ER-F-ER-F thin rectangular plate vibrating in finite fluid medium, the equation of the free motion becomes:

$$\omega_{mn}^2 - \omega^2\left(1 + \frac{2\rho H_{mn}\chi_{mn}}{M_p k_{mn}}\right) = 0 \tag{4}$$

where H_{mn} = coth(k_{mn}h) is the channel height parameter, k_{mn} is the modal wavenumber of a ER-F-ER-F plate and h is the channel height and χ_{mn} is the constants of the proportionality for the fluid loading effect of a plate with SS-F-SS-F edges. Hence, the in-fluid natural frequency (ω^f_{mn}) for a single-plate model is estimated as:

$$\omega^f_{mn} = \frac{\omega_{mn}}{\sqrt{1 + \frac{2\rho H_{mn}\chi_{mn}}{k_{mn}M_p}}} \tag{5}$$

where ρ is the fluid density. In Eq. (5), if the channel height goes infinite, the channel height parameter becomes unity and so the values of frequencies are the same as those of a plate in infinite fluid medium. In a similar manner, the in-fluid natural frequency for a two-plate model can be expressed in the form of the matrix as:

$$\begin{bmatrix} \omega^2\left(1+\dfrac{2\rho H_{mn}\chi_{mn}}{k_{mn}M_p}\right)-\omega_{mn}^2 & -\dfrac{\rho\chi_{mn}\omega^2}{k_{mn}M_pH_2} \\ -\dfrac{\rho\chi_{mn}\omega^2}{k_{mn}M_pH_2} & \omega^2\left(1+\dfrac{2\rho H_{mn}\chi_{mn}}{k_{mn}M_p}\right)-\omega_{mn}^2 \end{bmatrix}=0 \quad (6)$$

where $H_2 = -2\sinh(k_{mn}h)$. Equation (6) involves the effect of the motion of the adjacent plate. Likewise, for the multi-plate models, the in-fluid natural frequencies can be estimated by adding the effects of the plates in the model.

4. Results and Discussion

In this study, one to five plates for the fluid-structural interaction models are employed to investigate the effects of fluid loading and the plate-plate interactions. To express the in-fluid natural frequencies in terms of unit Hertz (Hz), the width (a) and thickness of a plate are given as 7.62 cm and 0.127 cm, respectively. For a single-plate and a two-plate models, respectively, Figs. 3 and 4 show the in-fluid natural frequency loci of the first mode (1,1) for various dimensionless channel height (h/a) are plotted versus the dimensionless rotational spring constants (D/aK_s), which is the ratio of the plate bending stiffness to the product of the plate width and the rotational spring stiffness

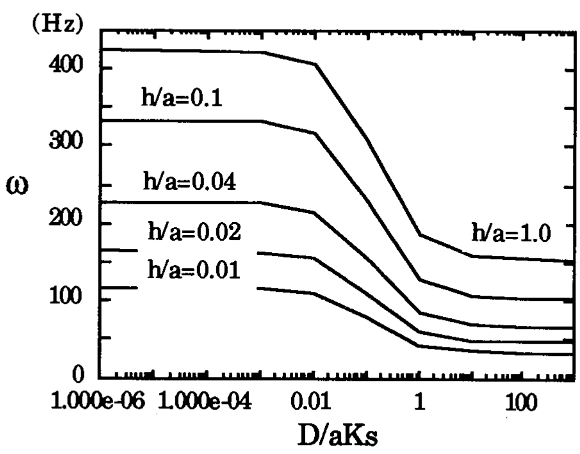

Fig. 3 In-fluid natural frequencies of mode(1,1) vs. D/aK_s, Single-plate, b/a=1

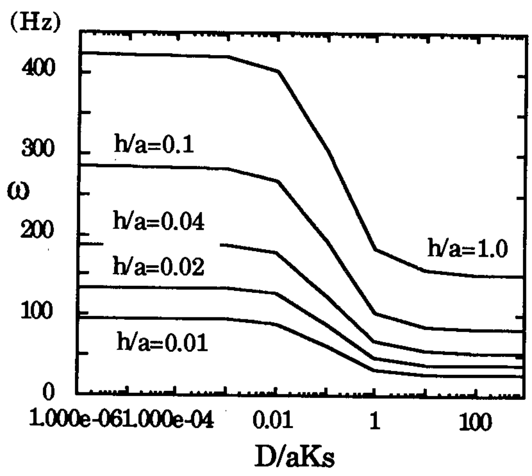

Fig. 4 In-fluid natural frequencies of mode(1,1) vs. D/aK_s, Two-plate, b/a=1

distributed along the two simply-supported edges. In Fig. 5, the in-fluid natural frequencies for the first mode of models for one through five plates stacked in parallel are drawn versus D/aK_s. The difference in the natural frequencies between the single-plate and the two-plate models is greater than that between the four-plate and the five-plate models. So, one can predict that the in-fluid natural frequencies for the fluid-structural model having more than five plates would not differ significantly from those of a four or five-plate models. In Fig. 6, the in-fluid natural frequencies for the first mode of a single-plate and a five-plate models for the case where $D/aK_s = 1000$ is plotted versus the dimensionless channel height. As the dimensionless channel height approaches 1, the in-fluid natural frequencies for both the single-plate and the five-plate models approach that of a single plate in an infinite medium.

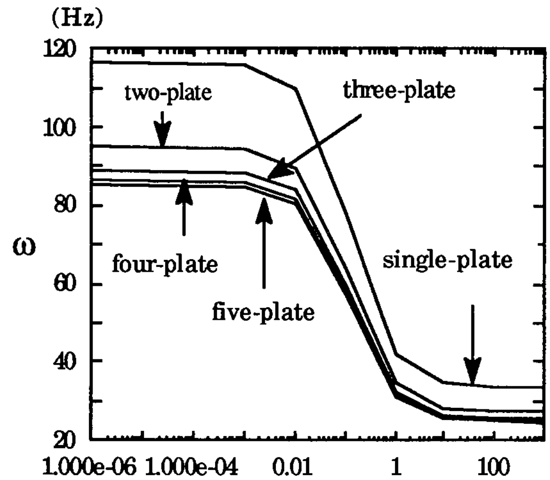

Fig. 5 ω^f_{11} vs. D/aK_s, h/a=0.01, b/a=1

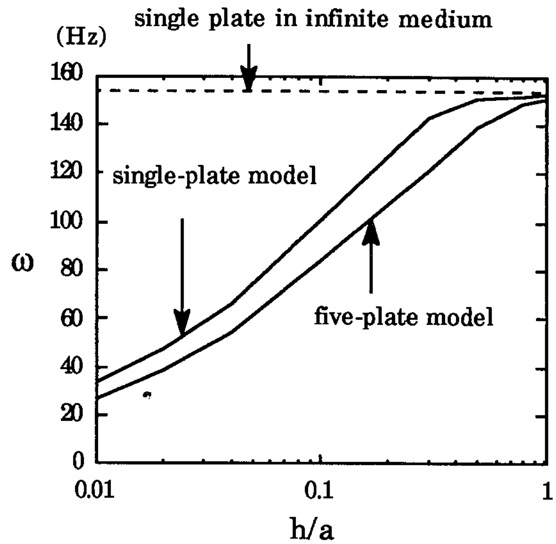

Fig. 6 ω^f_{11} vs. h/a, $D/aK_s = 1000$, b/a=1

5. Conclusions

On the basis of the analyses in this study, the conclusions are obtained as follows.

1. As the channel height (h) decreases, the in-fluid natural frequencies of the fluid-structural interaction model decrease.
2. In the case of the large channel height ($h \geq a$), the plate-plate interaction effects are very weak.
3. As more plates are stacked in parallel, the in-fluid natural frequencies are further reduced due to the plate-plate interactions.

6. References

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