STRESS ANALYSIS OF TWO-DIMENSIONAL PROBLEMS UNDER SIMULTANEOUS CREEP AND PLASTICITY

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ABSTRACT

In many reactor components which operate at high temperatures, thermally activated creep and plasticity occur simultaneously under cyclic conditions. Because of possible degradation of the fatigue life of such structures, it is necessary for the designer to determine, as an essential part of the design process, reasonably accurately the time-dependent response of these components under postulated load histories. With this objective in mind, a method of analysis is developed for plane and axisymmetric problems of arbitrary geometry subjected to arbitrary loading and transient non-uniform temperature fields.

In an attempt to introduce the dependence of the material response on past history, a phenomenon which is encountered in viscoelastic materials and which has been observed to exist in metals under high temperature, the constitutive equations are formulated on the basis of the memory theory of creep. Embodied in these equations is the time-independent elastic-plastic stress strain relations for isotropic materials, with kinematic strain-hardening. Ziegler's modification of Prager's rule is considered.

Because of limitations on presently available material data required for the memory theory of creep, an alternate formulation based on the mechanical equation of state is also considered. In this connection two engineering creep theories are used: the time-hardening rule and the strain-hardening rule.

The numerical analysis is obtained within the framework of the finite element method and a step-by-step solution procedure is used. Numerical examples illustrating the combined effect of creep and plasticity will be given.

1. INTRODUCTION

In many reactor components which operate at high temperatures, thermally activated creep and plasticity occur simultaneously under cyclic conditions, which generally result in a time-dependent inelastic response. The present paper deals with the combined creep and plasticity problem utilizing appropriate constitutive relations and making use of the finite element computational method.

In an attempt to introduce the dependence of the material response on past history, a phenomenon which is encountered in viscoelastic materials and which has been observed to exist in metals under high temperature, the constitutive equations are formulated on the basis of the memory theory of creep[1]. Because of the limited creep data, which is available only for single step simple extension experiments, the stress-strain relations can only be derived in the form of a nonlinear superposition integral[2]. However, instead of using direct superposition, the kernel function was derived as an integral transformation that reduced the time-temperature-stress relationship to a single quantity. The uniaxial stress-strain law is first discussed with emphasis on the creep problem. Then the incremental stress-strain relations for the combined creep and plasticity problem are derived. The time-temperature-stress transformation is derived for a particular class of material, namely, stainless steel.

2. UNIAXIAL CREEP RELATION

Consider a uniaxial creep experiment in which the measured creep strain ε is determined as a function of stress σ, temperature T and time t as shown in the following equation.

$$\varepsilon = F(\sigma, T, t) \tag{1}$$

This general relation can be recast in a simpler form as in equation (2)

$$\varepsilon = \sum_{i=1}^{n} f_i(\sigma, T)\, g_i(t) \tag{2}$$

or equation (3)

$$\varepsilon = \sum_{i=1}^{n} a_i(T)\, \sigma^i\, g_i(t) \tag{3}$$

Any one of these equations can adequately describe the creep response of a uniaxial bar under conditions of constant stress and constant temperature. In order to generalize the uniaxial creep strain-stress relation to time variable stresses and temperatures, we assume that the strain at any time t is a functional of the history of loading as shown in equation (4)

$$\varepsilon(t) = \underset{\tau=-\infty}{\overset{\tau=t}{F}}[\sigma(\tau)] \tag{4}$$

This equational can be expaned into Frechet series as follows

$$\begin{aligned}\varepsilon(t) = & \int_{-\infty}^{t} J_1(t-\tau_1) \frac{d\sigma(\tau_1)}{d\tau_1} d\tau_1 \\ & + \int_{-\infty}^{t}\int_{-\infty}^{t} J_2(t-\tau_1;\ t-\tau_2) \frac{d\sigma(\tau_1)}{d\tau_1}\frac{d\sigma(\tau_2)}{d\tau_2} d\tau_1\, d\tau_2 \\ & + \int_{-\infty}^{t}\int_{-\infty}^{t}\int_{-\infty}^{t} J_3(t-\tau_1;\ t-\tau_2;\ t-\tau_3) \frac{d\sigma(\tau_1)}{d\tau_1}\frac{d\sigma(\tau_2)}{d\tau_2}\frac{d\sigma(\tau_3)}{d\tau_3} d\tau_1\, d\tau_2\, d\tau_3 \\ & + \cdots\cdots\end{aligned} \tag{5}$$

where $J_i(t - \tau_1; \text{-----} t - \tau_i)$ are the creep compliance functions determined in a multi-step loading program. For constant loading history, equation (5) reduces to the following relation

$$\varepsilon(t) = \sigma J_1(t) + \sigma^2 J_2(t,t) + \sigma^3 J_3(t,t,t) + \text{-----} \tag{6}$$

which is seen to be a polynomial representation of a single-step creep test.

It is theoretically possible to represent, to any desired degree of accuracy, the creep response under any arbitrary loading history by means of equation (5). However, the experimental problems involved in determining these creep compliances and the resulting computational problems that would be encountered in stress analysis preclude the use of such a creep representation. As an alternative, non-linear superposition approximation to dequation (5), namely,

$$\varepsilon(t) = \int_{-\infty}^{t} \frac{\partial C(\sigma,t-\tau)}{\partial \sigma} \frac{d\sigma(\tau)}{d\tau} d\tau \tag{7}$$

has been used[3]. In terms of a single creep compliance function, equation (7) can be written as

$$\varepsilon(t) = \int_{-\infty}^{t} J_1(\sigma,t-\tau) \frac{d\sigma(\tau)}{d\tau} d\tau \tag{8}$$

where $J_1(\sigma,t)$ depends continuously on the stress σ. Equation (8) is exact only for linear material for which J_1 is independent of σ.

However, the simplicity of this equation in comparison with equation (5) encourages its use especially since it requires single step creep data only. Further improvement of equation (8) can be made by adding another term of the form

$$\int_{-\infty}^{t} \int_{-\infty}^{t} J_2[\sigma(\tau_1),\ \sigma(\tau_2);\ t-\tau_1, t-\tau_2] \frac{d\sigma(\tau_1)}{d\tau_1} \frac{d\sigma(\tau_2)}{d\tau_2} d\tau_1\, d\tau_2 \tag{9}$$

where $J_2(\sigma,t,t)$ is determined from a two-step creep experiment. Unfortunately, multi-step fundamental creep experiments are totally lacking for the materials of interest.

In an attempt to bridge the wide gap between equations (5) and (8), the uniaxial creep strain-stress relation is postulated in the following form

$$\varepsilon(t) = \int_{-\infty}^{t} J[\xi(t)-\xi(\tau)] \frac{d\sigma(\tau)}{d\tau} d\tau \tag{10}$$

where

$$\xi(t) = \int_{-\infty}^{t} \phi[\sigma(\tau)]\, d\tau \tag{11}$$

The physical interpretation of the transformation of equation (11) can be understood by considering a semi-log plot of the creep curves, Fig. 1, at various stress levels. As shown in the figure, creep curves for 10,000 psi and 12,000 psi can be obtained by displacing the 6000 psi curve rigidly to the left. The shifted curves, shown in dashed lines, are to be compared with the solid lines which are obtained directly from the formula. The accuracy is not of the same degree for both curves. Further analysis of the creep formula showed that the primary creep term which does not admit separation of variables in stress and time, as implied by equation (2), is responsible for the lack of accuracy. A plot of the primary creep as a function of log t in Fig. 2 indicates quite clearly that a shift of the base curve along an inclined line, shown dashed in the figure, results in exact match between the base and the original curves. This suggests the basis for the following derivation for the time stress-temperature transformation.

3. TIME-STRESS-TEMPERATURE TRANSFORMATION

For the material in question, namely stainless steel, the creep compliance can be written as

$$J = J_1 + J_2 \tag{12}$$

where

$$J_1 = A(\sigma,T) \quad [1 - e^{-t\ r(\sigma,T)}] \tag{13}$$

$$J_2 = B(\sigma,T)\ t \tag{14}$$

A measure of the horizontal shift of the base curve can be derived by first defining

$$J_1^* = J_1 \; F(\sigma_0,T_0)/F(\sigma,T) \tag{15}$$

$$J_2^* = J_2 \tag{16}$$

where

$$F(\sigma,T) = \sigma^n \; (a + bT) \tag{17}$$

n , a , b Constants

and then applying the shift hypothesis to J_1^* and J_2^*.

Introducing the shift factors ϕ_1^* and ϕ_2^* as follows

$$\phi_1^*(\sigma,T) = r(\sigma,T)/r(\sigma_0,T_0) \tag{18}$$

$$\phi_2^*(\sigma,T) = B(\sigma,T)/B(\sigma_0,T_0) \tag{19}$$

then the reduced times, by virtue of equation (11), are given by

$$\xi_i^*(t) = \int_0^t \phi_i^*[\sigma(\tau);\; T(\tau)] \; d\tau \tag{20}$$

$i = 1,\ 2$

Hence, equations (13) and (14) become

$$J_1 = A(\sigma_0,T_0) \; [1 - e^{-r(\sigma_0,T_0)\;\xi_1^*}] \cdot F(\sigma,T)/F(\sigma_0,T_0) \tag{21}$$

$$J_2 = B(\sigma_0,T_0) \; \xi_2^* \tag{22}$$

If we define

$$\eta = A(\sigma_0,T_0) \; [1 - e^{-r(\sigma_0,T_0)\;\xi_2^*}] \tag{23}$$

then equation (21) becomes

$$J_1 = F(\sigma,T)/F(\sigma_0,T_0) \cdot \eta(\xi_1^*) \tag{24}$$

Performing a second transformation on equation (24) using the following shift factor,

$$\phi_1 = F(\sigma,T)/F(\sigma_0,T_0) \tag{25}$$

we obtain

$$\xi_1 = \int_0^{\xi_1^*} \phi_1[\sigma(\xi_1^*)] \frac{\partial n(\bar{\xi}_1^*)}{\partial \bar{\xi}_1^*} d\bar{\xi}_1^* \tag{26}$$

A second transformation on J_2, equation (22), is not required. Therefore,

$$\xi_2 \equiv \xi_2^* \tag{27}$$

Substituting the above relations in equation (26) yields

$$\xi_1 = \int_0^{\xi_1^*} \frac{F(\sigma,T)}{F(\sigma_0,T_0)} \frac{r(\sigma,T)}{r(\sigma_0,T_0)} e^{-r(\sigma_0,T_0)} \xi_1(t)\, dt \tag{28}$$

The final form of the creep compliance becomes

$$J(\xi_1,\xi_2) = A(\sigma_0,T_0)\, r(\sigma_0,T_0)\, \xi_1 + B(\sigma_0,T_0)\, \xi_2 \tag{29}$$

4. COMBINED ELASTIC-PLASTIC-CREEP RELATIONS FOR MULTIAXIAL STRESS STATES

The following derivations leading to the incremental stress strain relations are obtained on the basis that the elastic and plastic strains occur instantaneously and that the total strain can be written as

$$\underset{\sim}{\varepsilon}(t) = \underset{\sim}{\varepsilon}(t)_{instantaneous} + \underset{\sim}{\varepsilon}(t)_{creep} \tag{30}$$

$$\underset{\sim}{\varepsilon}(t) = \int_0^t \underset{\sim}{C}(\sigma,\tau) \frac{\partial \underset{\sim}{\sigma}(\tau)}{\partial \tau} d\tau + \int_0^t J[\xi(t) - \xi(\tau)] \frac{\partial \underset{\sim}{s}(\tau)}{\partial \tau} d\tau \tag{31}$$

where

$\underset{\sim}{\varepsilon}(t)$:	Strain Vector	
$\underset{\sim}{\sigma}(t)$:	Stress Vector	
$\underset{\sim}{s}(t)$:	Deviatoric Stress Vector	(32)
$\underset{\sim}{C}(\sigma,t)$:	Inverse of Plasticity Matrix	
$J[\xi(t)]$:	Creep Compliance	
$\xi(t)$:	Stress-Temperature Equivalent Time	

The plasticity matrix $\underset{\sim}{H} = \underset{\sim}{C}^{-1}$ is derived next for von Mises material subject to kinematic hardening.

4.1 ELASTIC-PLASTIC INCREMENTAL STRESS-STRAIN RELATIONS

The instantaneous incremental strain $d\varepsilon_{ij}$ is the sum of the elastic strain $d\varepsilon^e_{ij}$ and the plastic strain $d\varepsilon^p_{ij}$, namely—

$$d\varepsilon_{ij} = d\varepsilon^e_{ij} + d\varepsilon^p_{ij} \tag{33}$$

where the elastic strains are related to the incremental stresses by—

$$d\varepsilon^e_{ij} = C_{ijk\ell}\, d\sigma_{k\ell} \tag{34}$$

or by the inverse relation—

$$d\sigma_{ij} = D_{ijk\ell}\, d\varepsilon_{k\ell} \tag{35}$$

For isotropic material, both the material tensors $C_{ijk\ell}$ and $D_{ijk\ell}$, respectively, reduce to—

$$C_{ijk\ell} = \frac{1-\nu}{E}\,\delta_{ik}\,\delta_{j\ell} - \frac{\nu}{E}\,\delta_{ij}\,\delta_{k\ell} \tag{36}$$

and

$$D_{ijk\ell} = \frac{E}{1+\nu}\,\delta_{ik}\,\delta_{j\ell} + \frac{\nu E}{(1+\nu)\,(1-2\nu)}\,\delta_{ij}\,\delta_{k\ell} \tag{37}$$

where

E = Young's modulus

ν = Poisson's ratio

δ_{ij} = Kronecker delta.

By Drucker's definition of work-hardening, the material is stable under plastic flow, i.e.—

$$(\sigma_{ij} - \sigma_{ij}{}^*)\, d\varepsilon^p_{ij} \geq 0$$

or (38)

$$d\sigma_{ij}\, d\varepsilon^p_{ij} \geq 0$$

where

σ_{ij} = stress state lying on the current yield surface

$\sigma_{ij}{}^*$ = previous stress state lying inside or on the yield surface

$d\sigma_{ij}$ = incremental stresses which occur as the stress state changes from $\sigma_{ij}{}^*$ to σ_{ij}.

From the above, the incremental plastic strain vector must be normal to the loading surface, therefore—

$$d\varepsilon^{p}_{ij} = \lambda \frac{\partial f}{\partial \sigma_{ij}}; \ \lambda > 0 \tag{39}$$

when

$$\frac{\partial f}{\partial \sigma_{ij}} d\varepsilon^{p}_{ij} > 0 \tag{40}$$

where

f = a suitable yield (or load) function to be defined later.

Equation (39) represents an associated flow rule since f is stipulated to be the yield function.

In view of equations (33), (35), and (39), the incremental stress-strain relations may be written as

$$d\sigma_{ij} = D_{ijk\ell}\left(d\varepsilon_{k\ell} - \lambda \frac{\partial f}{\partial \sigma_{k\ell}}\right) \tag{41}$$

In the above equation, the free parameter λ remains to be determined.

Consider a material element subjected to a stress state, σ_{ij}, which lies on the yield surface. Under a subsequent load increment, we assume plastic flow continues, and therefore the new stress state satisfies a yield condition of the following form:

$$f = f(\sigma_{ij}, \varepsilon^{p}_{ij}, \kappa) = 0 \tag{42}$$

where κ is a material parameter, assumed to be constant.

Differentiating (42) gives—

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \varepsilon^{p}_{ij}} d\varepsilon^{p}_{ij} = 0 \tag{43}$$

Eliminating $d\sigma_{ij}$ between (41) and (43) and substituting for $d\varepsilon^{p}_{ij}$ from (39) yields—

$$\frac{\partial f}{\partial \sigma_{ij}} D_{ijk\ell}\left(d\varepsilon_{k\ell} - \lambda \frac{\partial f}{\partial \sigma_{k\ell}}\right) + \lambda \frac{\partial f}{\partial \varepsilon^{p}_{ij}} \frac{\partial f}{\partial \sigma_{ij}} = 0 \tag{44}$$

Solving for λ—

$$\lambda = \Lambda_{k\ell}\, d\varepsilon_{k\ell} \tag{45}$$

where

$$\Lambda_{k\ell} = D_{ijk\ell} \frac{\partial f}{\partial \sigma_{ij}} \Bigg/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} - \frac{\partial f}{\partial \varepsilon^{p}_{mn}} \frac{\partial f}{\partial \sigma_{mn}}\right) \tag{46}$$

Finally, the incremental stress-strain law is found to be—

$$d\sigma_{ij} = D_{ijk\ell}\left(d\varepsilon_{k\ell} - \eta\,\frac{\partial f}{\partial \sigma_{k\ell}}\,\Lambda_{mn}\,d\varepsilon_{mn}\right) \tag{47}$$

where

$$\eta = 0 \qquad \text{when} \quad f < 0$$

$$\text{or when } f = 0 \text{ and } \frac{\partial f}{\partial \sigma_{ij}}\,d\sigma_{ij} < 0$$

and (48)

$$\eta = 1 \qquad \text{when} \quad f = 0 \text{ and } \frac{\partial f}{\partial \sigma_{ij}}\,d\sigma_{ij} > 0$$

We now apply the above results to the kinematic strain-hardening rule originally proposed by Prager.

The yield function is of the form—

$$f = f(\sigma_{ij} - \alpha_{ij}, \kappa) = 0 \tag{49}$$

where the tensor α_{ij} represents the translation of the yield surface. If the stress-strain curve is bilinear then α_{ij} is proportional to the plastic strain tensor, namely—

$$\alpha_{ij} = c\varepsilon^{p}_{ij} \tag{50}$$

where c is a material constant which, for linear hardening materials, is equal to 2/3 the hardening modulus in simple tension.

For (49) and (50) the yield surface becomes—

$$f(\sigma_{ij} - c\varepsilon^{p}_{ij}, \kappa) = 0 \tag{51}$$

The above results are used in conjunction with von Mises yield condition, namely—

$$f = \frac{1}{2}\,S'_{ij}\,S'_{ij} - \kappa^2 = 0 \tag{52}$$

where

$$S'_{ij} = (\sigma_{ij} - \alpha_{ij}) - \frac{1}{3}\,(\sigma_{kk} - \alpha_{kk})\,\delta_{ij} \tag{53}$$

Form (52) and (53), making use of (50)—

$$\frac{\partial f}{\partial \varepsilon^{p}_{ij}} = -\,c\,\frac{\partial f}{\partial \sigma_{ij}} \tag{54}$$

Equation (46) becomes—

$$\Lambda_{k\ell} = D_{ijk\ell} \frac{\partial f}{\partial \sigma_{ij}} \Bigg/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} + c \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{mn}} \right) \tag{55}$$

The substitution of (52) and (53) into (55) leads to—

$$\Lambda_{k\ell} = S'_{k\ell}/R \tag{56}$$

with

$$R = 2\kappa^2 \left(1 + \frac{c}{2\mu}\right) \tag{57}$$

where μ is the shear modulus.

Finally, from equation (47), the incremental stress-strain relations are given by—

$$d\sigma_{ij} = 2\mu \left(d\varepsilon_{ij} + \frac{\nu}{1-2\nu} \delta_{ij}\, d\varepsilon_{kk} - \eta \frac{S'_{ij} S'_{mn}}{R} d\varepsilon_{mn} \right) \tag{58}$$

The translation of the yield surface is given by—

$$d\alpha_{ij} = (\sigma_{ij} - \alpha_{ij})\, du \tag{59}$$

The scalar du is determined by the condition that the stress point remains on the yield surface during plastic flow. This condition is stated as follows:

$$(d\sigma_{ij} - d\alpha_{ij}) \frac{\partial f}{\partial \sigma_{ij}} = 0 \tag{60}$$

The scalar du is then determined from (59) and (60) as follows:

$$du = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \Bigg/ \left[(\sigma_{k\ell} - \alpha_{k\ell}) \frac{\partial f}{\partial \sigma_{k\ell}} \right] \tag{61}$$

Finally the plasticity matrix H , by virtue of equation (58), is given by

$$\underset{\sim}{H} = \underset{\sim}{H}_0 - \eta \underset{\sim}{H}_p \tag{62}$$

where

$\underset{\sim}{H}_0$ = the elasticity matrix

η = zero for elastic state or unity for plastic state

$\underset{\sim}{H}_p$ is given below

$$(h_{ijk\ell})_p = \frac{2\mu}{2\kappa^2 \ (1 + \frac{E'}{3\mu})} S'_{ik}\ S'_{j\ell} \tag{63}$$

In the above, $\kappa = \sigma_y/\sqrt{3}$

where

σ_y = uniaxial yield stress

μ = shear modules

E' = uniaxial hardening modules in a bilinear stress-strain curve.

4.2 THE COMBINED INCREMENTAL STRESS-STRAIN RELATIONS

The incremental stress-strain relations for the combined elastic-plastic-creep problem is of the following form:

$$\Delta\underset{\sim}{\sigma}_n = \underset{\sim}{M}_n \left[\Delta\underset{\sim}{\varepsilon}_n - \underset{\sim}{H}_2 \sum_{i=1}^{n-1} (J_{n,i} - J_{n-1,i}) \Delta\underset{\sim}{\sigma}_i\right] \tag{64}$$

where

n: Designates the n^{th} time step

$\Delta\underset{\sim}{\sigma}_n, \Delta\underset{\sim}{\varepsilon}_n$: Stress and Strain Increments at t_n

$$J_{n,i} = \frac{1}{\Delta t_i} \int_{t_{i-1}}^{t_i} J[\xi(t_n) - \xi(\tau)]\, d\tau \tag{65}$$

$$\underset{\sim}{M}_n = [\underset{\sim}{I} + \underset{\sim}{J}_{n,n} \underset{\sim}{H}_n \underset{\sim}{H}_2]^{-1} \underset{\sim}{H}_n \tag{66}$$

$\underset{\sim}{I}$: Identity Matrix

$\underset{\sim}{H}_2$: Defined by $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$, namely, $\underset{\sim}{S} = 2H_2\,\underset{\sim}{\sigma}$

$\underset{\sim}{H}_n$: Plasticity Matrix

5. EXAMPLE ANALYSIS

As an example to illustrate the present development, Figs. 3 and 4 show the response of a cylinder to the internal pressure history shown in the figures. It should be pointed out that the present method is relatively insensitive to the size of the time step. This offers a distinct computational advantage over the initial strain approaches that have generally been applied in creep and plasticity analyses.

6. REFERENCES

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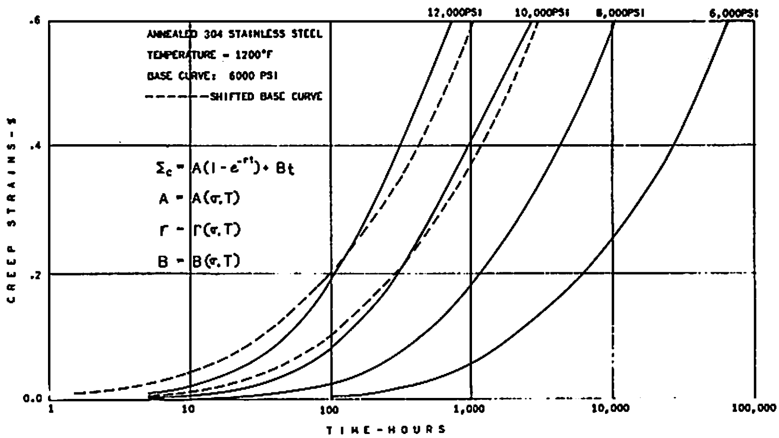


FIGURE 1 TOTAL CREEP CURVES

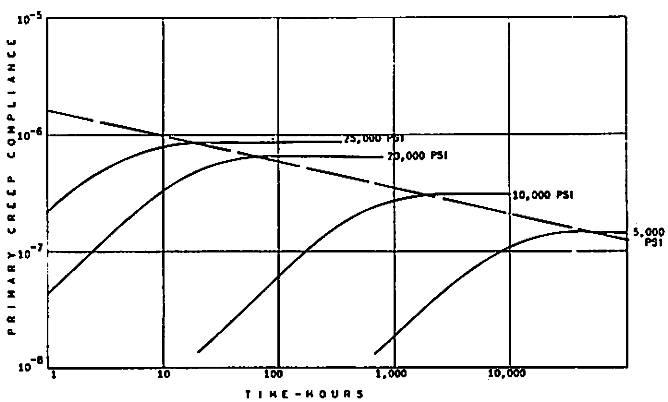


FIGURE 2 PRIMARY CREEP CURVES

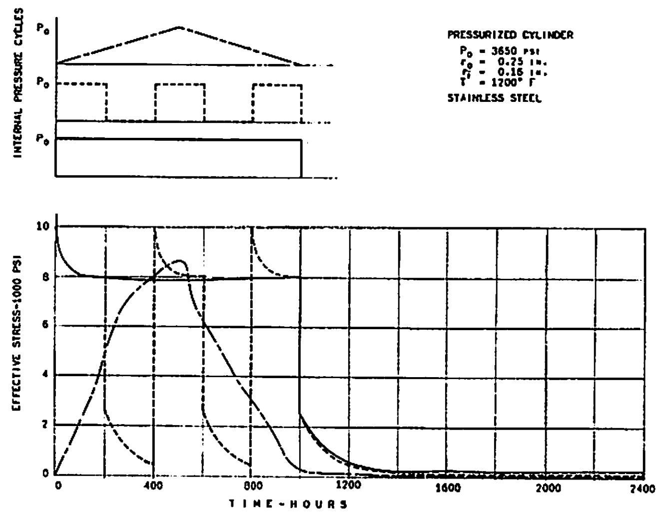


FIGURE 3 STRESS VS. TIME FOR PRESSURIZED CYLINDER

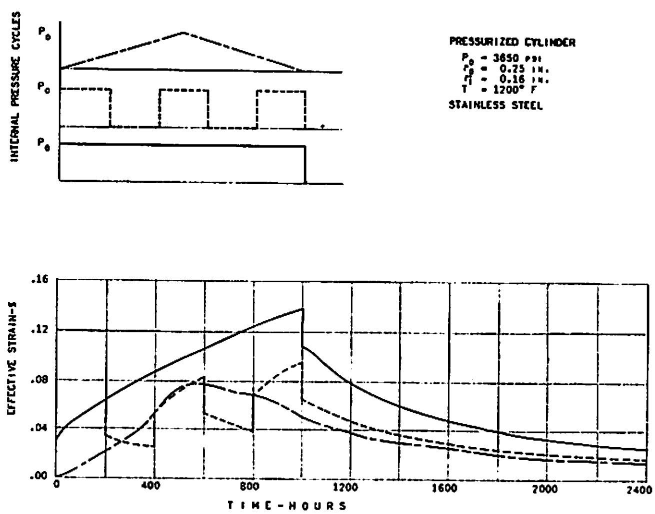


FIGURE 4 STRAIN VS. TIME FOR PRESSURIZED CYLINDER

DISCUSSION

Q P. R. W. GUMMERT, Germany

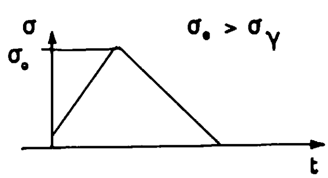
It may be allowed to do a little remark to this, I think, very good contribution: In the beginning of your paper you told about the constitutive equation, consisting of many multiple integrals. For the designers and constructors in this auditorium it could be shocking if they see such difficult terms. It can be shown that your expressions are transformable in a NEUMANN-series with special kernels (linear, square, etc.). NEUMANN-series are convergent if their kernels become monotonously smaller, as it is the case of the certainly justified assumption of a fading memory. Therefore, your uniaxial equation can be written, of course, with one kernel, the multiaxial equation with two kernels, if you regard, as later done, a homogeneous, isotropic "simple" material (compare COLEMAN/NOLL). These two kernels are determinable from two experiments - one uniaxial and at least one multiaxial. This, I think, is the right way to get a general theory (constitutive equation) without being forced to have certain and many experimental results before being able to calculate the problem. Later in the special case and for your special material you can and must have these specific material values. Then you determine the material coefficients and the special material functions (kernels).
This is also independent whether you take the reference stress and reference strain (II. invariant) of the tensors, how it is here always and everywhere done or whether you take generalized materials functions before the tensors that depend or can depend on all (without the first invariant in the special case of isochor and/or hydrostatic pressure independent material).

A Y. R. RASHID, U. S. A.

I must thank Prof. Gummert for his very constructive comment and I agree with his viewpoint that one should not follow a brute force approach to evaluate the necessary kernel functions. An analytical approach such as the one referred to by Prof. Gummert is definitely needed in view of the complexity of the general problem.

Q E. KREMPL, U. S. A.

In your nonlinear viscoelasticity theory can you predict permanent strain, e. g., uniaxial case ?



Permanent strain observed in real materials.

A Y. R. RASHID, U. S. A.

The answer is yes provided that the stress σ (t) at some time t exceeds σ_y, the yield stress. Since the problem is uniaxial loaded beyond the yield limit, permanent strain will be predicted in this case.