

Thermal Stresses in Pipes under Transient Heat Flow — A Semi-Analytical Approach

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ABSTRACT

Subsection NB-3600 of "ASME Boiler and Pressure Vessel Code" Section III furnishes approximate formulas for evaluating the stresses in class 1 nuclear piping. The stresses induced by thermal loads are given in terms of three quantities \bar{T} , ΔT_1 e ΔT_2 derived from the actual temperature distribution across the pipe wall. The objective of this paper is to determine these three quantities for any given temperature transient in the internal fluid. The pipe wall is considered to be flat, i.e., the one dimensional heat conduction equation in cartesian coordinates is used. This approximation was justified by considering a pipe with ratio between external and internal radii $r_o/r_i = 1.174$ and using the computer code ANSYS for both cases, i.e., axisymmetric and flat wall models. Both temperature and stress fields differ very little, showing that the flat wall approximation gives satisfactory accuracy.

The internal fluid temperature versus time curve can be arbitrary. It is approximated by a sum of step functions. Thus we have developed the analytical solution of the governing differential equation only for the unity step change in the fluid temperature. The actual solution is obtained by superposition of this solution.

A computer code was developed and the ASME code stresses obtained compared with exact elastic solution and finite element stresses (ANSYS).

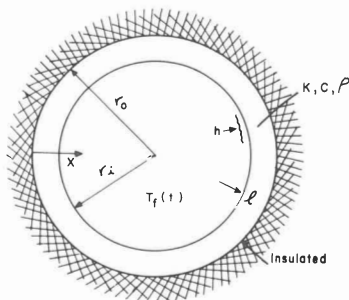


FIGURE 1 - Problem Geometry and Boundary Conditions

1. Introduction

Class 1 nuclear piping must be designed against fatigue failure. The ASME Code Section III under subsection NB-3600 gives simplified rules to fulfill the general requirements of NB-3200. Thermal stresses in the pipe caused by temperature transients in the fluid can be estimated by means of the quantities \bar{T} , ΔT_1 and ΔT_2 defined in the Code.

Here we propose a semi-analytical method to find the temperature field across the pipewall produced by an arbitrary temperature transient in the internal fluid. The pipewall is considered flat and the one dimensional heat conduction differential equation in cartesian coordinates is used.

The pipe is considered to be externally insulated although other boundary conditions could be treated as well.

2. Semi-Analytical Solution of Transient Heat Equation

Figure 1 shows the geometry and the boundary conditions of the problem. The internal and external radii are respectively r_i and r_o . The wall thickness is denoted by ℓ . The film coefficient, pipe thermal conductance, specific heat and mass density are h, k, c, ρ respectively. Initially the pipe is at temperature T'_0 and the fluid temperature time history is given as $T_f(t)$.

The governing differential equation is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (1)$$

where T and t denote temperature and time respectively. The variable x is measured from the pipe outside wall and the thermal diffusivity $\alpha = \frac{k}{\rho c}$. The boundary conditions are

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0 \quad (2)$$

$$k \frac{\partial T}{\partial x} = h(T - T_f) \text{ at } x = \ell \quad (3)$$

$$T = T'_0 \text{ at } t = 0 \quad (4)$$

For the general case of arbitrary fluid temperature time history a complete analytical solution is not possible. However considering $T_f(t)$ as a sum of step functions as shown in Figure 2 the solution to equation (1) with boundary conditions (2), (3), (4) is obtained as a sum of unit step solutions as follows.

Let $\Delta T^1(x, j)$ be the variation of pipe temperature at point x , at time $t = j\Delta t$ due to a unit step temperature variation in the fluid at time $t = 0$. Thus the temperature at point x at time $t = j\Delta t$ is approximately

$$\begin{aligned} T(x, t=j\Delta t) = & T'_0 + \Delta T^1(x, j)(T'_0 - T'_0) + \Delta T^1(x, j)(T_1 - T'_0)/2 \\ & + \Delta T^1(x, j-1)(T_2 - T'_0)/2 + \Delta T^1(x, j-2)(T_3 - T_1)/2 + \dots + \Delta T^1(x, 1)(T_j - T_{j-2})/2 \end{aligned} \quad (5)$$

or in compact form

$$T(x, t=j\Delta t) = T'_0 + \Delta T^1(x, j)(T'_0 + T_1 - 2T'_0)/2 + \sum_{k=1}^{j-1} \Delta T^1(x, j-k)(T_{k+1} - T_{k-1})/2 \quad (6)$$

Therefore for Δt sufficiently small equation (6) gives the temperature field in the pipe. It remains the determination of $\Delta T^1(x, j)$, i.e., solution of equation (1) with

boundary conditions (2), (3), (4) with $T_f = T'_0 + 1$. Using separation of variables we find the solution.

$$\Delta T^1(x, t) = 1 + \sum_{n=1}^{\infty} A_n^1 \exp \left[-\frac{\alpha}{\ell^2} (\lambda_n \ell)^2 t \right] \cos \left[(\lambda_n \ell) \frac{x}{\ell} \right] \quad (7)$$

where:

$$A_n^1 = -\frac{2 \sin(\lambda_n \ell)}{\lambda_n \ell + \sin \lambda_n \ell \cos \lambda_n \ell} \quad (8)$$

and $(\lambda_n \ell)$ are the roots of equations (9) below.

$$\cot \lambda_n \ell = \frac{\lambda_n \ell}{B_1} \quad (9)$$

where the Biot number $B_1 = \frac{\ell h}{K}$

Therefore equation (6) allow us to calculate \bar{T} , ΔT_1 , and ΔT_2 according to the definitions of subsection NB-3652 of ASME Code.

3. Numerical Solution of the Eigenvalue Equation

The roots of equation (9) can be easily found by means of Newton's method specially applied to the equation. Consider determining the roots of

$$F(\lambda \ell) = B_1 \cot \lambda \ell - \lambda \ell = 0 \quad (10)$$

Plotting $\cot \lambda \ell$ and $\frac{\lambda \ell}{B_1}$ it is clear that the i^{th} root of equation (10) is between $(i-1)\pi$ and $(i-1/2)\pi$. Also in this interval $F'(\lambda \ell) < 0$ and $F''(\lambda \ell) > 0$, i.e., $F(\lambda \ell)$ has the configuration shown in Figure 3.

The following algorithm is easily seem to be always convergent to determine de i^{th} root:

- 1) Let $x_1 = (i-1)\pi + \frac{\pi}{4i}$
- 2) Calculate $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$$= x_1 + \frac{B_1 \cot x_1 - x_1}{1 + B_1 \operatorname{cosec}^2 x_1}$$
- 3) If $x_2 < (i-1)\pi$ let $x_1 = 1/2 [x_1 + (i-1)\pi]$

and return to step 2. If not let $x_1 = x_2$, return to step 2 and repeat this process until the tolerance has been achieved, i.e., $||x_1| - |x_2|| < \text{tolerance}$.

Application of this scheme in the computer has shown excelent results.

4. Numerical Results

4.1 Temperature Distribution

Referring to Figure 1 we have considered a pipe with the following characteristics: $r_i = 71.65 \text{ mm}$; $r_o = 84.15 \text{ mm}$; $c = 550 \text{ Ws/kg}^\circ\text{C}$; $\rho = 7.82 \times 10^{-6} \text{ kg/mm}^3$; $h = 5,000.0 \times 10^{-6} \text{ W/mm}^2 \text{ oC}$. The thermal conductance k was considered differently by ANSYS and by the present method, i.e., ANSYS has the capability of input k as a function of the temperature and the present method used and average value of k . The general expression for k was $k = 45.73 \times 10^{-3} - 0.013 \times 10^{-3} T \text{ (W/mm}^\circ\text{C)}$.

The thermal boundary conditions were: T'_0 (initial pipe temperature) = 296°C ; $T_f(t)$ (fluid temperature time history) = 120°C .

Figure 4 presents the comparison between ANSYS temperature calculations (axisymmetric and planar models) and the present method using a value of k corresponding to the average temperature of the transients, i.e., $T_{ave} = (120^{\circ} + 296^{\circ})/2 = 208^{\circ}\text{C}$. This gives $k_{ave} = 43.026 \times 10^{-3} (\text{W}/\text{mm}^{\circ}\text{C})$. It is seen that at $t=5.0$ records, for example, the temperature are

LOCATION	TEMPERATURE ($^{\circ}\text{C}$) at $t= 5.0$ seconds		
	ANSYS (axisym)	ANSYS (planar)	Present Methods
INNER WALL	206.25	203.85	203.9
MIDDLE THICKNESS	255.06	252.51	251.8
OUTER WALL	271.90	270.03	268.5

Considering that ANSYS calculations were done with variable k the results above are in good agreement.

4.2 Stress Distribution

Subsection NB-3600 of ASME Code III gives approximate formulas to evaluate the thermal stresses in pipes. The peak thermal stress intensity in a straight pipe is given by

$$S_p = \frac{E \alpha}{2(1-\nu)} K_3 \Delta T_1 + \frac{E \alpha}{1-\nu} \Delta T_2 \quad (11)$$

where ΔT_1 and ΔT_2 are defined in ASME Code III, E, α and ν are Young's modulus, thermal expansion coefficient and Poisson's ratio, respectively. $k_3=1.0$ for straight pipe.

The stress was calculated in a cylindrical coordinates system where r, θ, z denote radial, circumferential and axial directions, respectively. Initially we have used ANSYS capability to calculate the actual thermal stresses which was done using a plane stress axisymmetric model for both previous cases, i.e., temperature distribution from axisymmetric and planar models. The values used were $E = 207,798.072 - 88.275T - 0.0031T^2$ (N/mm^2) where T is the temperature in degrees Celsius, $\nu = 0.3$ and $\alpha = 12.5 \times 10^{-6}$ ($1/^{\circ}\text{C}$). Since the pipe is in a plane strain state the ANSYS results must be transformed as follows.

The plane stress elastic solutions for a straight pipe with a radial temperature distribution $T(r)$ is given by (2).

$$\sigma_{rr} = \frac{\alpha E}{r^2} \left[\frac{r^2 - a^2}{b^2 - a^2} \int_a^b T(r) r dr - \int_a^r r T(r) dr \right] \quad (12)$$

$$\sigma_{\theta\theta} = \frac{\alpha E}{r^2} \left[\frac{r^2 + a^2}{b^2 - a^2} \int_a^b T(r) r dr + \int_a^r T(r) r dr - T(r) r^2 \right] \quad (13)$$

where a and b are the internal and external radii, respectively. Also by (2), the plane strain solution is obtained from the plane stress results by substitution for E, ν and α of the quantities $E_1 = E/(1-\nu^2)$, $\nu = \nu/(1-\nu)$ and $\alpha_1 = \alpha(1+\nu)$, respectively. Applying these in (12) and (13), is to replace αE by $\alpha_1 E_1 = \alpha E/(1-\nu)$. Also for the case of plane strain there is an additional non zero stress components, σ_{zz} ; when the end faces are free of traction it is given by the formula:

$$\sigma_{zz} = \frac{\alpha E}{1-\nu} \left[\frac{2}{b^2 - a^2} \int_a^b T(r) r dr - T(r) \right] = \sigma_{rr} + \sigma_{\theta\theta} \quad (14)$$

and the components σ_{rr} and $\sigma_{\theta\theta}$ are obtained dividing equations (12) and (13) by $(1-\nu)$. This last result was used to transform plane stress ANSYS results into plane strain solution.

The temperature distribution given by the present method at $t=5.0$ seconds was used to evaluate ASME Code III and elastic stress solutions. The results are summarized below:

LOCATION	STRESSES (N/mm^2) at $t = 5.0$ seconds			
	ANSYS (*)		Elastic. Solution (**)	ASME Code Method(**)
	Axisymmetric Model for Temp. Calc.	Planar Model for Temp. Calc.		
INNER WALL	$\sigma_{\theta\theta} = 149.81$	$\sigma_{\theta\theta} = 149.39$	$\sigma_{\theta\theta} = 146.96$	SI= 144.14 ($k_3 = 1.0$)
	$\sigma_{rr} = 0.0$	$\sigma_{rr} = 0.0$	$\sigma_{rr} = 0.0$	
	$\sigma_{zz} = 149.81$	$\sigma_{zz} = 149.39$	$\sigma_{zz} = 146.96$	
	SI = 149.81	SI = 149.39	SI = 146.96	
OUTER WALL	$\sigma_{\theta\theta} = -70.76$	$\sigma_{\theta\theta} = -72.46$	$\sigma_{\theta\theta} = -71.42$	
	$\sigma_{rr} = 0.0$	$\sigma_{rr} = 0.0$	$\sigma_{rr} = 0.0$	
	$\sigma_{zz} = -70.76$	$\sigma_{zz} = -72.46$	$\sigma_{zz} = -71.42$	
	SI = 70.76	SI = 72.46	SI = 71.42	

(*) Young's modulus $E = 207,798.072 - 88.275T - 0.003T^2$

(**) Young's modulus $E = (\bar{T} = 208^\circ C) = 189,307.0$

SI Stress intensity

5. Conclusions

The previous section showed that the ASME Code Method gives good estimates to the maximum stress intensity due to thermal effects. Comparison between the two ANSYS solutions, i.e., considering the temperatures found by axisymmetric and planar models into the same axisymmetric model to find the stresses, showed very little difference in the stress intensities. This justifies the use of planar models to temperature calculations.

The slight unconservatism found in the ASME Code method compared with the elastic solution is due to the fact that for a straight pipe, k_3 in equation (11) should be $k_3 = 1 + \frac{b-a}{3(b+a)}$. Since the quantity $\frac{b-a}{3(b+a)}$ is much smaller than 1, it can be neglected. Here $\frac{b-a}{3(b+a)} = .0267$.

Although we have considered only one set of boundary conditions, other boundary conditions could be treated identically. This will affect essentially the eigenvalue equation (9).

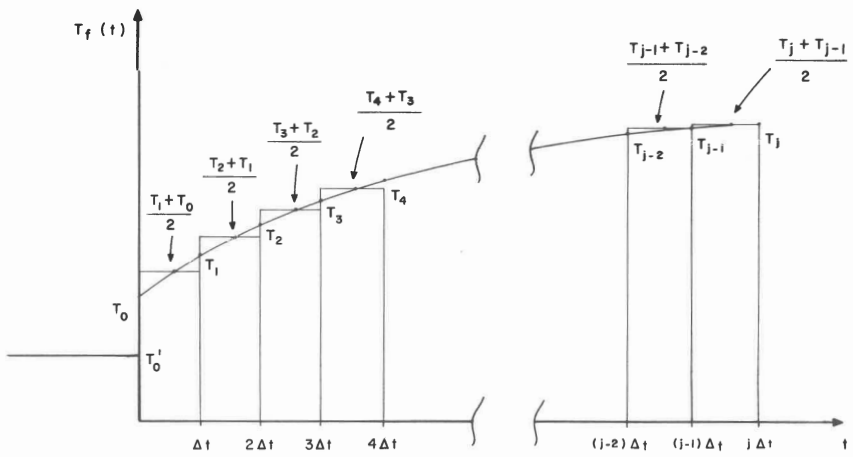


FIGURE 2 - Discretization of $T_f(t)$

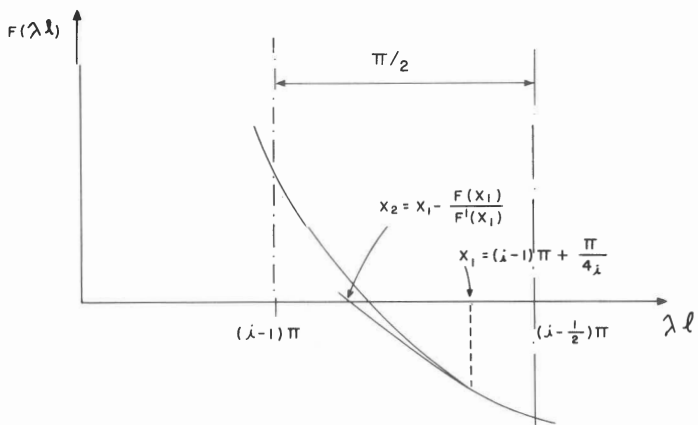


FIGURE 3 - Configuration of $F(\lambda\ell)$ in the Vicinity of the i^{th} root

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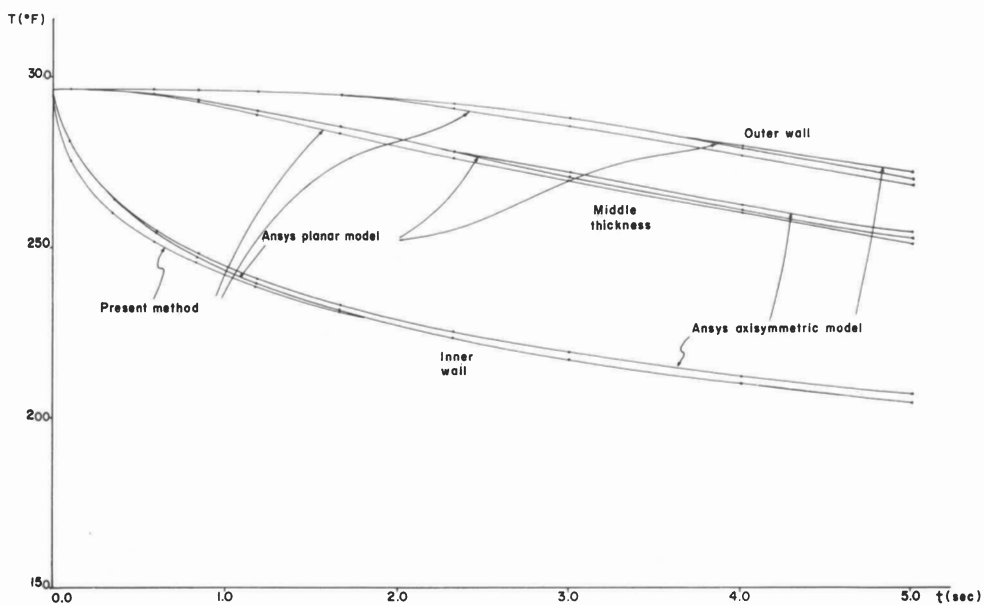


FIGURE 4 - Temperature Time Histories

