ABSTRACT

ALEXANDER, KRISTY. An industrial application of Time Series Forecasting of lumber demand. (Under the direction of Professor Robert B. Handfield.)

Forecasting lumber demand is vital for operational purposes in the Distribution Centers of Home Improvement retail chains. This paper describes econometric time series analyses applied to specific lumber skus from the largest Home Improvement chain in the United States. We propose simple univariate smoothing models and examine the causal relationship between temperature, housing starts, price and lumber demand. We find that complicated ARIMA models are not necessary; simple smoothing models are more appropriate. The results indicate that monthly seasonal models fit better that weekly moving average models and that even though the Point-of-Sale time series and Housing Starts time series show similar trends, the relationship is not strong enough for housing starts to be used as a short-term predictor. Also, the local maxima of the Point-of-Sale time series trends in the Spring, Summer and Fall result in low correlations between that series and the average monthly temperature or price series. So, temperature and price cannot be used as short-term predictors either.

AN INDUSTRIAL APPLICATION OF TIME SERIES FORECASTING OF LUMBER DEMAND.

by
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DEDICATION

This work is dedicated to Mr. Nyuiawodze Kodjo Adovor. Without his love, wisdom, and patient support, this work would not have been possible. Thank you.

BIOGRAPHY

Kristy Alexander was born in the twin island Republic of Trinidad and Tobago on July 7th, 1979. In Trinidad, She attended Tunapuna Government Secondary School and St. George's College where she completed her high school education and sat the Caribbean Examinations Council (CXC) and Cambridge Advanced Level examinations with concentrations in Chemistry, Physics and Mathematics.

Before joining the Operations Research Program at North Carolina State University, she attended North Carolina Central University (NCCU) with the aid of a full academic university scholarship. At NCCU she received a Bachelor of Science degree in Physics with a minor in Mathematics. During her tenure at NCCU she participated in two internships, one at NASA Langley Research Center and the other at Fermi National Accelerator Laboratory.

At North Carolina State University, she took coursework in Operations Research and Supply Chain Management. She also worked on various industry-related problems in the area of supply chain management under the supervision of Dr. Robert Handfield. Her research interests and experience include supply chain management, simulation and financial engineering.

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Firstly, I would like to thank my Lord and Savior, Jesus Christ, for teaching me to number my days so that I may apply my heart to wisdom. I wish to also thank my parents Mr. Trevor and Annette Alexander and my siblings Kevin and Kory for their continued encouragement. Thanks to my relatives and friends Eileen O'Brady, Jemma Lewis-Todd, Earl Bonas, George and Linda Jones, Harris Johnson and their families, for ensuring that I was never in need. I am also grateful to my church families at Arouca Revival Tabernacle, Cary Church of God, Harvest Church Plainfield and NC State's Graduate Inter-Varsity Christian Fellowship.

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1 INTRODUCTION

Charles F. Kettering eloquently stated, "My concern is with the future since I plan to spend the rest of my life there." Forecasting is the process of predicting the future. It is an important activity in economics, commerce, marketing and various branches of science. Statistical Forecasting techniques are widely used in production and inventory systems, quality and process control, financial planning, marketing, investment analysis and distribution planning.

Firms can benefit from good forecasting, and also pay the price for poor forecasting. Compaq Computer became a market leader in the early 1980s because they were able to properly forecast consumer demand for a portable version of the IBM PC, which gained great popularity. Forecasting also played a major role in Ford Motors's demise when they failed to forecast that customers would tire of their open Model T design.

Enduring principles developed by Dorn and Armstrong have been applied in this study. Dorn concluded that forecasters should use the longest time series available (Dorn, 1950). He also argued that forecasting models should be fairly simple. Armstrong (1984) also determined that in many cases, simple methods are often as good as sophisticated ones.

Time Series forecasting is the use of data that are obtained from observations of a phenomenon over time to predict future observations. There have been many time series forecasting applications to industrial problems. Liao-Shih-Jen used time series to forecast waste-water treatment applications and to ensure the full compliance of discharge requirements at the wastewater treatment facilities at the University of Pennsylvania[1]. Wu-Hong also used time-series forecasting to forecast carrot exports from the United States to Canada[2].

There have been several suggestions as to the causal relationships between housing starts and price with lumber demand. Many Associations such as WWPA (Western Wood Products Association) suggest that housing starts are a major predictor of lumber demand. Rich (1970)

[1] Volume 59-11B Dissertation Abstracts International, page 6014.
[2] Volume 36-03 Dissertation Abstracts International, page 702.

also suggests that home ownership affects lumber demand. Wongcharupan-Metha[3] suggests that there are causal relations among price and quantity variables in the US West Coast lumber market.

1.1 BACKGROUND

The Home Depot is currently the largest Home Improvement retail chain in the United States. The company continues to grow at a steady pace. To satisfy the demand from new stores, the location, size and goods carried by Distribution Centers (DCs) must be continuously evaluated. The problem is that, when new stores are added, some of the DC evaluations suggest that, larger bulk DCs should replace existing ones. However, the current low inventory turns of some of the existing DCs do not warrant the cost that will be incurred if they are expanded. The company is, therefore, now faced with the challenge of making more storage space available in the existing DCs. The decision was consequently made to increase inventory turns in the existing DCs, to utilize space more efficiently, before exploring the option of expansion. The current baseline average inventory turns for all the Bulk DCs is 19.6. The best in class DCs have average turns for all skus in the low 30s. The goal of the inventory turns project at the company is to take the average turns for all the Bulk DCs to 30. To more efficiently manage the inventory turns in the bulk DCs, better forecasts of the DC lumber inventory needed to service the stores is required. The goal of this study is to determine a forecasting methodology that can be used to forecast the monthly lumber demand from the bulk DCs. The company requires that this methodology meet the following criteria:

1. The methodology must be as simple as statistically and legitimately possible.
2. The company must also be able to implement the methodology using the software it currently has.
3. The forecast should have an upper bound using a 99% tolerance interval to ensure that the current fill rate of 98.25% is maintained.

[3] Volume 61-04B Dissertation Abstracts International, page 1717.

1.2 CHOOSING SKUS

The company currently has 30 DCs that service hundreds of stores. There are also thousands of lumber Store Keeping Units (skus). The decision was, therefore, made to do in-depth time-series analysis of a subset of these skus and DCs. Prior to this paper, a preliminary study was done to determine which DCs promised the most savings if there was an improvement in lumber inventory turns. Three of the most promising DCs from this analysis were chosen. One DC was picked from the North East, Mid West and South West to observe any variation in demand in these regions and the effect of causals on these variations. Two of the highest volume skus were chosen from each DC. In addition, one sku was the same for the Mid West and South West DCs and the demand for that common sku was also evaluated in two additional DCs (another in the North East and one in the South East) to establish the seasonal effects. Table 1 summarizes the skus and DCs that were observed. To protect the privacy of the company the DCs and skus are numbered arbitrarily. The sku descriptions are also given in Table 1.

DC-REGION	SKU	SKU DESCRIPTION
DC1-NE	Sku1	Douglas Fir studs
	Sku2	Furring Boards
DC2-MW	Sku3	Whitewood Studs
	Sku4	Sheathing Plywood
DC3-SW	Sku4	Sheathing Plywood
	Sku5	Timbers Landscape
DC4-NE	Sku4	Sheathing Plywood
DC5-SE	Sku4	Sheathing Plywood

Table 1: Summary of skus and DCs chosen.

1.3 CHOOSING DATA

The purpose of this study is to forecast the DC transfers, that is, the movement of inventory from the DCs into the stores. However, there is only one year and 4 months of historical transfer data available and since most of the skus exhibit seasonality, the transfer data was not sufficient to do analyses. The company has 24 months of Point-of-Sale data available. Some analysis was done to evaluate how well the Point-of-Sale data from the stores mimics the transfer data from the DCs. An analysis of the Point-of-Sale data and Transfer data for the period from September 2001 to January 2003, suggests that the two sets of data are well correlated. Therefore, the decision was made to forecast actual customer demand using the Point-of-Sale Data and use these forecasts to represent transfers. The correlations between the two series are greater than 0.75 for all but two of the skus studied (see Table 2). Also, autocorrelation and normality tests revealed that the residuals between the transfers and POS series are white noise (that is, the residuals are independent and normally distributed). Therefore, there are no structural or systematic differences. The normality test used is the Ryan-Joiner Test, which is based on the Shapiro-Wilk Test (Shapiro, 1965). The test is correlation based and determines whether a random sample comes from a normal distribution (that is, how well correlated the normal scores of the residuals are with the residuals themselves). The Ryan-Joiner correlations in Table 2, suggest that we would not reject normality at the 0.1 significance level (or less).

The two series that had correlations that were less than 0.75 showed that the Transfer series trailed the POS series (see Appendix 1). One reason for this may be that the DCs take some time to respond to drastically increased or decreased demand during season changes. A monthly forecast would serve to alleviate the delayed response times, since the company will be able to plan ahead for these seasonal demand changes.

DC-SKU	Correlation	Ryan-Joiner Correlation
DC1-Sku1	0.97	0.951
DC1-Sku2	0.19	0.983
DC2-Sku3	0.84	0.961
DC2-Sku4	0.85	0.972
DC3-Sku4	0.75	0.966
DC3-Sku5	0.82	0.973
DC4-Sku4	0.19	0.981
DC5-Sku4	0.83	0.958
Ryan-Joiner Correlation is the Pearson Correlation between the normal scores and residuals		

Table 2: Comparison between the POS and Transfers series

1.4 CURRENT STATE OF AFFAIRS

Currently, the forecast of lumber demand, at the company, is made using a Moving Average (MA) of the most recent historical 7 weeks. The company experiences several problems with this forecasting model. First, the forecast is made on a weekly basis, while the lead-time for incoming lumber is at least 3 weeks (more in some cases). Therefore, any forecast changes cannot be easily implemented because of the long response time. The biggest problem is that, the forecast does not predict when seasonal changes may occur. The time-series analyses in this paper show that all the skus studied demonstrate marked seasonal patterns. It would, therefore be beneficial to the company, if they could predict in what months demand is expected to increase or decrease, that is when the skus become "in-season" or go "out-of-season". Thirdly, forecasts made for weekly periods prove to have a greater degree of error because of the increased randomness for such a short period. In other words, it is "easier" to give a ball-park figure for an entire month's demand based on historical data, than it is to determine what the demand is for each week, because there is high variation at the weekly level (see Appendix 2 for the weekly time series plots and observe decreased variability in the monthly time series plots in Appendix 3).

Unless otherwise mentioned, all the analyses were done using Minitab software. Minitab is used because it is the statistical software currently in use at the company. The accuracy of the current Moving Average forecasting method used at the company was evaluated. The details of this analysis are available in Appendix 2. The Moving Average length was 7 weeks. Table 3 shows a summary of the fit results obtained. The Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD) and Mean Square Deviation (MSD) are used to evaluate the fit accuracy. The formulas for the fit statistics are as follows:

$$\text{MAPE} = \frac{\sum_{t=1}^{n} | Z_t - F_t |}{n} \qquad (1)$$

$$\text{MAD} = \frac{\sum_{t=1}^{n} | (Z_t - F_t) / Z_t |}{n} \text{ x } 100 \qquad (2)$$

$$\text{MSD} = \frac{\sum_{t=1}^{n} (Z_t - F_t)^2}{n} \qquad (3)$$

Where Z_t is the actual number of units sold in week t and F_t is the forecasted (or predicted) number of units to be sold in week t.

DC-SKU	MAPE	MAD	MSD
DC1-Sku1	39	781	1190011
DC1-Sku2	8	2731	14587294
DC2-Sku3	11	12268	22600000
DC2-Sku4	17	1242	2564230
DC3-Sku4	13	3078	16459446
DC3-Sku5	21.4	193.4	60445.9
DC4-Sku4	13	812	1055816
DC5-Sku4	17	1706	4874870

Table 3: Summary of AS-IS forecasting fit at the company.

2 ANALYSIS

Simple time series techniques are used to fit the historical sales data. Specifically, deterministic models are fitted and evaluated and then exponential smoothing techniques are used. Finally, regression techniques are used to determine the effect of causals such as housing starts, price and weather on the forecast.

2.1 DETERMINISTIC MODELS

For each of the skus studied, an attempt was first made to fit deterministic models to the time series since these models would be the most simple for the company to implement and they would also shed some light on the nature or structure of the series.

We assume the deterministic model $Z_t = \mu_t + X_t$,

Where Z_t is the number of units sold in month t, μ_t is the mean constant function and X_t is the stochastic or random component.

2.1.1 Seasonal Means Model

The time series plots show blatant seasonal patterns (See Appendix 3). As a result of this observed seasonality, a seasonal means deterministic model is fitted to the series. The seasonal means model (Cryer, 1986) assumes,

$$\mu_t = \mu_{t+12} \quad \text{for all } t$$

We assume that there are 12 parameters, $\mu_1, \mu_2, \ldots, \mu_{12}$, giving the expected average monthly lumber demand for each of the twelve months. For this model, in the absence of any predictor variables, the seasonal means estimate of β_j is just the average of all data at period j for all the seasons.

2.1.2 Cosine Trend Model

Another deterministic model, the cosine trend model, is also fitted to the time series. This model is also chosen because of the seasonality in the series. The cosine trend model (Cryer, 1986) assumes,

$$\mu_t = \beta_0 + \beta_1 \cos(2\pi t/12) + \beta_2 \sin(2\pi t/12) \qquad (5)$$

An adaptation of the cosine trend model that improved the fit is done. This adapted cosine trend model assumes,

$$\mu_t = \beta_{0t} + \beta_{1t} \cos(2\pi t/12) + \beta_{2t} \sin(2\pi t/12) \qquad (6)$$

The optimal β_{1t}'s and β_{2t}'s were determined using quadratic programming (Winston, 1994) to minimize the sum of the square residuals for each month,

$$\text{Minimize } \Sigma e_t^2$$

$$\text{Subject to:}$$

$$F_t - Z_t - e_t = 0 \qquad \text{for each } t$$

$$F_t - \beta_{0t} + \beta_{1t} \cos(2\pi t/12) + \beta_{2t} \sin(2\pi t/12) = 0 \qquad \text{for each } t$$

$$F_{t+12} - \beta_{0t} + \beta_{1t} \cos(2\pi t/12) + \beta_{2t} \sin(2\pi t/12) = 0 \qquad \text{for each } t$$

$$F_t \geq 0,\ \beta_{0t},\ \beta_{1t},\ e_t \text{ free}$$

Where Z_t is the number of units sold in month t, F_t, a variable, is the forecasted number of units sold in month t and e_t is the residual.

2.2 SMOOTHING MODELS

If the deterministic models were not appropriate, exponential smoothing models were fitted.

2.2.1 Winters Additive Model

An observation of the time series, shows that the amplitude of the seasonal pattern is not proportional to the average height of the series. Therefore an additive seasonal model is in order (Montgomery et al., 1990). This approach to forecasting a seasonal time series is due to Holt (Holt, 1957) and Winters (Winters, 1960) and is called the Winters additive method. In keeping with the notation already used, we will assume that the time series are adequately represented by the model,

$$Z_t = b_1 + b_2 t + s_t + X_t$$

where b_1 is the level or permanent component, b_2 is the linear trend component and s_t is the additive seasonal factor. The following iterative procedure, specifically tailored to the available data set of 24 POS monthly observations, is used to fit data at the end of each month, t, and forecast at the end of the sample:

1. The starting (time 0) level and trend are obtained by solving 2 linear equations:

$$b_1(0) \sum_{t=1}^{24} t + b_2(0) \sum_{t=1}^{24} t^2 = \sum_{t=1}^{24} t Z_t \qquad (7)$$

$$24\, b_1(0) + 300\, b_2(0) = \sum_{t=1}^{24} Z_t \qquad (8)$$

2. The estimate of the permanent component is updated:

$$b_1(t) = [Z_t - s_t(t - 12)] + (1 - \alpha)[b_1(t - 1) + b_2(t - 1) \qquad (9)$$

where $0< \alpha <1$ is the level smoothing constant.

3. The estimate of the trend component is updated.

$$b_2(t) = \gamma\, [b_1(t) - b_1(t - 1)] + (1 - \gamma)b_2(t - 1) \qquad (10)$$

where $0<\gamma<1$ is the trend smoothing constant.

4. The estimate of the seasonal factor is updated.

$$s_t(t) = \delta\, [Z_t - b_1(t)] + (1 - \delta)\, s_t(t-12) \qquad (11)$$

where $0<\delta<1$ is the seasonal smoothing constant.

5. To forecast the observation in any future time period $t + \tau$ we compute:

$$Z_{t+\tau}(t) = b_1(t) + b_2(t)\tau + s_{t+\tau}(t + \tau-12) \qquad (12)$$

2.2.2 Seasonal Exponential Smoothing Model

The Seasonal Exponential Smoothing model is identical to the Winters Additive Method, except that there is no trend component. Therefore, the model equation is,

$$Z_t = b_1 + s_t + X_t$$

This model only uses the level, α, and seasonal, δ, smoothing constants.

For both Smoothing models, SAS/ETS software[4] is used to optimize the smoothing constants by minimizing the sum of squared one-step-ahead prediction errors.

2.2.3 Log Transforms

In some cases, logarithm transformations are done before the smoothing methodology is applied, to obtain approximately constant variance. The logarithmic transform is,

$$W_t = \ln (Z_t) \qquad (13)$$

2.3 MODEL EVALUATION

Each model is evaluated based on:

1. Appropriateness: The model is deemed appropriate if its residuals are independent, that is, randomly distributed (Chatfield, 2001). Randomness is determined by observing the autocorrelations. The sample autocorrelation function, r_k is defined by,

$$r_k = \frac{\sum_{t=k+1}^{n} [Z_t - Y][Z_{t-k} - Y]}{\sum_{t=1}^{n} [Z_t - Y]^2} \quad \text{for } k = 0,1,2,\ldots \qquad (14)$$

   Where k is the lag, Z_t is the number of units sold in month t and Y is the sample mean. The model is appropriate if the residual autocorrelations do not differ significantly from zero.

[4] SAS/ETS Software: Time Series Forecasting System, Version 6, 1995.

2. Fit: The fit statistic used is MAPE (Mean Absolute Percentage Error). This fit statistic is chosen since its unit, percentage, is scale independent (Chatfield, 2001). The MAPE values from the forecasting models recommended in this study, can be compared to the MAPE values achieved in the current moving average forecasting that the company uses. The other fit statistics that Minitab computes, MAD (Mean Absolute Deviation) and MSD (Mean Square Deviation), have values in the units of the data being fit. Since we are comparing weekly data (of the moving averages) to monthly data (of this study), the MAD and MSD are not appropriate.
3. Comparison: The February 2003 data point was used as the out-of-sample validation point. The forecasting model that has the smallest MAPE for this point is deemed the "best" (Chatfield, 2001).

2.4 FORECAST LIMITS

Ryan-Joiner (Ryan, 1976 and Filliben, 1975) normality tests are done on the residuals of the "best" models. Minitab has three tests for normality: Anderson-Darling, Ryan-Joiner and Kolmogorov-Smirnov. The Anderson-Darling and Ryan-Joiner tests have similar power for detecting non-normality. The Kolmogorov-Smirnov test has lesser power because it does not fit the distribution tails well (see D'Augustino, 1986 for discussions of these tests for normality). Therefore, the Ryan-Joiner test was chosen to do the normality analyses.
The common null hypothesis for the Ryan-Joiner tests is,
H0: data follow a normal distribution. If the p-value of the test is less than the significance level, reject H0.

If the normality tests are successful (that is, there is not sufficient evidence to reject normality, with p-value ≥ 0.10), the 99% tolerance interval is determined for each of the forecasts. The upper limits of the interval are used as the forecasts that would maintain the current fill rates (98.25%) at Home Depot. If the residuals are not normally distributed, further smoothing is done until the final residuals are normally distributed.

2.5 CAUSAL VARIABLES

Causal models exploit the relationship between the time series of interest and one or more other time series (Montgomery et al, 1990). During preliminary discussions at Home Depot, the Global Product Merchants and DC Inventory Managers suggested that weather and price affect the lumber demand. Trade Associations such as Western Wood Products Association (WWPA) suggest that consumer tolerance and housing starts are indicators of U.S. lumber demand. Based in Portland, WWPA compiles lumber industry statistics and provides business information services to mills. WWPA president, Michael O'Halloran, says that, "Residential construction is the largest market for lumber, so any reduction in the number of new homes built will affect lumber consumption." Based on this "domain knowledge", stepwise regression is used to find the effects of average temperature, housing starts and price on the demand for lumber. Minitab's stepwise regression (McCullagh and Nelder, 1992) performs stepwise, forward selection, or backward elimination, which add or remove variables from a model in order to identify a useful subset of predictors. It uses a least square estimation method where the sum of squared errors is minimized.

3 RESULTS

3.1 TIME SERIES PLOTS

The time series plots for all the DC-Skus show striking seasonal patterns. These plots are shown in Appendix 3. Except for DC1-Sku1, all the time series show demand peaks in April, July and October. As will be shown in Section 3.4, these local maxima are also seen in the housing starts series. DC1-Sku1 has peaks in July, October and December. Also, DC2-Sku1 has its maximum yearly demand in January.

As mentioned in Section 1.2, sku 4 was chosen for DCs in different climatic regions (namely Mid-West, South West, North East and South East) so the climatic effect on the demand could be observed. We find that the seasonal demand changes are more exaggerated in the regions where there are more severe climatic changes. In fact in the Mid-West and North East regions, the demand increases in peak time are as much as 75% and 80% from the previous months. Conversely, in the South West and South East regions, the demand changes are more modest with the highest increases being 47% and 46% respectively.

3.2 BEST FITS

The methodology put forth in the Analysis section was implemented for each of the DC-Sku combinations. The Seasonal Means and Cosine Trend Models, even though they were good fits in most cases, were found to not be appropriate for any of the time series studied because the residuals are not independent; the correlograms for these models had correlation coefficients that differed significantly from zero. A DC-Sku (DC1-Sku1) example of the fit of the Seasonal Means and Cosine Trend models are shown in Figures 1 and 2.
Since the deterministic models are not appropriate, the smoothing models are applied. Table 4 shows a summary of the best three smoothing models for the skus studied and the optimal smoothing constants for each model, obtained using SAS/ETS software. The Time series plots of the Winters Additive and Seasonal Exponential Smoothing models

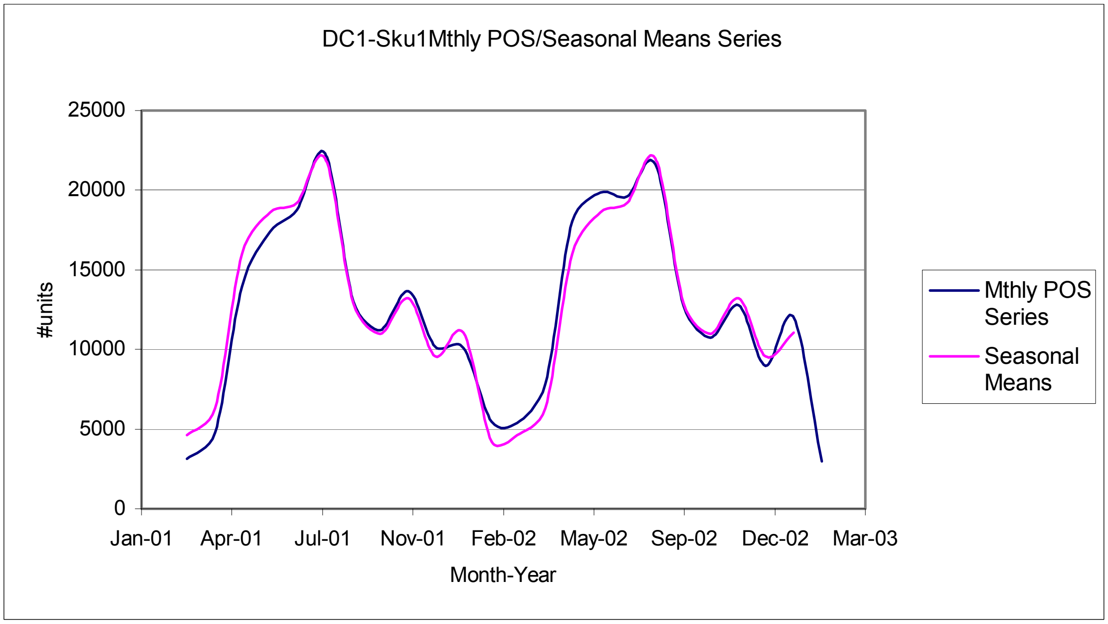

Figure 1: Fit of Seasonal Means model to DC1-Sku1 time series

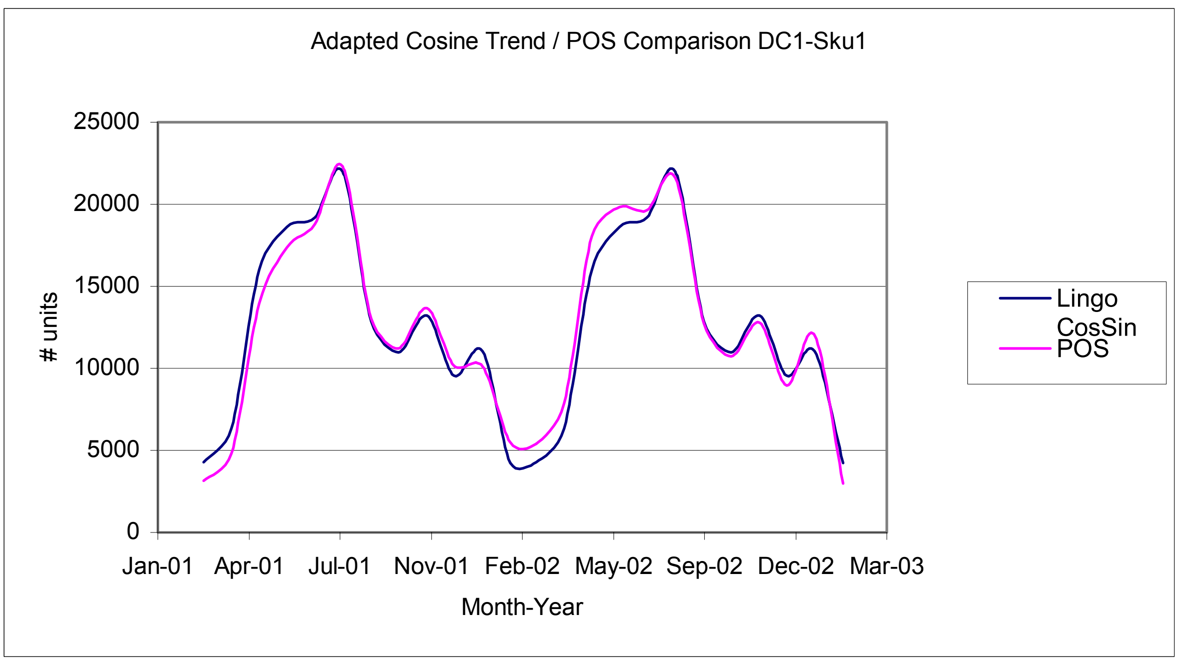

Figure 2: Fit of Cosine Trend model to DC1-Sku1 time series

are shown in Appendix 4. The comparison of the MAPEs from these models to those of the current moving average model in use shows that the proposed Winters Additive and Seasonal Exponential Smoothing models provide much better fits. These comparisons are shown in Table 5. The proposed models improved MAPEs by as much as 19%. The smallest improvement was 3% for DC1-Sku2, which is still a substantial improvement. The average improvement is 11.2%. The proposed improvement in fit accuracy is enough to warrant the extra "trouble" of using the simple exponential smoothing models. In most cases the Seasonal Exponential Smoothing models and Winters Additive models behaved similarly in terms of fit. In the instances when one was better than the other, the MAPEs only differed by 1%. In addition, a smaller MAPE was achieved if the actual POS series was logarithmically transformed (see equation (13)), before the smoothing model was applied. But even in these cases, the improvements were not more than 1%.

DC-SKU	MODEL	α	γ	δ
DC1-Sku1	Winters Additive	0.5504	0.0010	0.0010
	Seasonal Exponential Smoothing	0.6100		0.0010
	Log Seasonal Exponential Smoothing	0.6369		0.0010
DC1-Sku2	Log Winters Additive	0.7445	0.9302	0.9990
	Winters Additive	0.8995	0.1872	0.9990
	Seasonal Exponential Smoothing	0.9065		0.9990
DC2-Sku3	Seasonal Exponential Smoothing	0.9397		0.9990
	Winters Additive	0.9336	0.0018	0.9990
	Log Seasonal Exponential Smoothing	0.9314		0.9990
DC2-Sku4	Log Winters Additive	0.9990	0.0010	0.9990
	Winters Additive	0.9990	0.0010	0.0010
	Seasonal Exponential Smoothing	0.9629		0.9990
DC3-Sku4	Log Winters Additive	0.8600	0.0010	0.0010
	Winters Additive	0.8543	0.0010	0.0010
	Seasonal Exponential Smoothing	0.9990		0.0010
DC3-Sku5	Seasonal Exponential Smoothing	0.8630		0.0010
	Winters Additive	0.8543	0.0010	0.0010
	Log Seasonal Exponential Smoothing	0.9990		0.0010
DC4-Sku4	Log Winters Additive	0.1014	0.0010	0.0010
	Winters Additive	0.0921	0.0010	0.0010
	Seasonal Exponential Smoothing	0.2915		0.0010
DC5-Sku4	Log Seasonal Exponential Smoothing	0.9990		0.0010
	Seasonal Exponential Smoothing	0.7829		0.0010
	Winters Additive	0.1993	0.9222	0.1065

Table 4: Summary of Best 3 Smoothing Models and Optimal Constants.

DC-SKU	MODEL	MAPE
DC1-Sku1	Winters Additive	10
	Seasonal Exponential Smoothing	9
	Log Seasonal Exponential Smoothing	10
	Moving Average	39
DC1-Sku2	Log Winters Additive	4
	Winters Additive	5
	Seasonal Exponential Smoothing	5
	Moving Average	8
DC2-Sku3	Seasonal Exponential Smoothing	7
	Winters Additive	7
	Log Seasonal Exponential Smoothing	7
	Moving Average	11
DC2-Sku4	Log Winters Additive	5
	Winters Additive	5
	Seasonal Exponential Smoothing	5
	Moving Average	17
DC3-Sku4	Log Winters Additive	5
	Winters Additive	5
	Seasonal Exponential Smoothing	5
	Moving Average	13
DC3-Sku5	Seasonal Exponential Smoothing	9
	Winters Additive	9
	Log Seasonal Exponential Smoothing	9
	Moving Average	21
DC4-Sku4	Log Winters Additive	3
	Winters Additive	3
	Seasonal Exponential Smoothing	4
	Moving Average	13
DC5-Sku4	Log Seasonal Exponential Smoothing	5
	Seasonal Exponential Smoothing	6
	Winters Additive	6
	Moving Average	17

Table 5: Summary of MAPEs for Proposed Models and Comparison with MA models

3.3 OUT-OF-SAMPLE FORECAST BEHAVIOR

3.3.1 Behavior of Forecast Interval

Appendix 5 shows the correlograms and normality plots for the log-transformed series. All of the residuals from these methods were independent, because the autocorrelation functions were close to zero up to lag 23, and all but one (DC3-Sku4) are normally distributed. Similarly, the residuals from the non-transformed models were all independent and normally distributed (see Appendix 6 for normality plots). We capitalize on the normally distributed residuals of the non-transformed series to obtain a 99% tolerance interval for the forecasts using the standard deviation of the residuals and multiplying that value by the z-score 2.58. Table 5 shows that the Actual lumber demand for February 2003 fell within this 99% tolerance interval for all the DC-Skus.

3.3.2 Accuracy Comparison of Forecast

The February 2003 forecasts produced by the non-transformed Winters Additive and Seasonal Exponential Smoothing models are compared to the actual lumber demand observed for that month. The non-transformed series are used because these are simpler to actually implement, and provided similar, if not better fits than the log transformed models (see Table 5). The Absolute Percentage Error (APE) is also compared to the MAPE obtained from the 7-week Moving Average (MA) model discussed in Section 1.4. The APE equation is identical to the MAPE equation (1), when n=1.

As Table 6 shows, when the two smoothing models are compared, the Winters additive method performed better for three (3) of the DC-Skus, the Seasonal Exponential Smoothing models performed better for four (4) of the DC-Skus and the two models performed equally well for one (1) of the DC-Skus. Therefore, just as the two models performed similarly in the in-sample fits (section 3.1), this result shows that the two models perform similarly in forecasting the out-of-sample February 2003 data point.

The Moving Average February 2003 forecast is compared to the forecast obtained using the two smoothing models in Table 6. There was a drastic decrease in demand for DC1-Sku1 in February 2003, the demand was 1598 units compared to February 2001 and February 2002, where the demand was 3135 units and 5395 units respectively. The Smoothing models, therefore, do not perform well in forecasting the DC1-Sku1 February 2003 demand. Because the average is obtained every week, the moving average model performs better, but still very badly in forecasting the DC1-Sku1 February 2003 demand. For the other seven DC-Skus, the smoothing models performed better for three (3) and the Moving Average model performed better for three (3), while for DC5-Sku4, the Winters Additive method performed the best and the Moving Average model performed better than the Seasonal Exponential Smoothing model. Therefore, no conclusive statement can be made about which model performs better in this out-of-sample analysis. We can say, however, that the Moving Average model responds more quickly to drastic deviations from the historical trend.

3.4 CAUSAL VARIABLES RESULTS

The time series of the causal variables, suggested by the DC Inventory Managers and trade associations such as WWPA, are studied. The causal factors observed are non-seasonally adjusted new housing starts in the DC region[5], monthly average temperature in the DC region[6] and price[7]. Step-wise regressions were performed on causals because the causal factors may be interdependent; step-wise regression first removes these interdependencies. The regression was done for the DC-Skus in the North East, Mid-West and South West.

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[5] The housing starts data was obtained from the US Census Bureau Website
http://www.census.gov

[6] The monthly average temperature was obtained from the National Weather Services Website
http://www.weather.gov

[7] Random Lengths price data was used
http://www.randomlengths.com

DC-SKU	MODEL	FORECAST	ACTUAL	ERROR	99% TOLERANCE INTERVAL UPPER	LOWER	APE	MA MAPE
DC1-Sku1	Winters Additive	3583	1598	-1985	5907	1259	124	75
	Seasonal Exponential Smoothing	3518	1598	-1920	5830	1206	120	75
DC1-Sku2	Winters Additive	111594	105082	-6512	133115	90073	6	22
	Seasonal Exponential Smoothing	121198	105082	-16116	143479	98917	15	22
DC2-Sku3	Winters Additive	537972	560733	22761	643709	432235	4	6
	Seasonal Exponential Smoothing	538818	560733	21915	644382	433254	4	6
DC2-Sku4	Winters Additive	16966	20368	3402	21581	12351	17	5
	Seasonal Exponential Smoothing	17024	20368	3344	21740	12308	16	5
DC3-Sku4	Winters Additive	82514	78826	-3688	100191	64837	5	21
	Seasonal Exponential Smoothing	82542	78826	-3716	99914	65170	5	21
DC3-Sku5	Winters Additive	2104	3192	1088	3395	813	34	17
	Seasonal Exponential Smoothing	2102	3192	1090	3393	811	34	17
DC4-Sku4	Winters Additive	20809	22872	2063	24080	17538	9	5
	Seasonal Exponential Smoothing	20789	22872	2083	24300	17278	9	5
DC5-Sku4	Winters Additive	23676	23031	-645	32052	15300	3	17
	Seasonal Exponential Smoothing	28526	23031	-5495	36004	21048	24	17

Table 6: Comparison of Out-of-Sample Model behavior for February 2003

3.4.1 Housing Starts Series

The plots of the housing starts time series in each of the DC regions observed are shown in Figures 3 through 5. The plots show local maxima in the Spring, Summer and Fall, however, the maxima do not always take place in April, July and October, as is the case for the Point-of-Sale local maxima. For example, for DC2, the housing starts Spring peaks occur in April 2001 and March 2002. Similarly, the Summer peaks occur in July 2001 and June 2002. Since the local maxima did not occur at the same time, none of the correlations between the POS series and the housing starts series exceeded 0.71 and for four of the six DC-Skus observed, the correlation was less than 0.4.

3.4.2 Average Monthly Temperature Series

The monthly average temperature varies as expected, with highs in the summer months and lows in the winter. These series did not show the lumpiness of the local maxima in the Spring, Summer and Fall as observed in the Point-of-Sale and Housing Starts Time Series. The monthly average temperature series are shown in Appendix 7. Because there was no lumpiness in the monthly average temperature series, the correlations of this series with the POS series did not exceed 0.78 and in three of the six cases, it was less than 0.4.

3.4.3 Price Series

There was very little variation in the random lengths price data during 2001 and 2002. In some cases, the price remained exactly the same throughout the entire period. In other cases, only one month recorded any change and even in these cases, the change was not substantial. Therefore, there was no correlation between the price series and the POS series over the two-year observation period.

3.4.4 Step-wise regression

None of the step-wise regressions performed produced R^2 (coefficient of determination) values greater that 65%. In many cases, these values were less than 40%. The Minitab output from this regression study is shown in Appendix 8. One interpretation of R^2 is that it is the square of the sample correlation coefficient between the observed series and the assumed trend (Cryer, 1986, page 40). Therefore, a R^2 value of 65% means that, 65% of the variation in the monthly POS

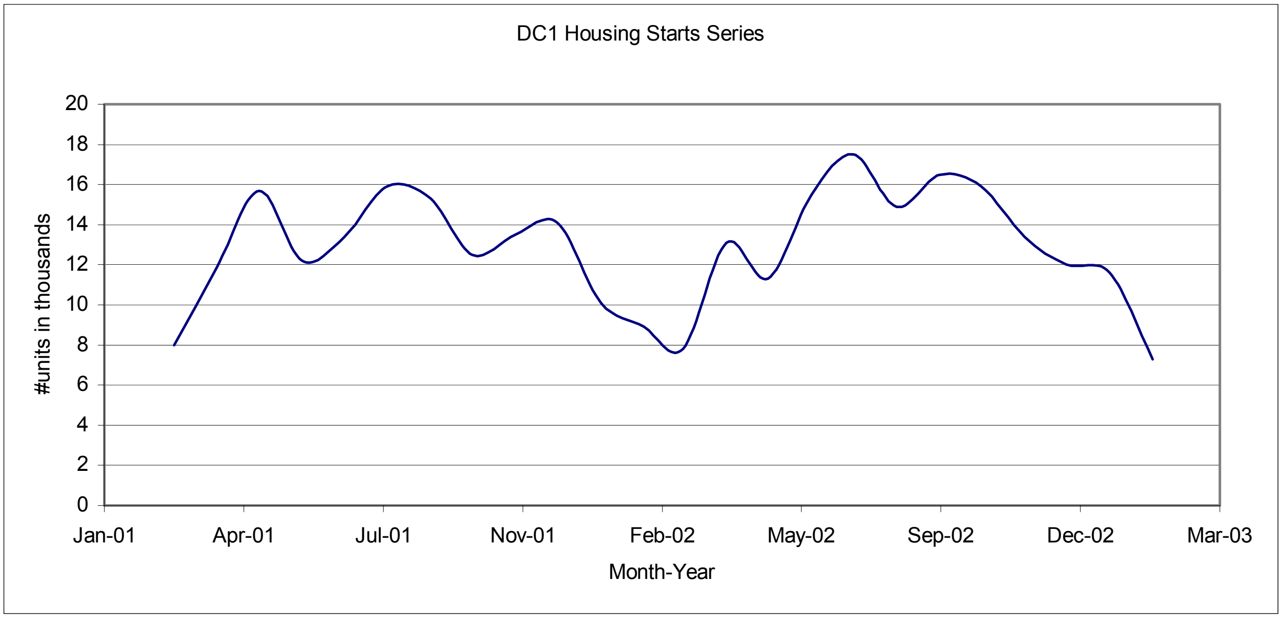

Figure 3: Housing Starts Time Series for DC1[8]

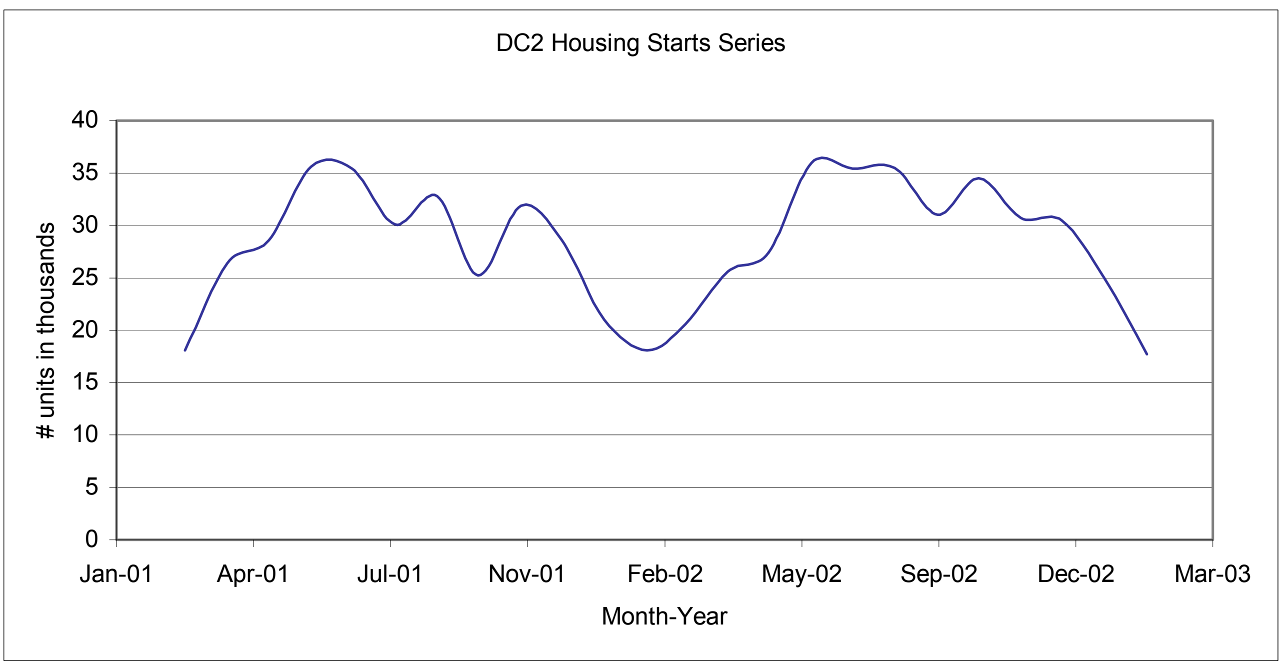

Figure 4: Housing Starts Time Series for DC2[9]

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[8] The DC1 Housing Starts Series has peaks in April, July, November of 2001 and March, June, August and December of 2002.

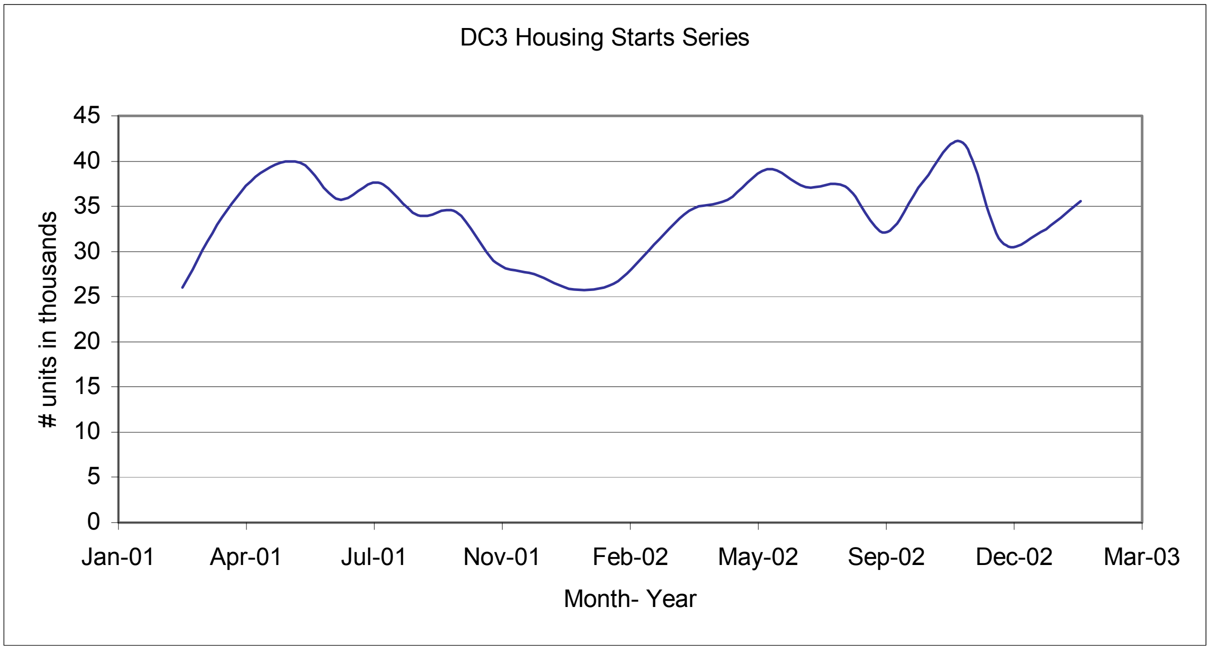

Figure 5: Housing Starts Time Series for DC3[10]

time series is explained by the proposed regression model. The R^2 values obtained suggest that none of the proposed causals should be used as predictor variables.

[9] The DC2 Housing Starts Series has peaks in June, August and October of 2001 and May, September and November of 2002.

[10] The DC1 Housing Starts Series has peaks in May, July and September of 2001 and May, July and October of 2002.

4 CONCLUSION AND RECOMMENDATIONS

The Smoothing models proposed in this paper, meet the criteria set forth by the company (see section 1.1); The Seasonal Exponential Smoothing and Winters Additive models are simple, easy to implement in the existing Minitab software available at the company, and since the residuals obtained from these models were normally distributed, the 99% tolerance interval to maintain the fill rate could be found. The out-of-sample forecasts for the smoothing models all fell within the 99% tolerance interval proposed.

The monthly forecasting would serve to facilitate decision making in light of the long lead times and does well to predict season changes. The smoothing models have better "ramp-up" and "ramp-down" properties, when the skus are coming into season and going out of season respectively, as observed in the time series plots in Appendix 4. The smoothing models also have better MAPE fit statistics than the Moving Average models as discussed in section 3.

No conclusive statement can be made, from the current out-of-sample analyses done, about whether any of the models (smoothing or Moving Average) behave better, except that the smoothing models would allow the DC inventory Manager to "look further ahead" more accurately. Both the Seasonal Exponential Smoothing and Winters Additive models have similar out-of-sample performance. They both perform badly when there is a drastic shift away from the historical trend as is the case with DC1-Sku1 February 2003. The Moving Average model captures the drastic shifts more quickly (although it still behaved badly in the DC1-Sku1 February 2003 case) because it is reviewed weekly. The author, therefore, recommends that the monthly smoothing models be used to plan ahead for seasonality changes, and the actual demand be reviewed weekly to determine whether the historical trend is being followed. For example if the actual demand for the first week of a particular month is only 10% of the forecasted demand for that month, then one can predict that that month is going to have unusually low demand and adjust shipment decisions accordingly.

The step-wise regressions performed suggest that neither housing starts, weather – specifically average monthly temperature, or price should be used as a short-term predictor of the lumber demand. However, the similar lumpiness in the housing starts and POS series in the Spring, Summer and Fall, suggest that the general trend for the two skus is similar. Therefore, if there is a dramatic increase or decrease in housing starts, or if at any time the series varies dramatically from the trend, this variation may directly affect the lumber demand. No statement can be made, however, about which month in each POS season will be affected. Similarly, for example, marked price increases may affect already seasonally low or high demands negatively. But, for the purposes of forecasting on a monthly basis, the price data observed for the period of February 2001 to February 2003 is not a good indicator of lumber demand at the company.

5 FUTURE WORK

Further observation of the out-of-sample behavior of the forecasting smoothing models proposed is required. As more demand data becomes available, the forecasting accuracy of the model will be continuously evaluated and updated.

The methodology would be implemented at the company (theoretically) for all the skus at the "problem" DCs where there are space constraints to determine the realizable average inventory turns possible.

The company plans to develop a tool that considers the on-hand lumber inventory, the expected inbound inventory and the forecasted demand to predict overages and shortages. The process will then be piloted (physically implemented) at one of the DCs. If the pilot is successful, the process would then be implemented at other "problem" DCs.

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7 APPENDICES

APPENDIX 1

POS-Transfer Comparisons for skus that showed little correlation

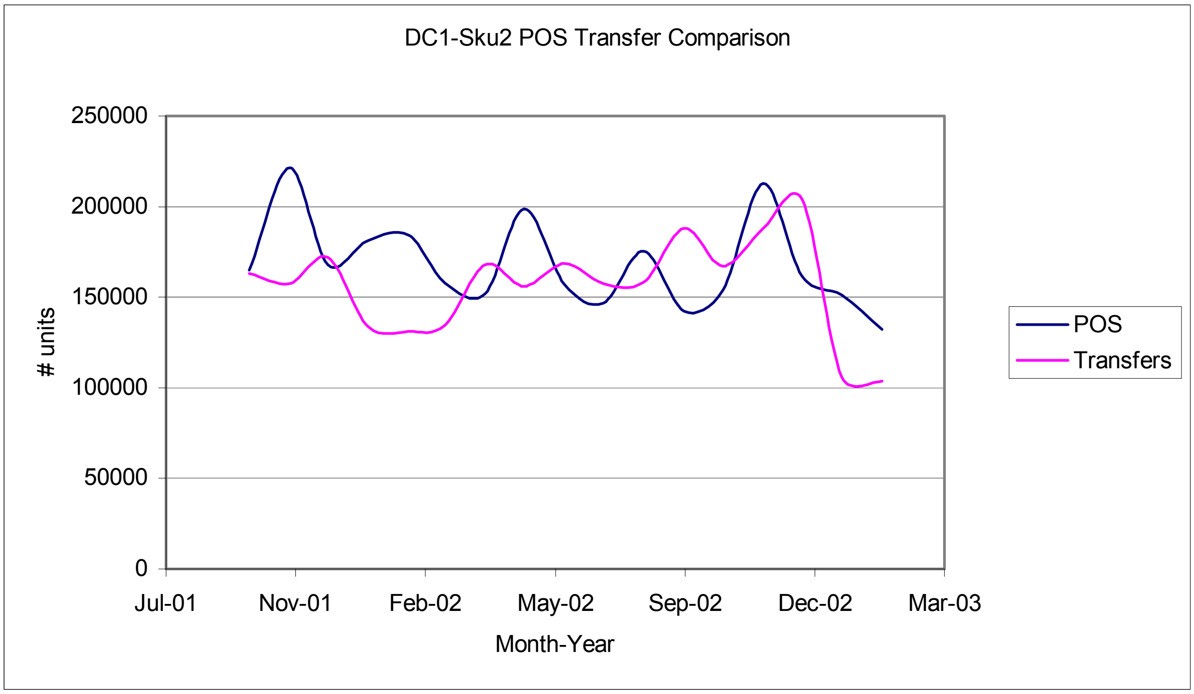


Figure 6: Point-of-Sale and Transfer Comparison for DC1-Sku2

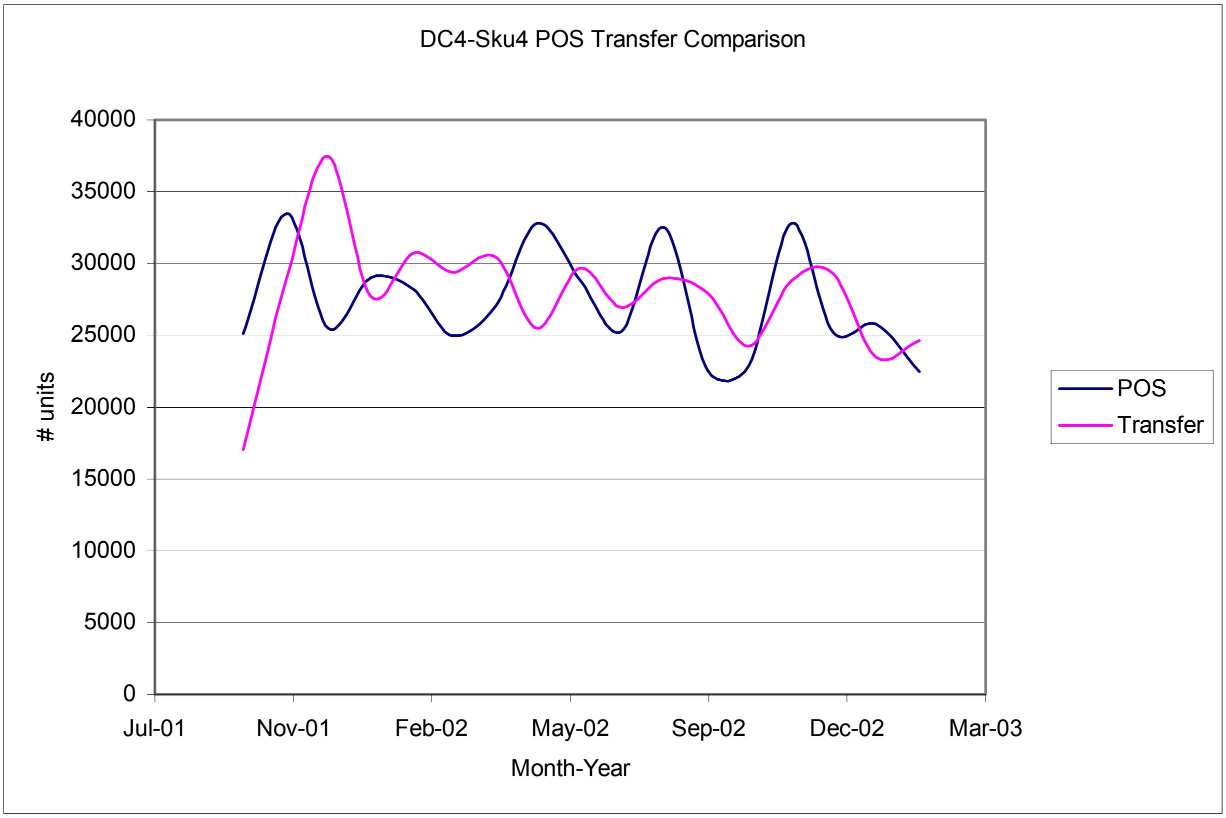


Figure 7: Point-of-Sale and Transfer Comparison for DC4-Sku4

APPENDIX 2

Time Series Plots of AS-IS Moving Average Forecasting models[11]

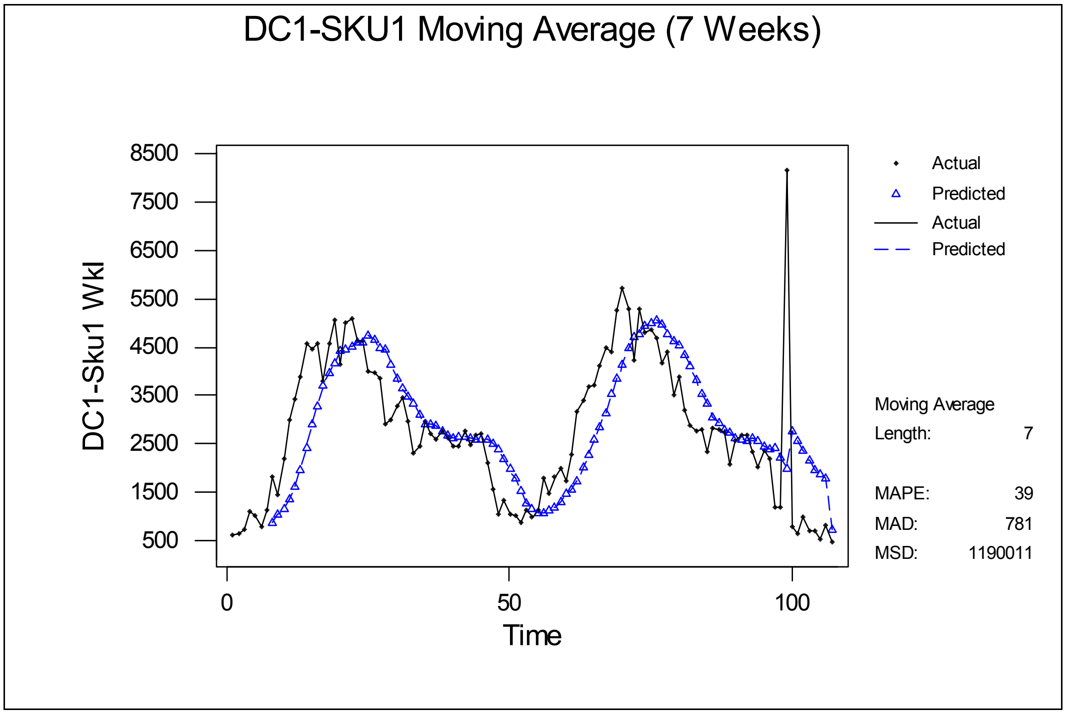


Figure 8: 7-Week Moving Average Time Series for DC1-Sku1

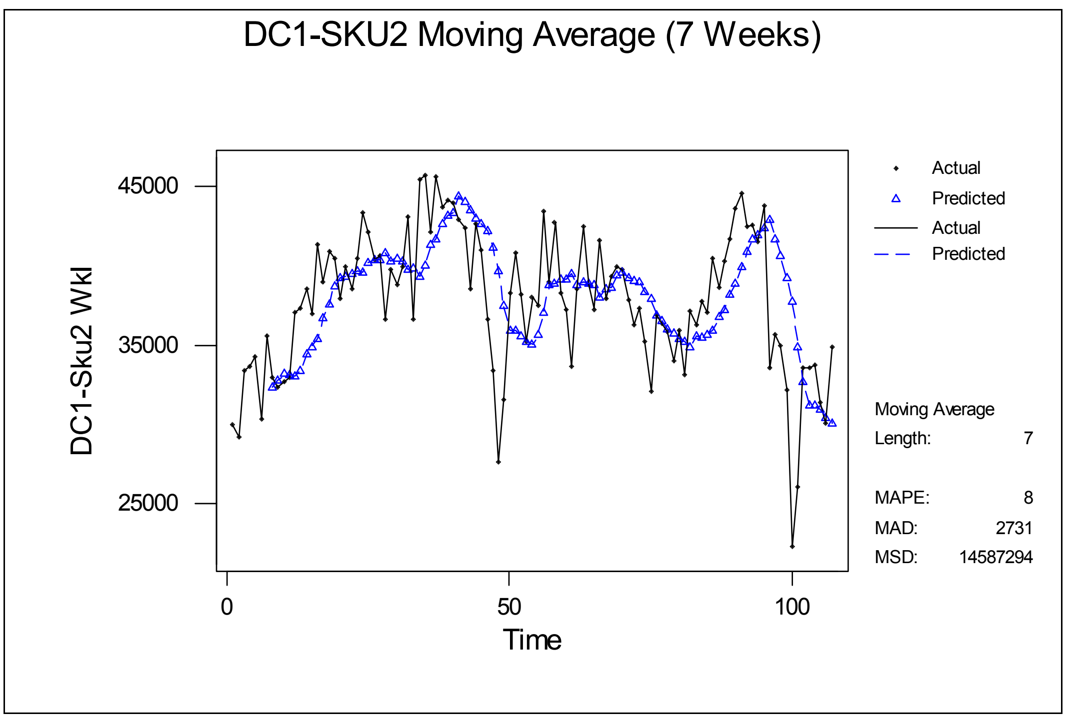


Figure 9: 7-Week Moving Average Time Series for DC1-Sku2

[11] The moving plots show weekly demand (week 1 being the first week in February 2001).

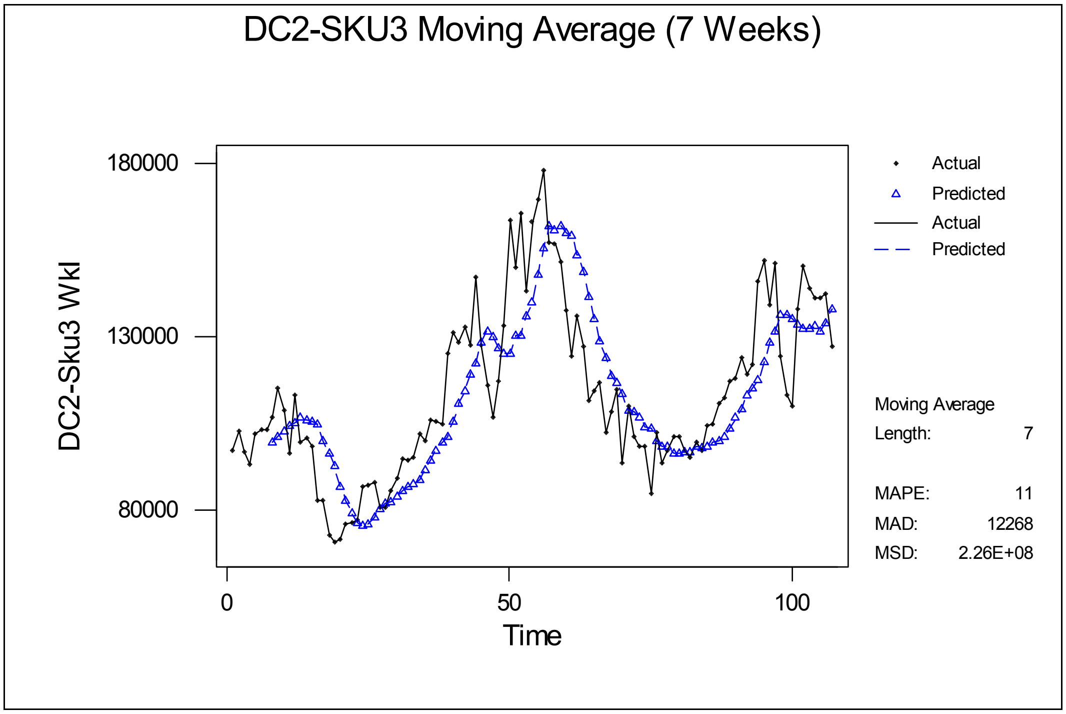


Figure 10: 7-Week Moving Average Time Series for DC2-Sku3

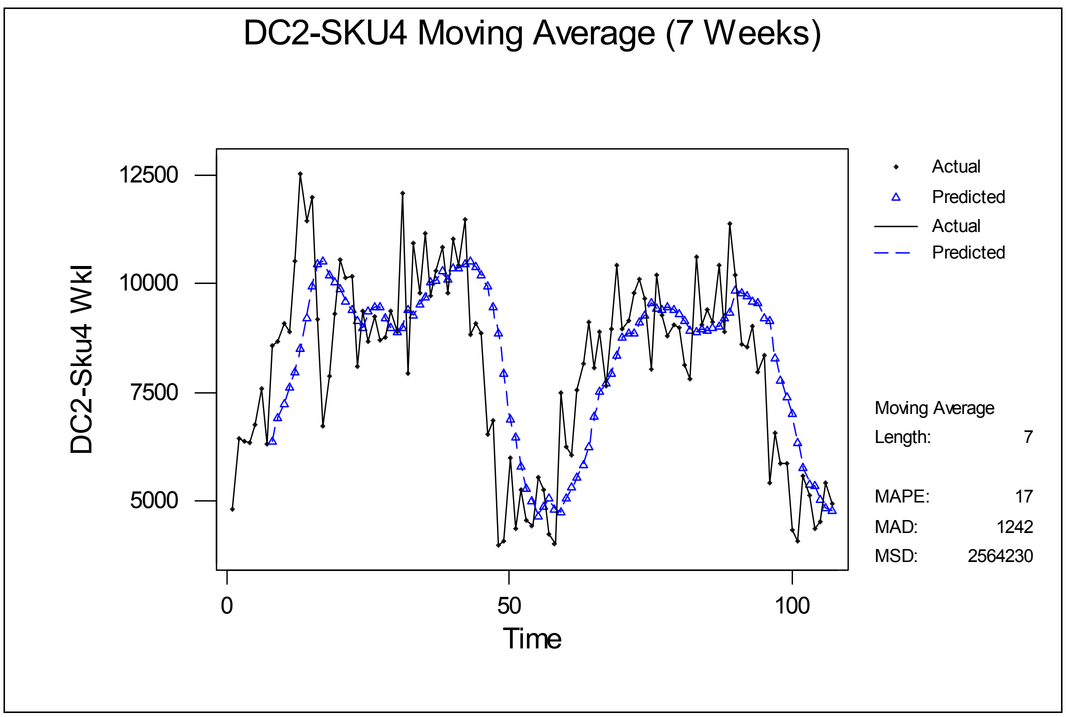


Figure 11: 7-Week Moving Average Time Series for DC2-Sku4

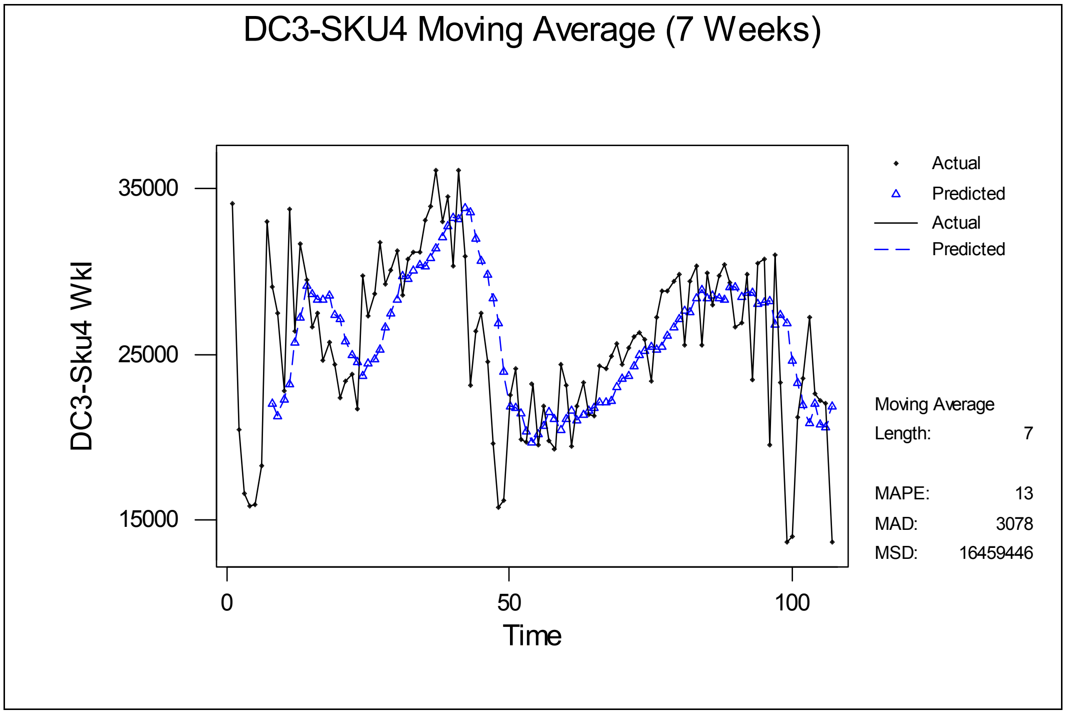


Figure 12: 7-Week Moving Average Time Series for DC3-Sku4

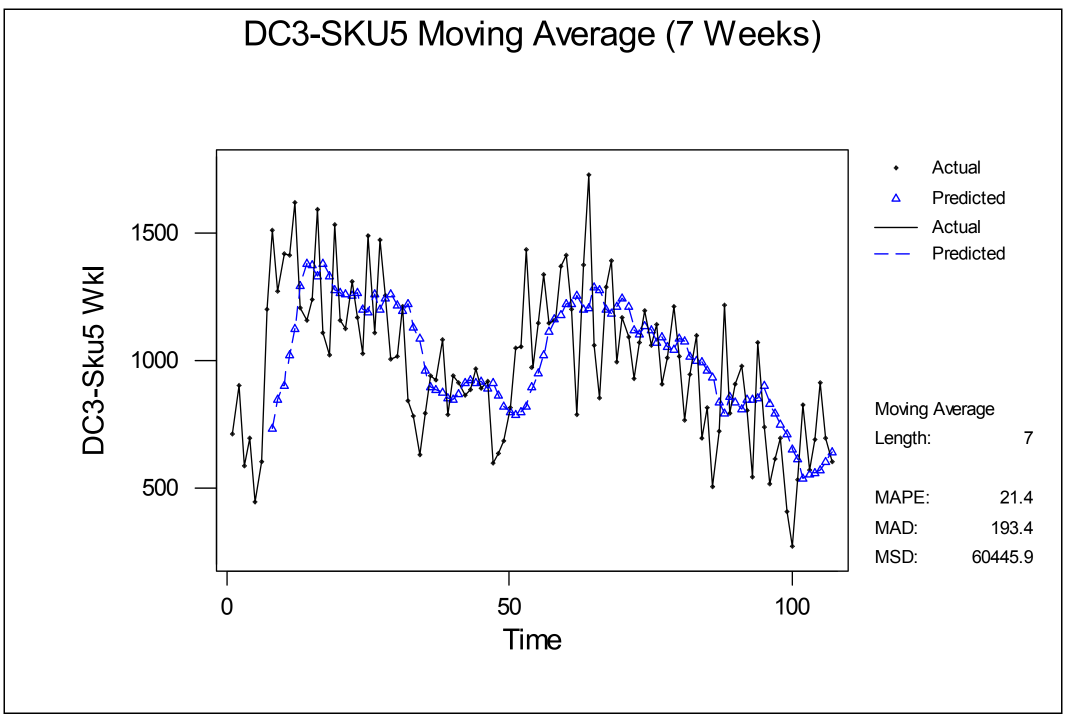


Figure 13: 7-Week Moving Average Time Series for DC3-Sku5

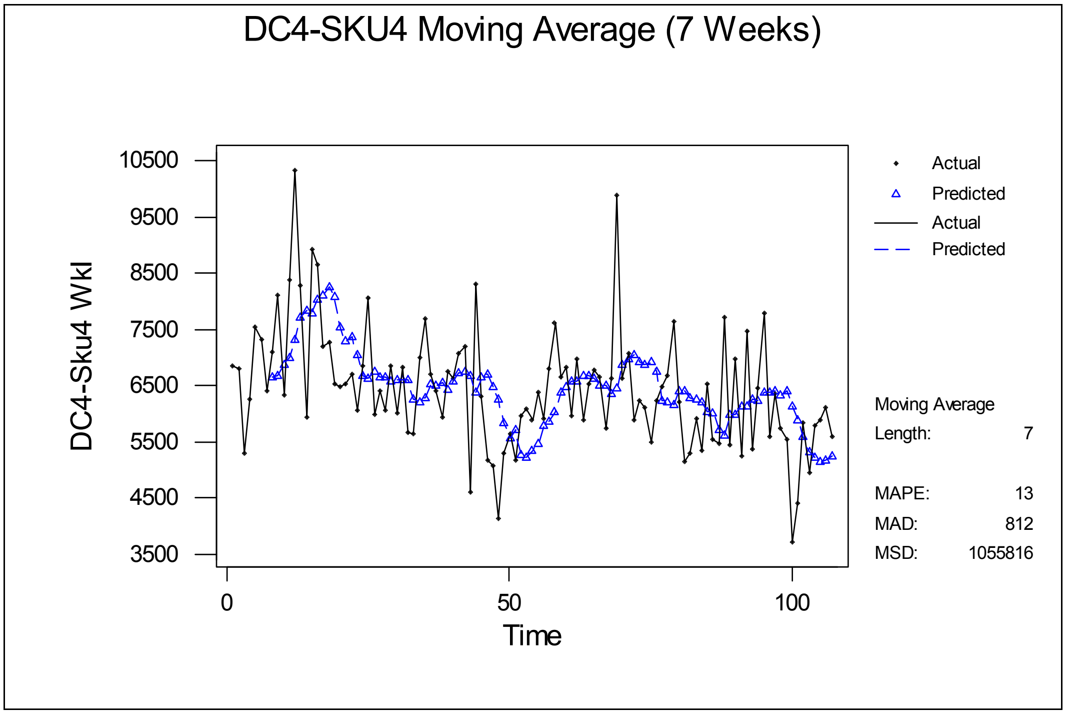


Figure 14: 7-Week Moving Average Time Series for DC4-Sku4

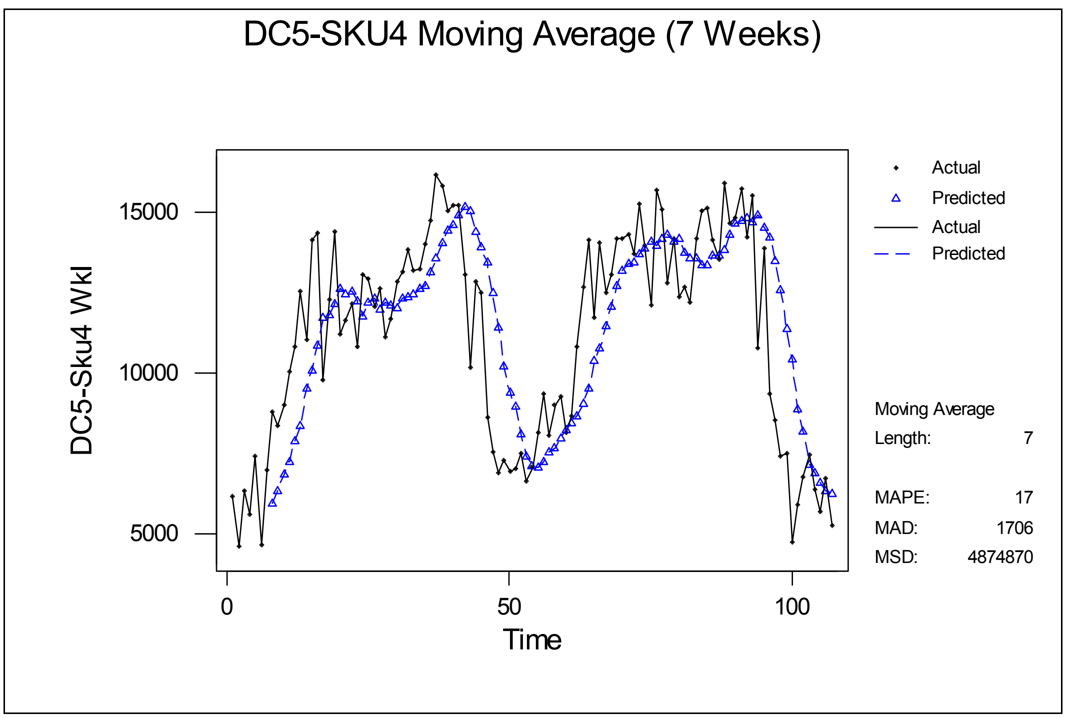


Figure 15: 7-Week Moving Average Time Series for DC5-Sku4

APPENDIX 3

Monthly Point-of-Sale Time Series Plots for the DC-Skus

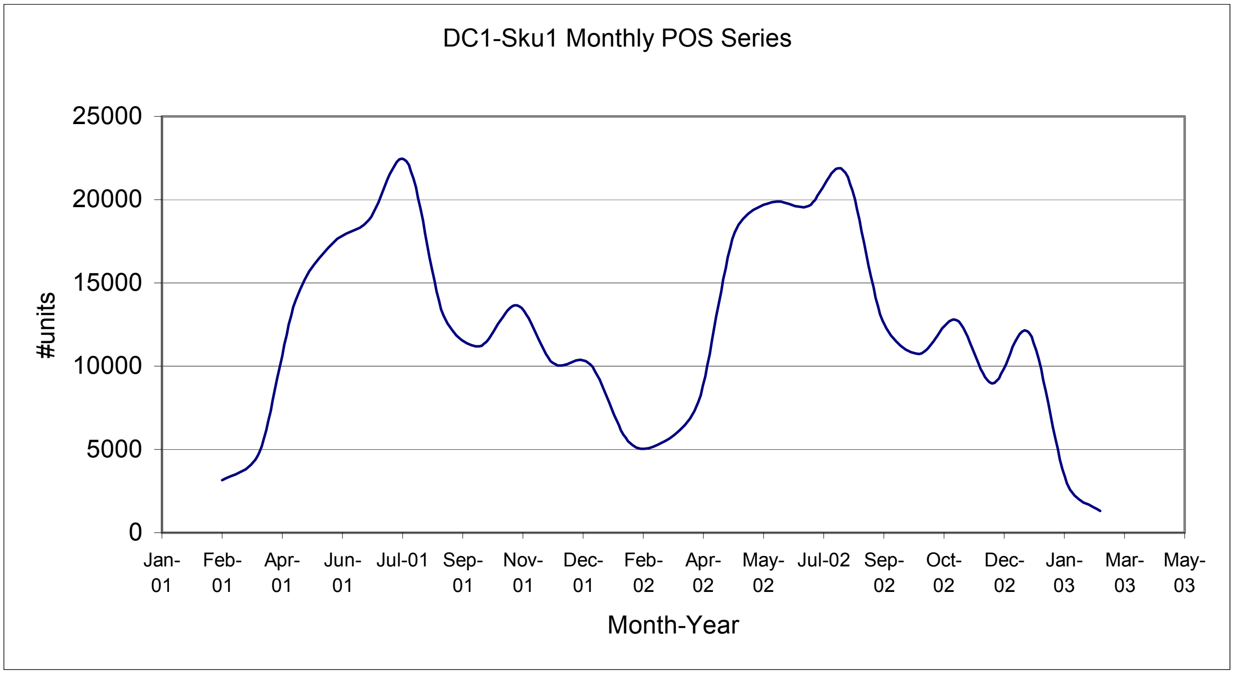


Figure 16: Monthly POS time series for DC1-Sku1

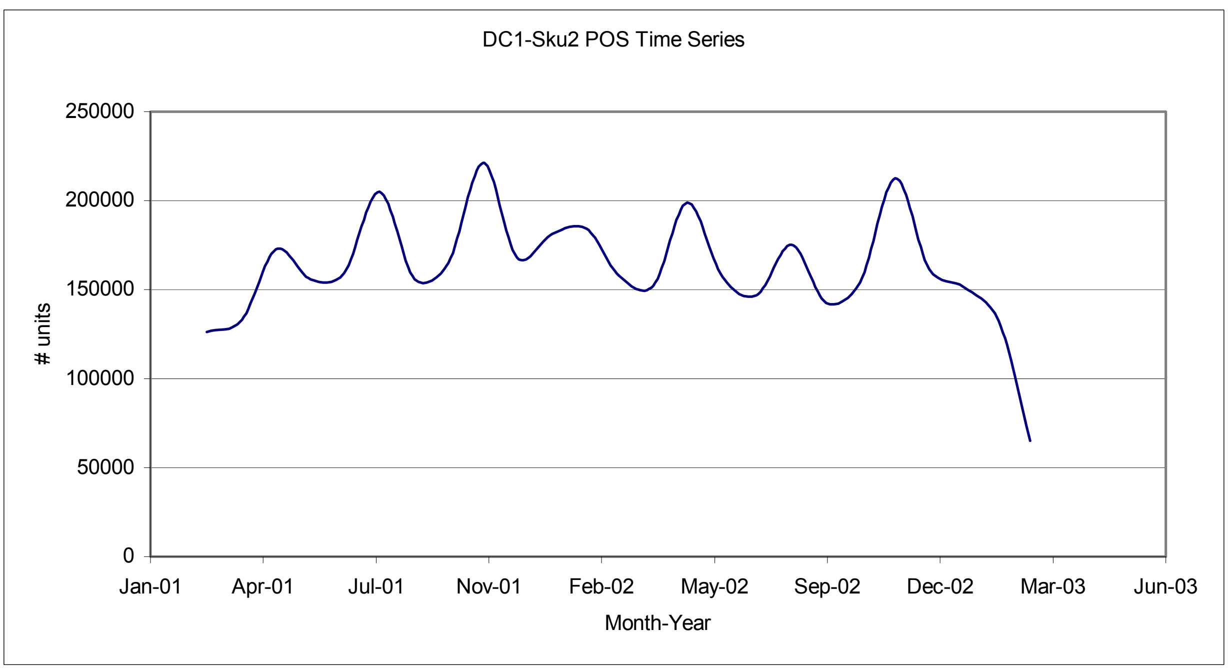


Figure 17: Monthly POS time series for DC1-Sku2

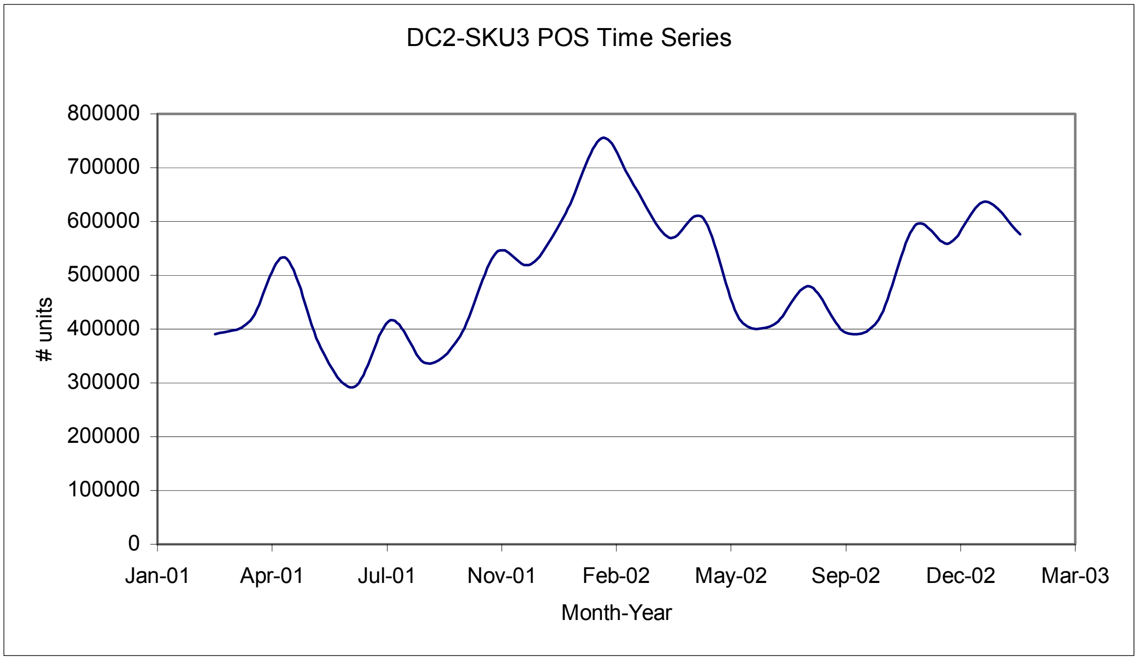


Figure 18: Monthly POS time series for DC2-Sku3

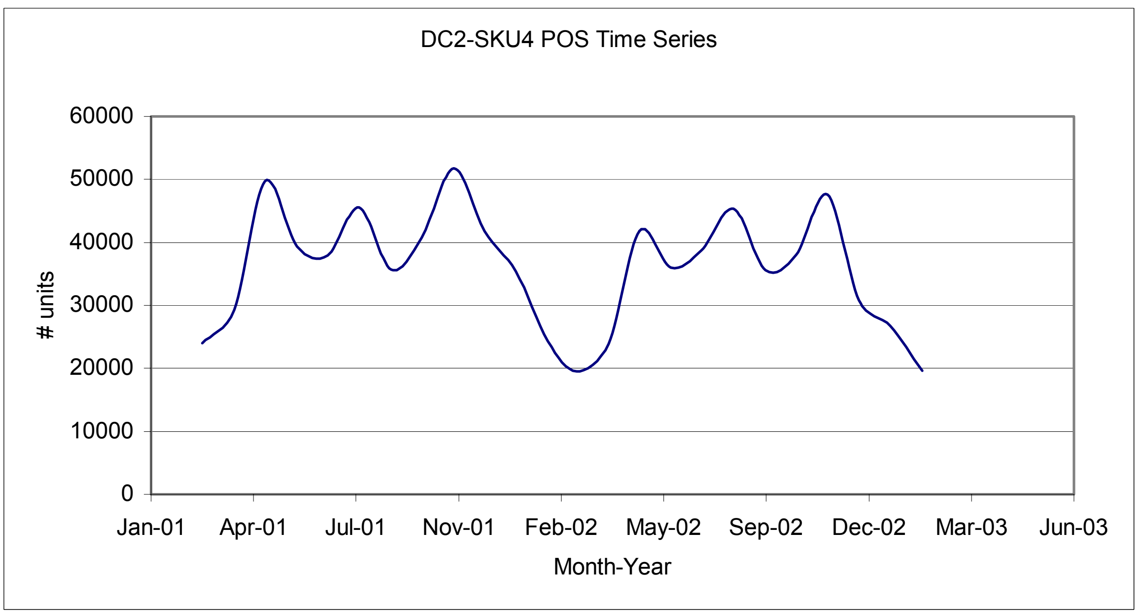


Figure 19: Monthly POS time series for DC2-Sku4

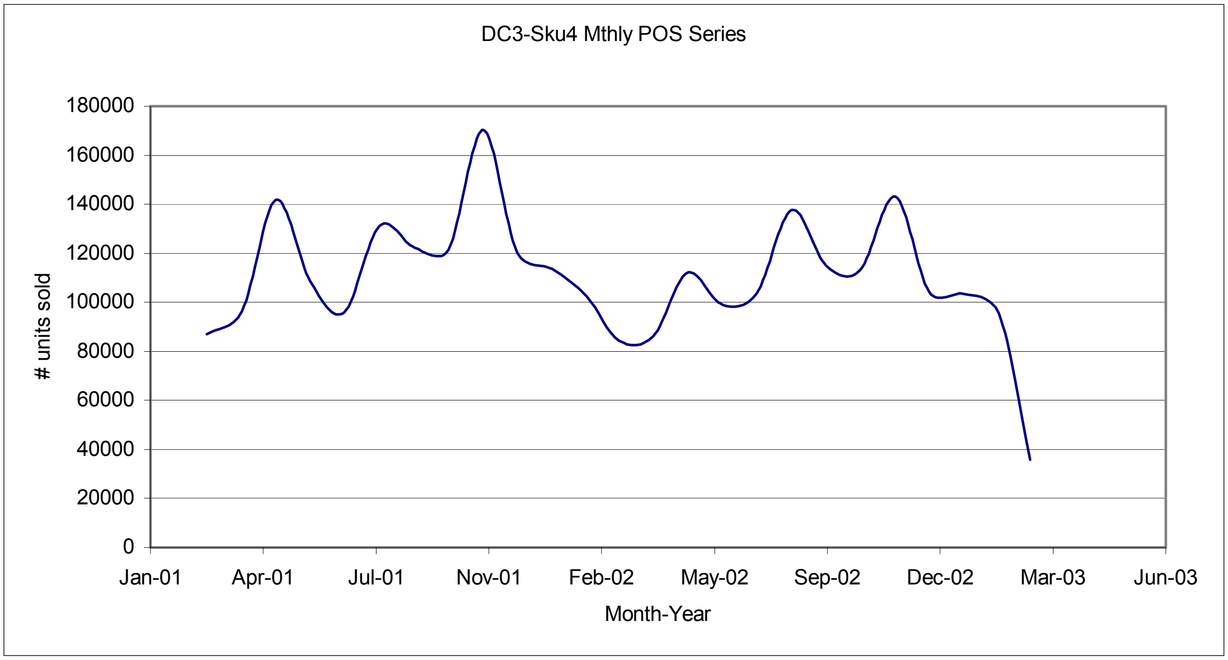


Figure 20: Monthly POS time series for DC3-Sku4

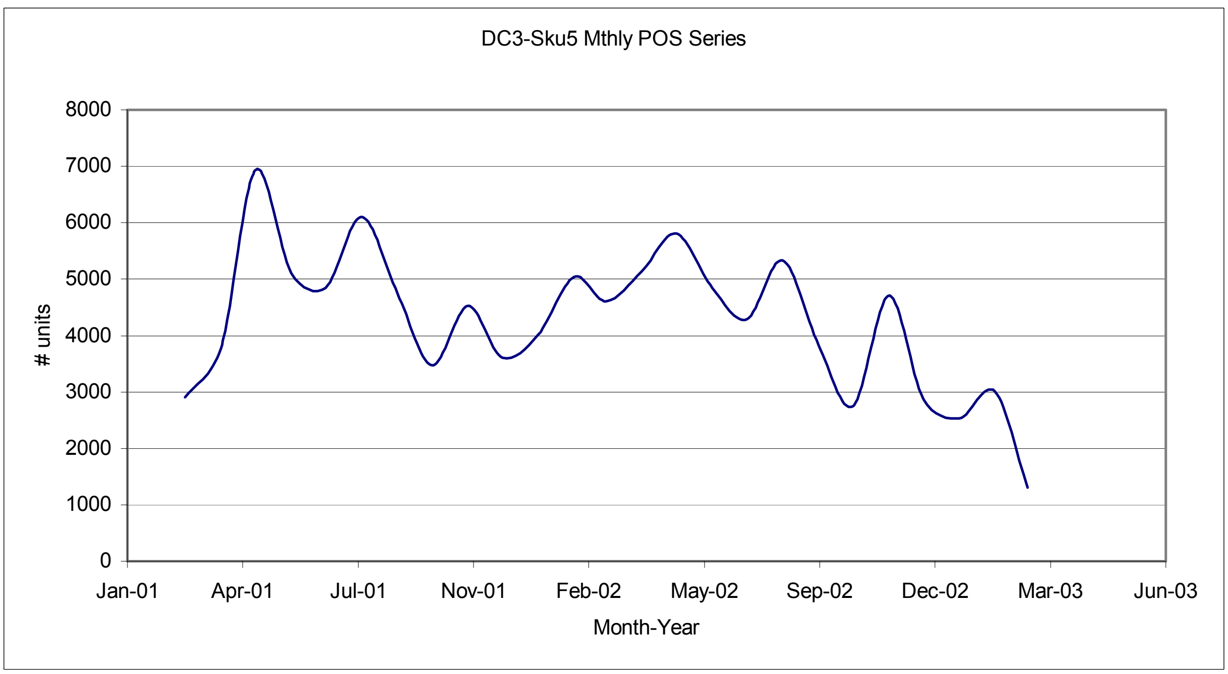


Figure 21: Monthly POS time series for DC3-Sku5

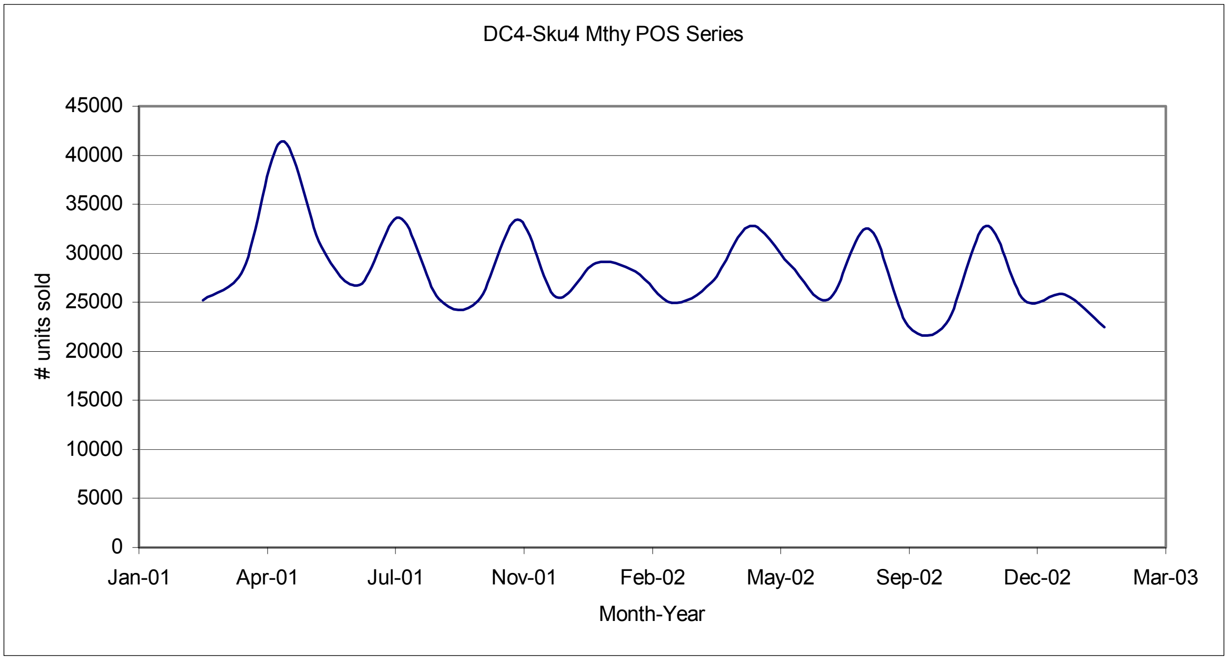


Figure 22: Monthly POS time series for DC4-Sku4

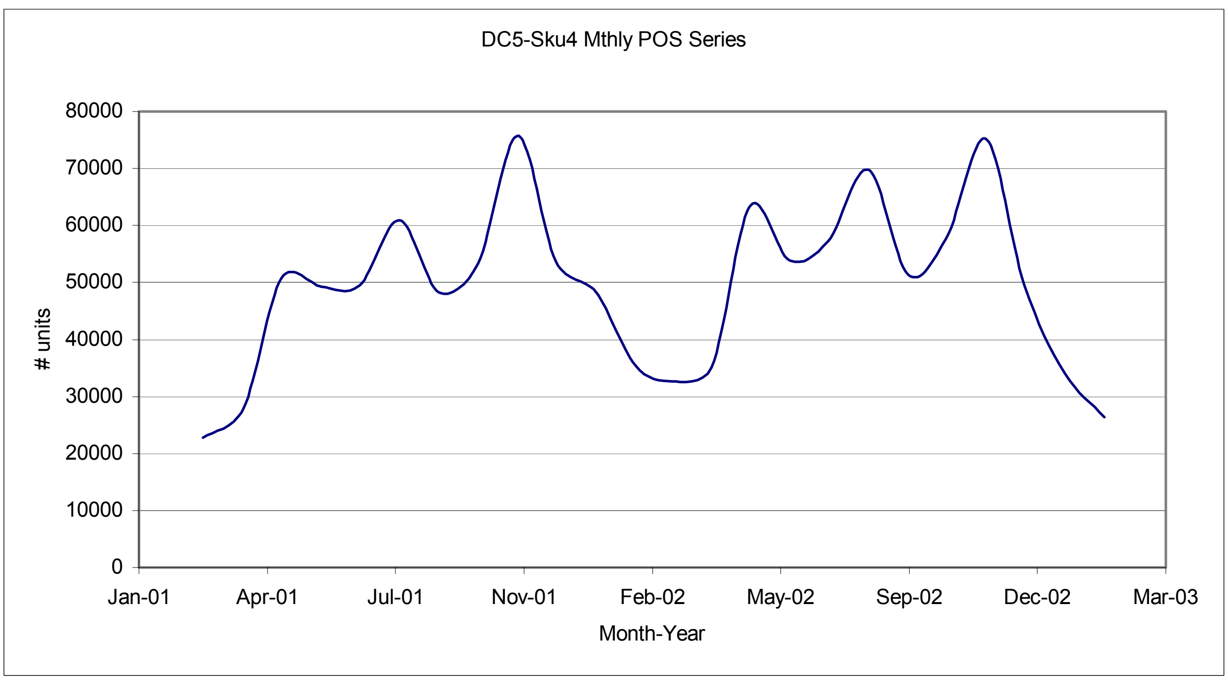


Figure 23: Monthly POS time series for DC5-Sku4

APPENDIX 4

Time Series Plots for the Winters Additive and Seasonal Exponential Smoothing Models in Table 4[12]

Below each plot is the table of the actual monthly Point of Sale, the smoothed values and the predicted values. Time 1 in the table represents February 2001, time 2 represents March 2001 etc. On the plot, the actual values are plotted in black and the predicted are plotted in blue. The smoothing constants used in each model are given in each plot. The corresponding MAPE, MAD and MSD values are also given in each plot.

[12] The Smoothing Models use Monthly Point-of-Sale data (time 1 being February 2001)

DC1-SKU1

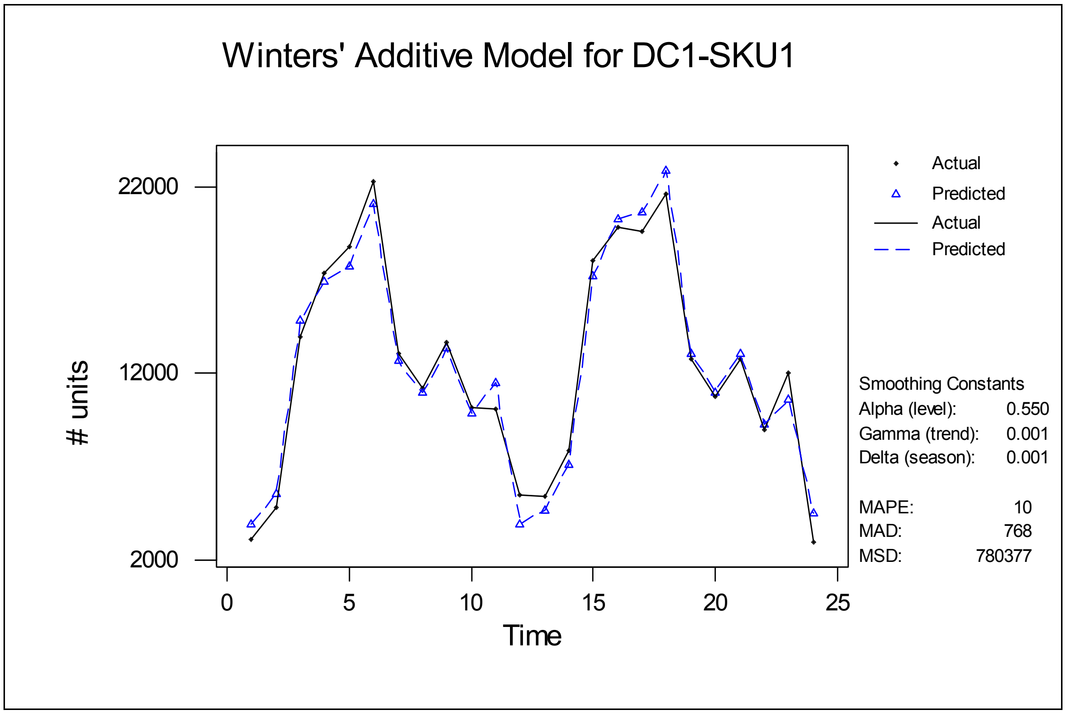


Figure 24: Winters Additive Model for DC1-SKU1

Row	Time	DC1-SKU1 POS	Smooth	Predict	Error
1	1	3135	3929.5	3917.2	-782.24
2	2	4797	5554.0	5541.3	-744.27
3	3	13952	14825.4	14812.3	-860.29
4	4	17395	16963.0	16949.5	445.53
5	5	18779	17789.5	17776.2	1002.84
6	6	22310	21107.9	21095.2	1214.84
7	7	13036	12686.1	12674.0	361.98
8	8	11210	10978.5	10966.6	243.36
9	9	13655	13347.9	13336.1	318.90
10	10	10177	9872.4	9860.8	316.18
11	11	10105	11538.1	11526.7	-1421.71
12	12	5483	3928.9	3916.7	1566.26
13	13	5395	4694.9	4683.5	711.46
14	14	7846	7142.4	7131.5	714.51
15	15	18057	17218.5	17208.0	849.05
16	16	19841	20300.2	20290.1	-449.07
17	17	19625	20637.9	20627.6	-1002.58
18	18	21632	22855.6	22844.7	-1212.72
19	19	12729	13099.1	13087.5	-358.55
20	20	10744	10995.4	10983.7	-239.67
21	21	12772	13099.1	13087.2	-315.18
22	22	8950	9274.5	9262.4	-312.42
23	23	12007	10593.0	10580.7	1426.28
24	24	2977	4551.9	4540.5	-1563.47

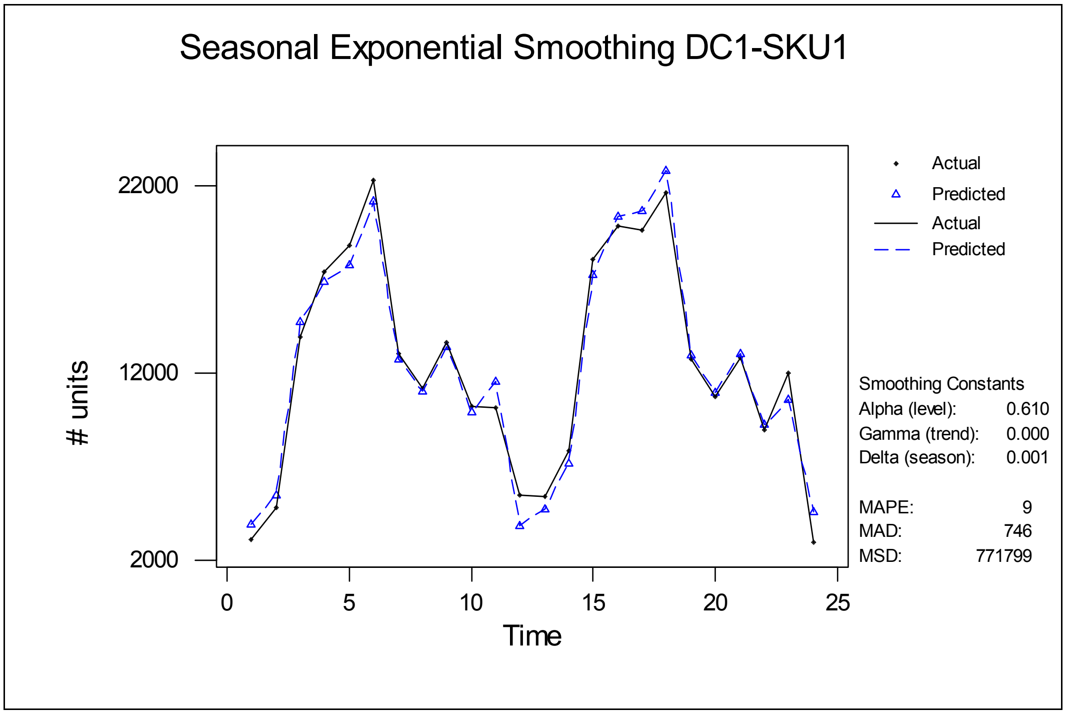


Figure 25: Seasonal Exponential Smoothing Model for DC1-SKU1

Row	Time	DC1-SKU1 POS	Smooth	Predict	Error
1	1	3135	3929.5	3917.2	-782.24
2	2	4797	5507.4	5495.1	-698.10
3	3	13952	14763.0	14750.8	-798.78
4	4	17395	16887.8	16875.6	519.45
5	5	18779	17787.2	17774.9	1004.06
6	6	22310	21167.2	21154.9	1155.06
7	7	13036	12781.8	12769.6	266.45
8	8	11210	11037.4	11025.1	184.89
9	9	13655	13385.2	13372.9	282.08
10	10	10177	9905.8	9893.5	283.49
11	11	10105	11569.7	11557.5	-1452.47
12	12	5483	3856.2	3844.0	1639.01
13	13	5395	4759.9	4747.6	647.40
14	14	7846	7209.8	7197.6	648.43
15	15	18057	17286.9	17274.6	782.40
16	16	19841	20376.7	20364.4	-523.40
17	17	19625	20640.1	20627.8	-1002.84
18	18	21632	22796.0	22783.7	-1151.69
19	19	12729	13003.1	12990.8	-261.84
20	20	10744	10936.4	10924.1	-180.11
21	21	12772	13061.6	13049.3	-277.31
22	22	8950	9240.9	9228.7	-278.68
23	23	12007	10561.3	10549.0	1457.97
24	24	2977	4624.4	4612.1	-1635.12

DC1-SKU2

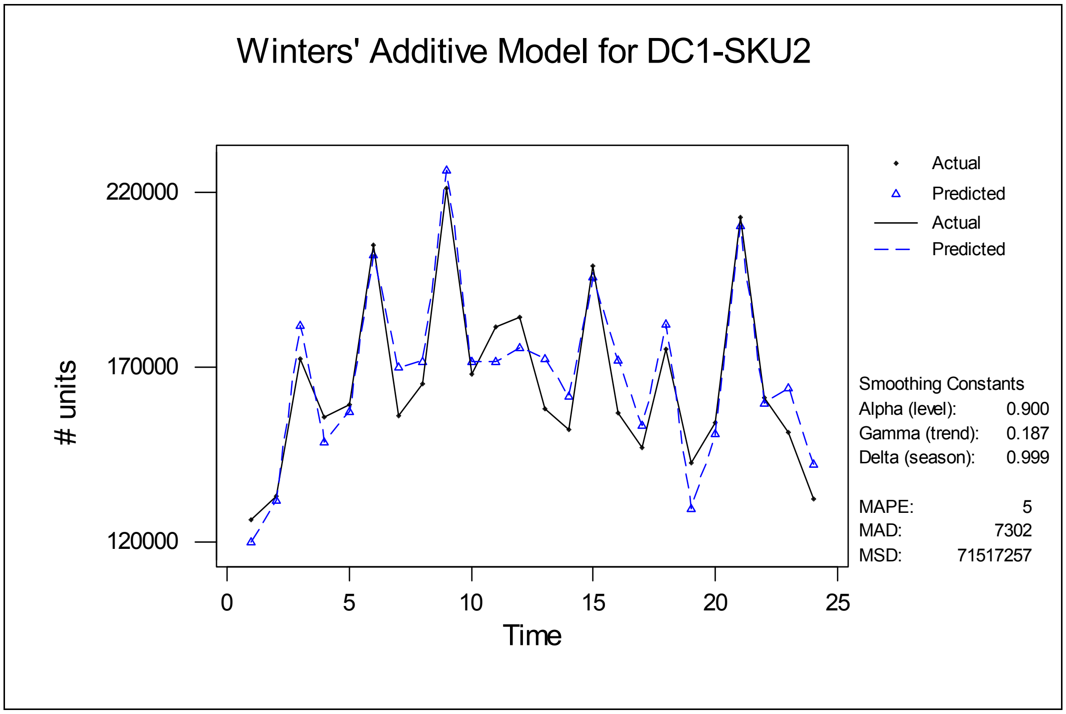


Figure 26: Winters Additive Model for DC1-SKU2

Row	Time	DC1-SKU2 POS	Smooth	Predict	Error
1	1	126290	115443	120109	6181.1
2	2	133121	126094	131800	1320.9
3	3	172506	176060	181989	-9482.8
4	4	155828	144211	148543	7284.6
5	5	159271	151670	157229	2042.4
6	6	204987	196187	202089	2897.9
7	7	155916	163722	170112	-14196.0
8	8	165112	167668	171668	-6556.1
9	9	221296	223258	226155	-4858.8
10	10	167861	169401	171480	-3618.8
11	11	181353	169913	171382	9970.5
12	12	184201	172289	175437	8763.9
13	13	158002	167915	172538	-14536.3
14	14	152043	159400	161576	-9532.8
15	15	198824	194988	195560	3264.3
16	16	157029	170931	172052	-15023.4
17	17	146756	154586	153178	-6421.9
18	18	175304	184608	182119	-6814.6
19	19	142449	133299	129662	12786.8
20	20	153926	152257	150773	3153.5
21	21	212739	211267	210314	2425.2
22	22	161430	160237	159692	1738.1
23	23	151337	164308	164056	-12718.6
24	24	132387	144432	142038	-9651.3

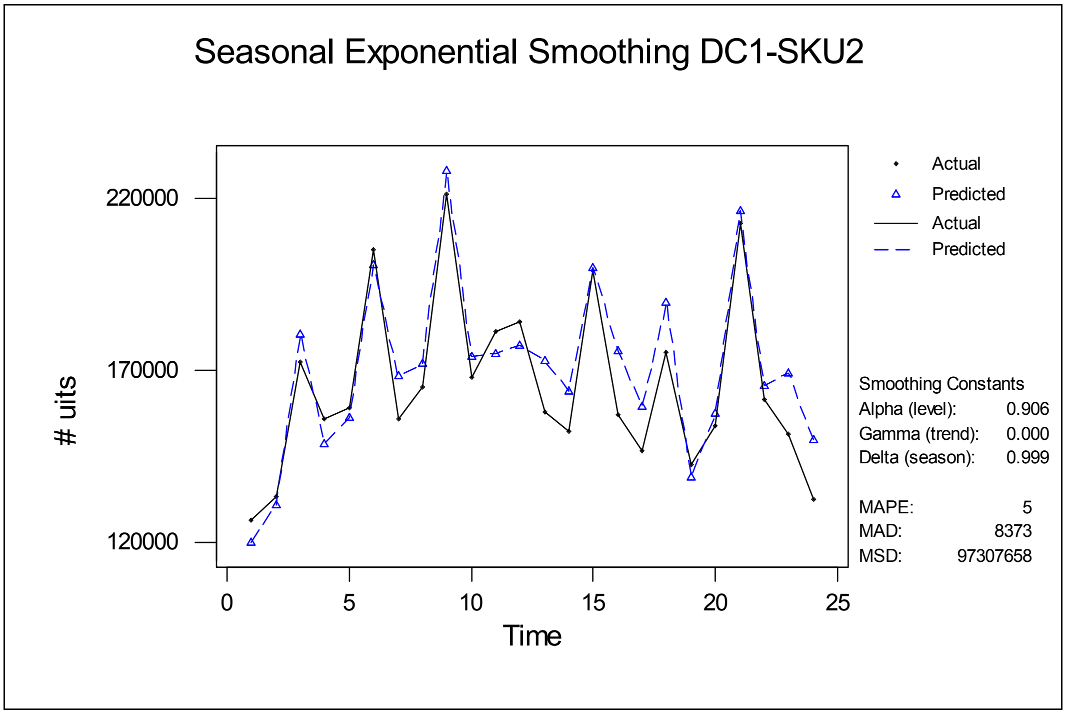


Figure 27: Seasonal Exponential Smoothing Model for DC1-SKU2

Row	Time	DC1-SKU2 POS	Smooth	Predict	Error
1	1	126290	115443	120109	6181.1
2	2	133121	126137	130803	2318.5
3	3	172506	175976	180642	-8135.8
4	4	155828	144019	148685	7143.0
5	5	159271	151734	156400	2871.4
6	6	204987	196123	200789	4198.0
7	7	155916	163620	168286	-12370.0
8	8	165112	167398	172064	-6951.9
9	9	221296	223250	227915	-6619.2
10	10	167861	169532	174198	-6336.6
11	11	181353	170142	174808	6545.3
12	12	184201	172679	177344	6856.5
13	13	158002	168111	172777	-14775.0
14	14	152043	159448	164113	-12070.5
15	15	198824	195267	199933	-1108.7
16	16	157029	171108	175773	-18744.2
17	17	146756	154957	159622	-12866.5
18	18	175304	185204	189870	-14565.8
19	19	142449	134144	138810	3639.1
20	20	153926	152940	157606	-3680.0
21	21	212739	211789	216454	-3715.3
22	22	161430	160730	165395	-3965.5
23	23	151337	164693	169358	-18021.5
24	24	132387	144989	149655	-17268.1

DC2-SKU3

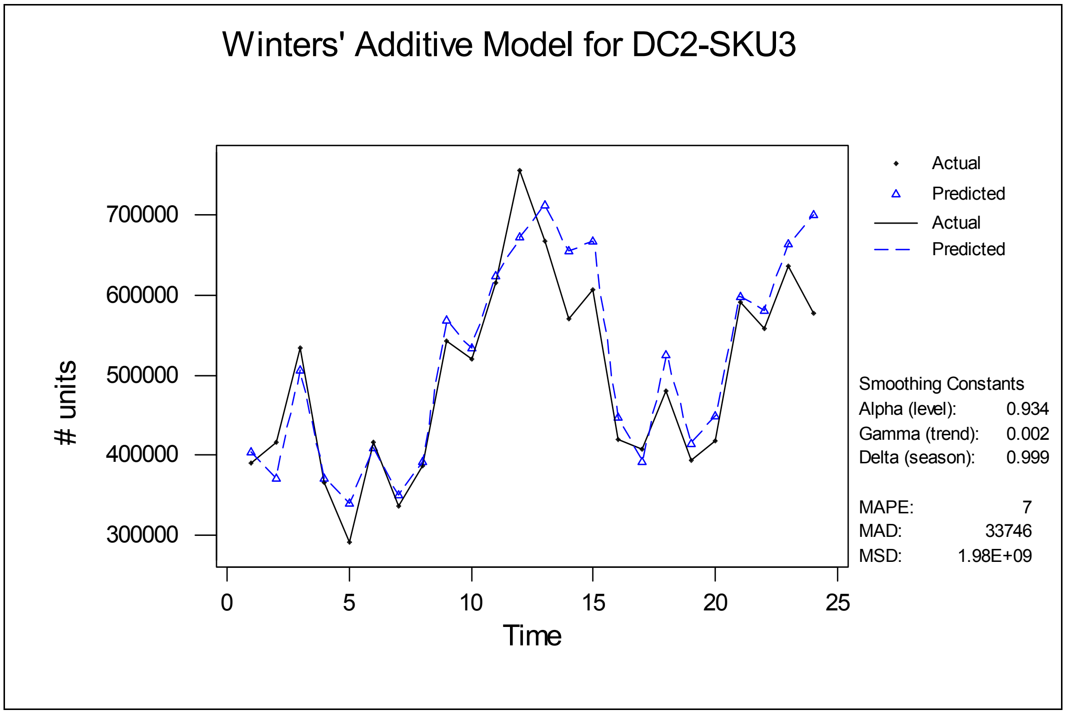


Figure 28: Winters Additive Model for DC2-SKU3

Row	Time	DC2-SKU3 POS	Smooth	Predict	Error
1	1	390733	381297	405065	-14332
2	2	415522	348461	372205	43317
3	3	533331	482521	506339	26992
4	4	365729	347431	371295	-5566
5	5	292264	317013	340867	-48603
6	6	416518	386372	410144	6374
7	7	337475	326649	350431	-12956
8	8	387072	368203	391964	-4892
9	9	542111	544681	568433	-26322
10	10	519962	509781	533488	-13526
11	11	614979	600282	623967	-8988
12	12	755184	648738	672407	82777
13	13	667602	688183	711993	-44391
14	14	570221	631154	654889	-84668
15	15	606023	644641	668231	-62208
16	16	419269	423889	447374	-28105
17	17	408418	369194	392631	15787
18	18	479778	501897	525361	-45583
19	19	393786	392078	415465	-21679
20	20	417780	425629	448979	-31199
21	21	591166	575715	599012	-7846
22	22	559013	558458	581741	-22728
23	23	636695	640246	663491	-26796
24	24	576669	677728	700928	-124259

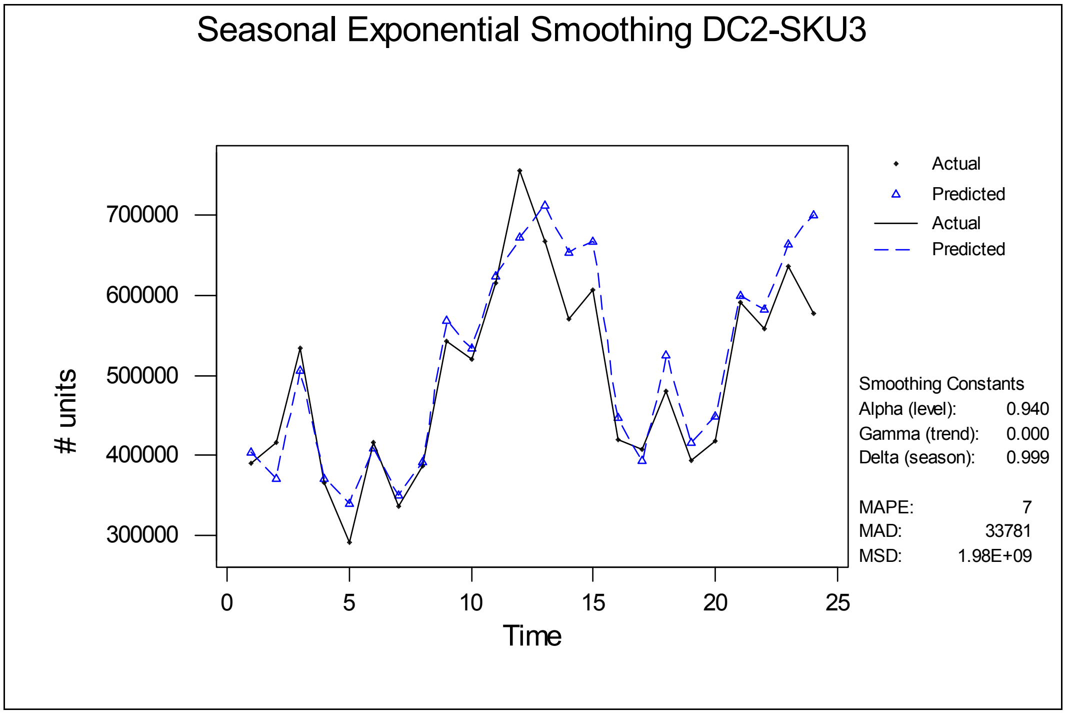


Figure 29: Seasonal Exponential Smoothing Model for DC2-SKU3

Row	Time	DC2-SKU3 POS	Smooth	Predict	Error
1	1	390733	381297	405065	-14332
2	2	415522	348373	372141	43381
3	3	533331	482784	506552	26779
4	4	365729	347610	371379	-5650
5	5	292264	316983	340752	-48488
6	6	416518	386066	409835	6683
7	7	337475	326669	350438	-12963
8	8	387072	368124	391892	-4820
9	9	542111	544647	568415	-26304
10	10	519962	509618	533386	-13424
11	11	614979	600193	623961	-8982
12	12	755184	648682	672450	82734
13	13	667602	688783	712552	-44950
14	14	570221	630564	654333	-84112
15	15	606023	644170	667938	-61915
16	16	419269	423697	447465	-28196
17	17	408418	369303	393071	15347
18	18	479778	501695	525463	-45685
19	19	393786	391904	415672	-21886
20	20	417780	425463	449232	-31452
21	21	591166	575667	599435	-8269
22	22	559013	558361	582130	-23117
23	23	636695	640096	663864	-27169
24	24	576669	677019	700787	-124118

DC2-SKU4

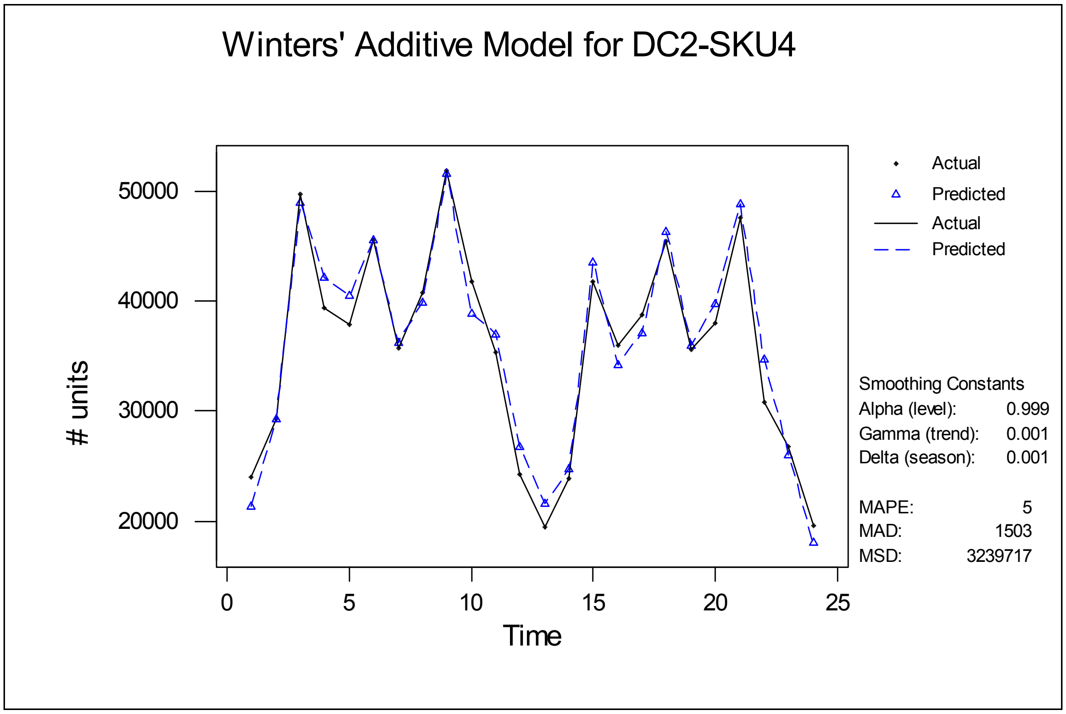


Figure 30: Winters Additive Model for DC2-SKU4

Row	Time	DC2-SKU4 POS	Smooth	Predict	Error
1	1	24014	21138.6	21362.6	2651.37
2	2	29252	29050.5	29277.2	-25.19
3	3	49707	48680.2	48906.8	800.16
4	4	39377	41893.9	42121.3	-2744.31
5	5	37910	40235.4	40460.1	-2550.11
6	6	45587	45325.2	45547.4	39.63
7	7	35728	36004.1	36226.3	-498.32
8	8	40794	39718.7	39940.4	853.64
9	9	51799	51353.8	51576.4	222.64
10	10	41789	38622.4	38845.2	2943.78
11	11	35386	36822.2	37047.9	-1661.94
12	12	24322	26540.3	26764.4	-2442.38
13	13	19540	21420.6	21642.3	-2102.25
14	14	23891	24581.3	24800.8	-909.78
15	15	41803	43320.1	43538.7	-1735.68
16	16	36019	33992.4	34209.3	1809.73
17	17	38708	36872.9	37091.5	1616.46
18	18	45367	46119.1	46339.3	-972.35
19	19	35571	35785.1	36004.5	-433.46
20	20	37996	39561.6	39780.5	-1784.49
21	21	47623	48558.4	48775.6	-1152.56
22	22	30791	34447.8	34663.8	-3872.78
23	23	26777	25831.0	26043.1	733.88
24	24	19657	17928.9	18141.8	1515.25

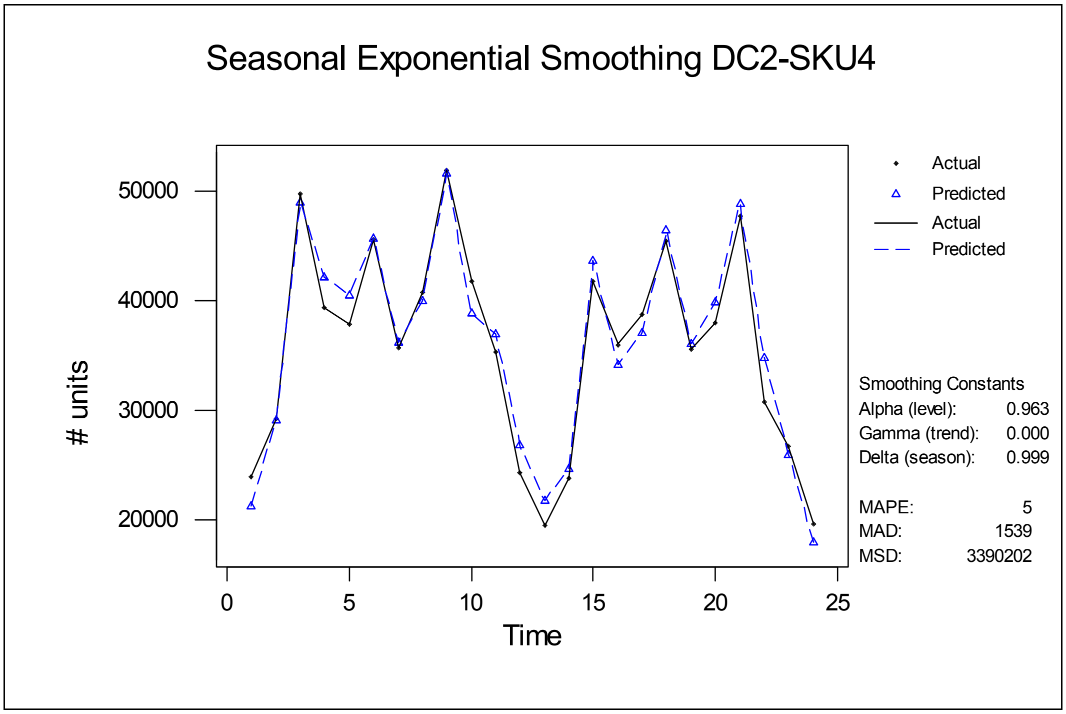


Figure 31: Seasonal Exponential Smoothing Model for DC2-SKU4

Row	Time	DC2-SKU4 POS	Smooth	Predict	Error
1	1	24014	21138.6	21362.6	2651.37
2	2	29252	28954.9	29179.0	73.05
3	3	49707	48677.5	48901.5	805.52
4	4	39377	41864.8	42088.8	-2711.84
5	5	37910	40333.1	40557.2	-2647.17
6	6	45587	45420.7	45644.8	-57.77
7	7	35728	36006.3	36230.3	-502.33
8	8	40794	39736.8	39960.8	833.20
9	9	51799	51323.8	51547.8	251.18
10	10	41789	38613.4	38837.4	2951.62
11	11	35386	36715.8	36939.8	-1553.83
12	12	24322	26596.2	26820.3	-2498.26
13	13	19540	21608.9	21832.9	-2292.92
14	14	23891	24568.7	24792.7	-901.71
15	15	41803	43379.7	43603.7	-1800.71
16	16	36019	33927.2	34151.2	1867.79
17	17	38708	36807.9	37031.9	1676.12
18	18	45367	46154.4	46378.4	-1011.43
19	19	35571	35805.2	36029.2	-458.21
20	20	37996	39627.6	39851.6	-1855.60
21	21	47623	48603.9	48827.9	-1204.90
22	22	30791	34591.3	34815.3	-4024.29
23	23	26777	25809.5	26033.5	743.47
24	24	19657	17867.2	18091.2	1565.82

DC3-SKU4

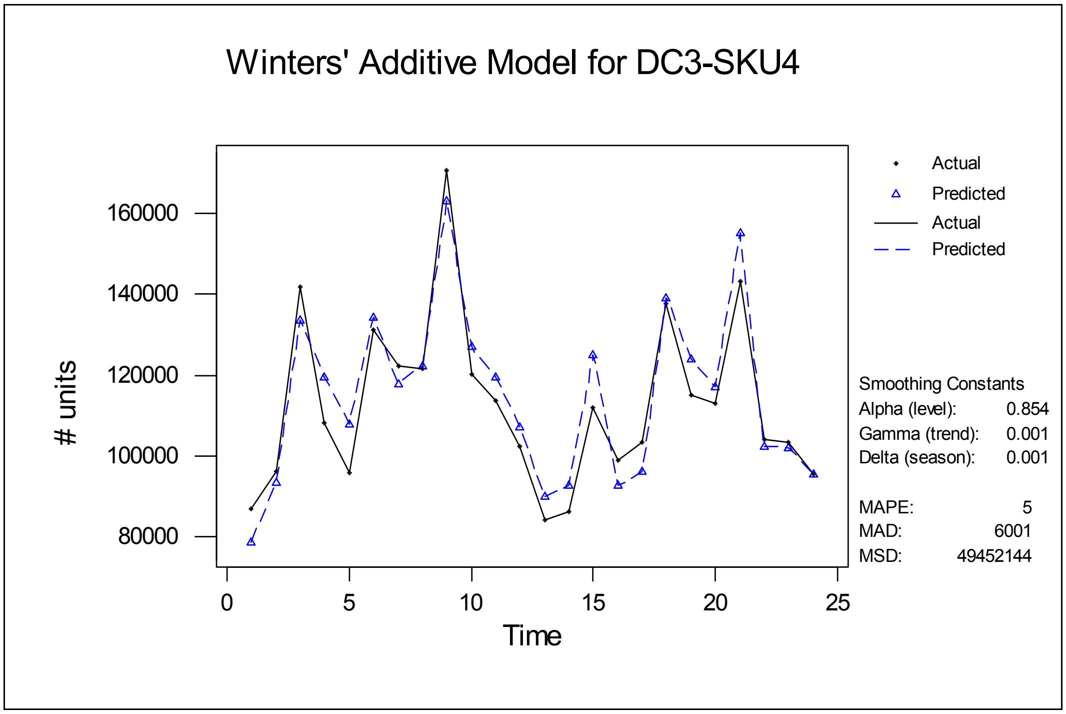


Figure 32: Winters Additive Model for DC3-SKU4

Row	Time	DC3-SKU4 POS	Smooth	Predict	Error
1	1	87055	77040	78979	8076.4
2	2	96317	91553	93498	2818.9
3	3	141938	131822	133769	8168.6
4	4	108337	117575	119530	-11192.6
5	5	95904	106167	108113	-12208.5
6	6	131214	132512	134447	-3232.6
7	7	122261	116080	118012	4249.3
8	8	121543	120510	122446	-902.9
9	9	170459	161258	163193	7266.3
10	10	120411	125082	127023	-6612.3
11	11	113758	117780	119715	-5957.0
12	12	102468	105294	107224	-4756.0
13	13	84474	88115	90041	-5567.1
14	14	86242	90959	92880	-6638.1
15	15	112133	123125	125041	-12908.0
16	16	99069	90837	92742	6326.8
17	17	103583	94347	96257	7325.8
18	18	137613	137346	139263	-1649.9
19	19	115037	122249	124164	-9127.3
20	20	113173	115234	117141	-3968.5
21	21	143105	153336	155240	-12134.5
22	22	104198	100553	102446	1752.0
23	23	103343	100348	102243	1099.8
24	24	95650	93851	95747	-96.7

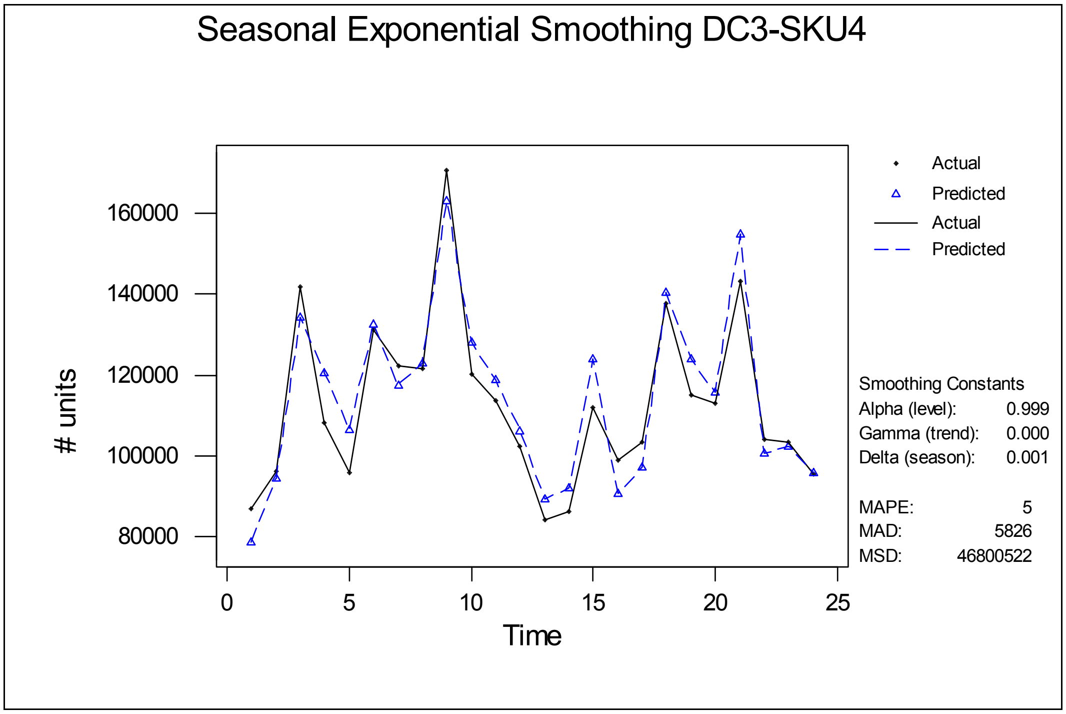


Figure 33: Seasonal Exponential Smoothing Model for DC3-SKU4

Row	Time	DC3-SKU4 POS	Smooth	Predict	Error
1	1	87055	77040	78979	8076.4
2	2	96317	92721	94660	1657.3
3	3	141938	132231	134169	7768.9
4	4	108337	118757	120696	-12358.5
5	5	95904	104549	106488	-10583.6
6	6	131214	130744	132682	-1468.4
7	7	122261	115610	117549	4712.2
8	8	121543	121125	123063	-1520.1
9	9	170459	161128	163066	7392.7
10	10	120411	126133	128072	-7660.9
11	11	113758	116824	118762	-5004.4
12	12	102468	104431	106369	-3901.3
13	13	84474	87425	89363	-4889.2
14	14	86242	90153	92092	-5849.7
15	15	112133	122163	124102	-11968.6
16	16	99069	88972	90910	8158.8
17	17	103583	95261	97199	6383.9
18	18	137613	138406	140344	-2731.4
19	19	115037	122011	123949	-8912.0
20	20	113173	113914	115853	-2679.7
21	21	143105	152759	154697	-11592.5
22	22	104198	98798	100737	3461.1
23	23	103343	100600	102538	804.7
24	24	95650	94010	95948	-298.5

DC3-SKU5

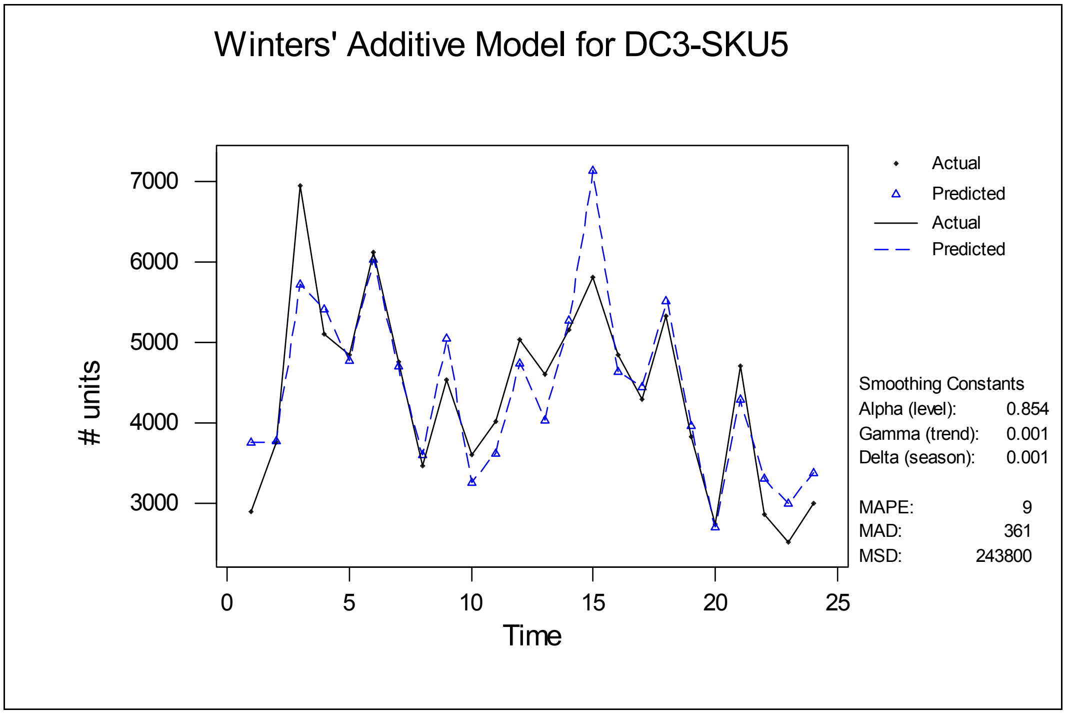


Figure 34: Winters Additive Model for DC3-SKU5

Row	Time	DC3-SKU5 POS	Smooth	Predict	Error
1	1	2904	3779.20	3758.49	-854.49
2	2	3769	3792.84	3771.41	-2.41
3	3	6938	5745.71	5724.28	1213.72
4	4	5101	5424.05	5403.65	-302.65
5	5	4842	4796.45	4775.80	66.20
6	6	6110	6047.22	6026.62	83.38
7	7	4755	4729.72	4709.19	45.81
8	8	3476	3627.19	3606.70	-130.70
9	9	4531	5065.90	5045.30	-514.30
10	10	3608	3283.79	3262.74	345.26
11	11	4013	3651.57	3630.82	382.18
12	12	5040	4766.19	4745.77	294.23
13	13	4602	4065.02	4044.85	557.15
14	14	5145	5285.32	5265.63	-120.63
15	15	5808	7139.11	7119.32	-1311.32
16	16	4850	4661.67	4640.76	209.24
17	17	4291	4470.93	4450.20	-159.20
18	18	5332	5529.06	5508.18	-176.18
19	19	3830	3989.52	3968.50	-138.50
20	20	2746	2729.01	2707.87	38.13
21	21	4712	4311.25	4290.14	421.86
22	22	2870	3328.53	3307.78	-437.78
23	23	2532	3027.64	3006.52	-474.52
24	24	3002	3409.98	3388.45	-386.45

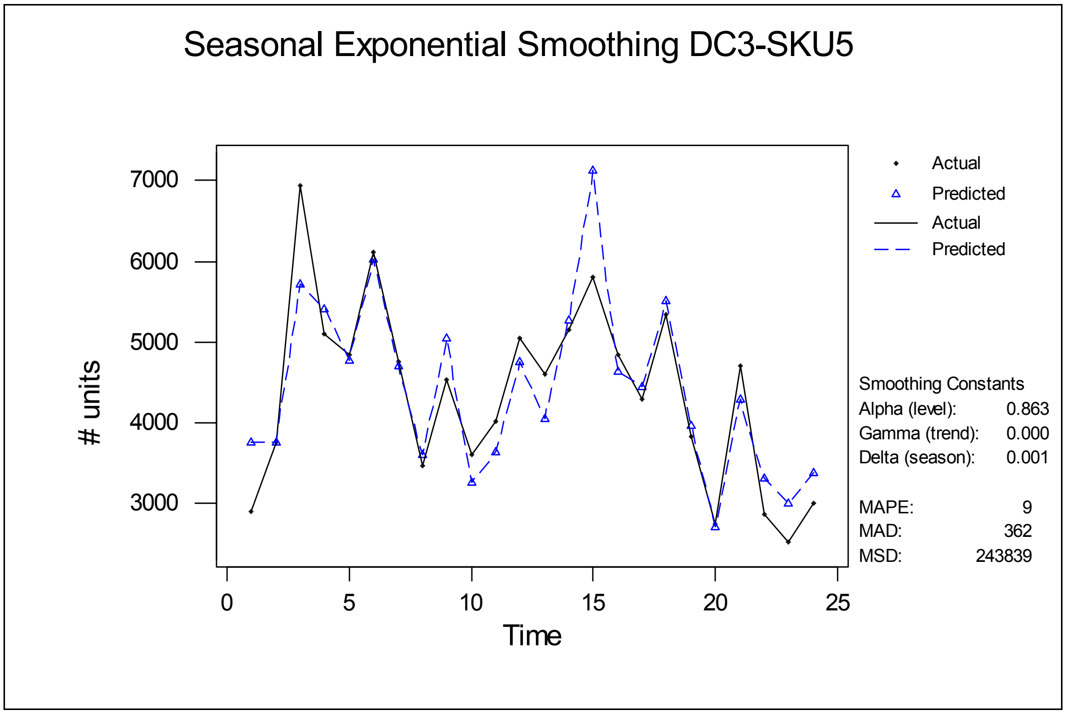


Figure 35: Seasonal Exponential Smoothing Model for DC3-SKU5

Row	Time	DC3-SKU5 POS	Smooth	Predict	Error
1	1	2904	3779.20	3758.49	-854.49
2	2	3769	3785.46	3764.76	4.24
3	3	6938	5744.78	5724.08	1213.92
4	4	5101	5434.51	5413.80	-312.80
5	5	4842	4795.23	4774.53	67.47
6	6	6110	6047.62	6026.91	83.09
7	7	4755	4730.48	4709.77	45.23
8	8	3476	3627.67	3606.96	-130.96
9	9	4531	5064.81	5044.11	-513.11
10	10	3608	3279.18	3258.48	349.52
11	11	4013	3653.96	3633.26	379.74
12	12	5040	4769.82	4749.12	290.88
13	13	4602	4068.03	4047.32	554.68
14	14	5145	5290.47	5269.77	-124.77
15	15	5808	7138.63	7117.92	-1309.92
16	16	4850	4650.17	4629.46	220.54
17	17	4291	4471.19	4450.49	-159.49
18	18	5332	5527.72	5507.02	-175.02
19	19	3830	3987.84	3967.14	-137.14
20	20	2746	2727.63	2706.93	39.07
21	21	4712	4311.46	4290.75	421.25
22	22	2870	3332.25	3311.55	-441.55
23	23	2532	3024.38	3003.67	-471.67
24	24	3002	3405.49	3384.79	-382.79

DC4-SKU4

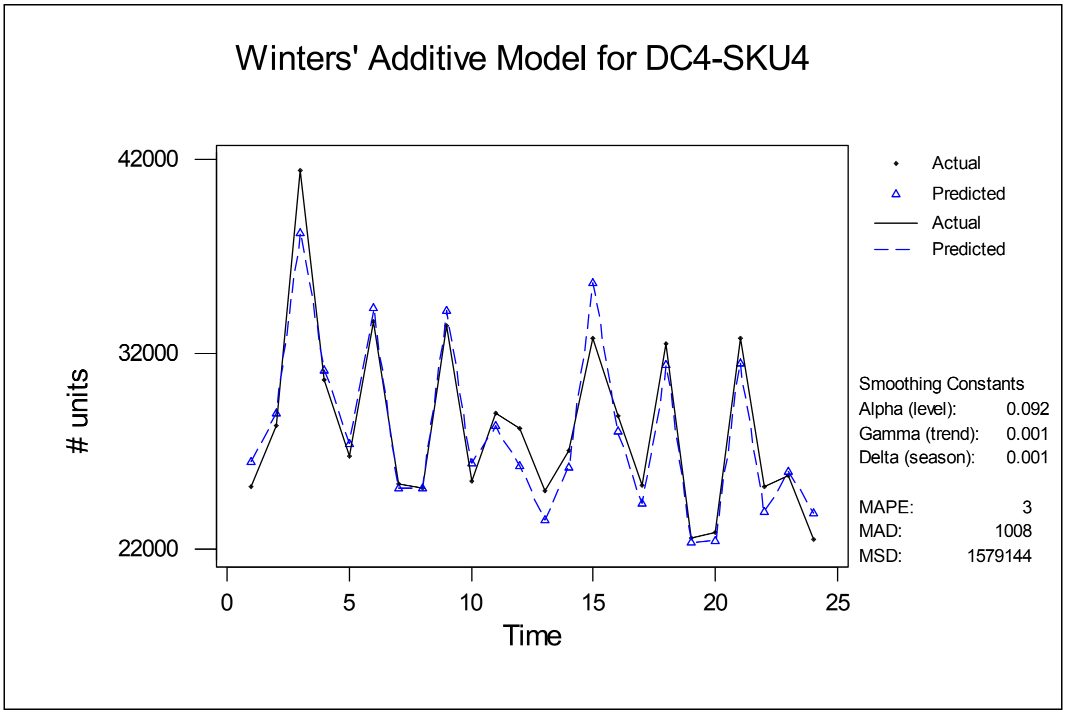


Figure 36: Winters Additive Model for DC4-SKU4

Row	Time	DC4-SKU4 POS	Smooth	Predict	Error
1	1	25211	26735.0	26481.5	-1270.48
2	2	28370	29201.1	28947.5	-577.48
3	3	41395	38505.4	38251.8	3143.24
4	4	30707	31452.9	31199.5	-492.51
5	5	26795	27637.8	27384.4	-589.35
6	6	33641	34600.7	34347.2	-706.22
7	7	25322	25374.3	25120.8	201.23
8	8	25119	25413.9	25160.4	-41.41
9	9	33455	34513.7	34260.2	-805.20
10	10	25512	26645.2	26391.6	-879.56
11	11	29008	28555.2	28301.5	706.50
12	12	28170	26521.2	26267.6	1902.42
13	13	24971	23745.8	23492.4	1478.59
14	14	27037	26465.8	26212.5	824.46
15	15	32786	35903.0	35649.8	-2863.75
16	16	28849	28294.3	28040.8	808.17
17	17	25280	24598.8	24345.4	934.61
18	18	32527	31702.0	31448.7	1078.35
19	19	22582	22640.9	22387.7	194.31
20	20	22886	22680.0	22426.8	459.22
21	21	32816	31825.5	31572.3	1243.66
22	22	25229	24145.9	23892.9	1336.12
23	23	25774	26262.0	26009.1	-235.07
24	24	22473	24143.1	23890.2	-1417.18

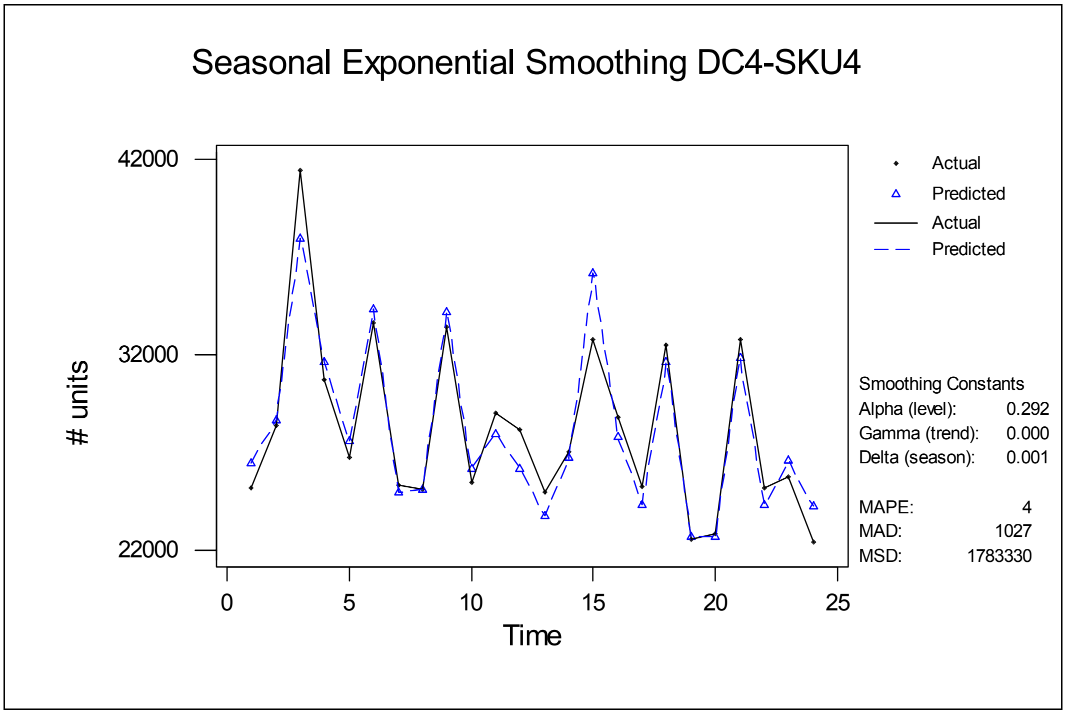


Figure 37: Seasonal Exponential Smoothing Model for DC4-SKU4

Row	Time	DC4-SKU4 POS	Smooth	Predict	Error
1	1	25211	26735.0	26481.5	-1270.48
2	2	28370	28947.7	28694.2	-324.24
3	3	41395	38210.8	37957.3	3437.65
4	4	30707	31871.1	31617.6	-910.64
5	5	26795	27835.8	27582.3	-787.30
6	6	33641	34623.4	34369.9	-728.91
7	7	25322	25249.5	24996.1	325.95
8	8	25119	25365.7	25112.2	6.79
9	9	33455	34471.3	34217.8	-762.82
10	10	25512	26454.6	26201.1	-689.07
11	11	29008	28244.8	27991.3	1016.68
12	12	28170	26442.3	26188.8	1981.16
13	13	24971	24069.7	23816.2	1154.79
14	14	27037	26990.1	26736.7	300.34
15	15	32786	36438.0	36184.5	-3398.51
16	16	28849	28102.3	27848.8	1000.18
17	17	25280	24624.1	24370.6	909.39
18	18	32527	31906.4	31652.9	874.10
19	19	22582	23000.6	22747.1	-165.10
20	20	22886	22973.4	22719.9	166.13
21	21	32816	32124.9	31871.4	944.61
22	22	25229	24605.9	24352.5	876.55
23	23	25774	26853.8	26600.3	-826.33
24	24	22473	24514.7	24261.2	-1788.25

DC5-SKU4

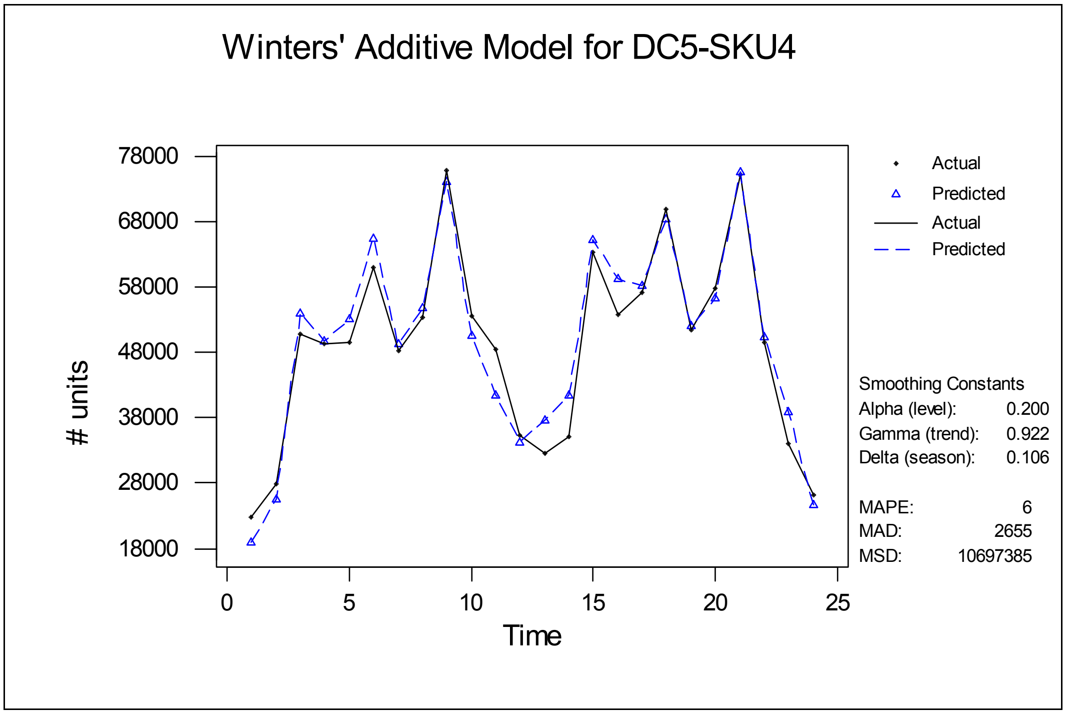


Figure 38: Winters Additive Model for DC5-SKU4

Row	Time	DC5-SKU4 POS	Smooth	Predict	Error
1	1	22755	17441.5	19103.2	3651.76
2	2	27826	23267.2	25602.3	2223.70
3	3	50774	51310.8	54055.9	-3281.86
4	4	49244	47548.6	49688.6	-444.56
5	5	49444	51037.6	53095.6	-3651.56
6	6	60953	64114.9	65499.6	-4546.59
7	7	48225	48686.0	49232.4	-1007.36
8	8	53341	54449.9	54810.5	-1469.49
9	9	75705	74090.6	74180.3	1524.72
10	10	53599	50223.0	50593.8	3005.16
11	11	48376	40582.1	41507.0	6869.01
12	12	35415	32172.2	34363.7	1051.32
13	13	32662	35345.5	37730.8	-5068.80
14	14	35118	40029.6	41480.3	-6362.32
15	15	63357	65003.1	65280.7	-1923.72
16	16	53844	59286.7	59209.6	-5365.62
17	17	57179	59301.5	58235.1	-1056.09
18	18	69828	69697.1	68435.9	1392.08
19	19	51406	53111.2	52106.7	-700.68
20	20	57787	57346.1	56212.4	1574.57
21	21	75230	76356.2	75512.9	-282.87
22	22	49471	51320.3	50424.8	-953.82
23	23	34128	39950.7	38879.3	-4751.32
24	24	26332	26725.6	24778.2	1553.82

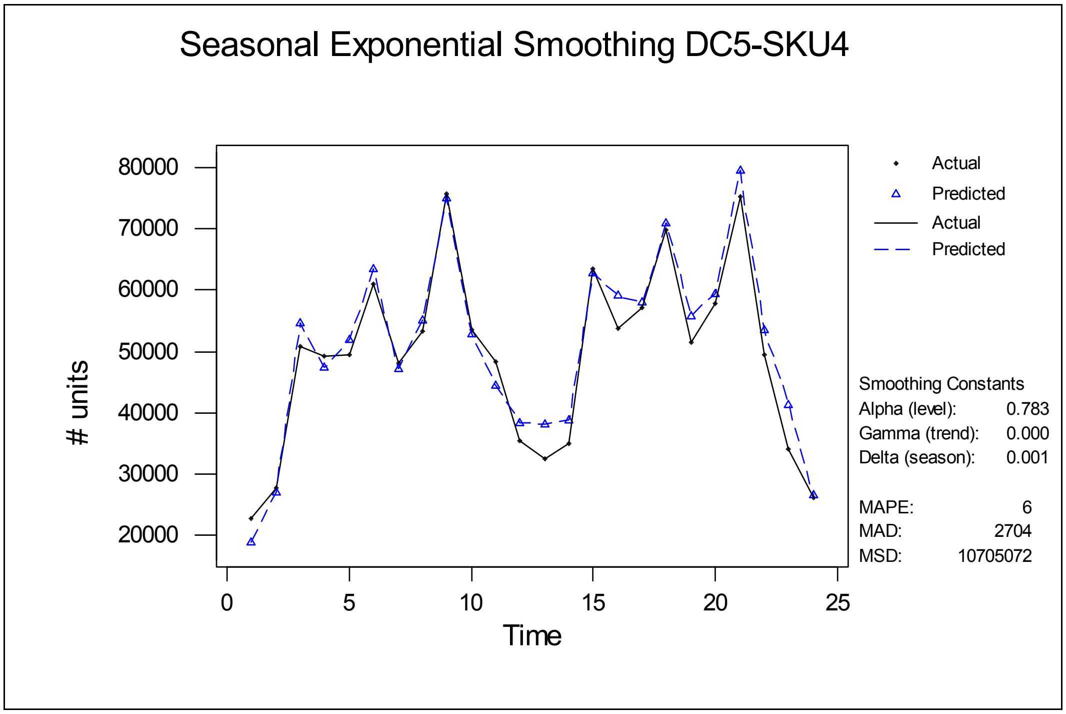
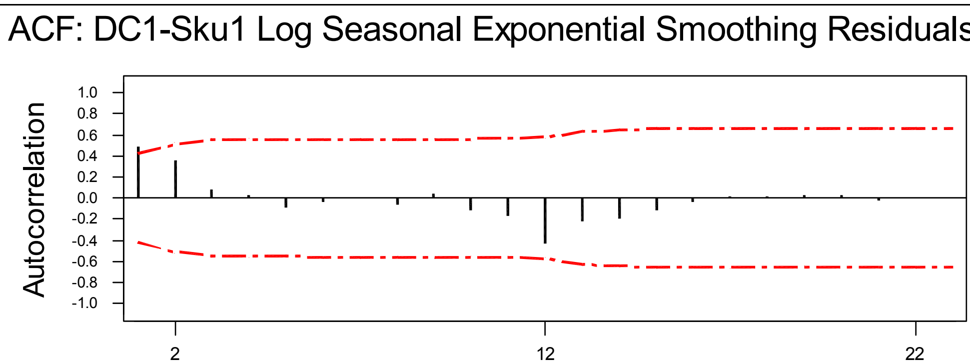


Figure 39: Seasonal Exponential Smoothing Model for DC5-SKU4

Row	Time	DC5-SKU4 POS	Smooth	Predict	Error
1	1	22755	17441.5	19103.2	3651.76
2	2	27826	25396.1	27057.8	768.15
3	3	50774	52923.1	54584.9	-3810.87
4	4	49244	45750.2	47412.0	1832.04
5	5	49444	50284.2	51945.9	-2501.90
6	6	60953	61736.6	63398.3	-2445.28
7	7	48225	45579.3	47241.0	984.01
8	8	53341	53430.3	55092.0	-1751.01
9	9	75705	73295.0	74956.8	748.23
10	10	53599	51280.5	52942.2	656.81
11	11	48376	42843.8	44505.5	3870.46
12	12	35415	36827.6	38489.4	-3074.37
13	13	32662	36543.7	38205.5	-5543.47
14	14	35118	37298.7	38960.5	-3842.47
15	15	63357	61215.1	62876.8	480.18
16	16	53844	57402.9	59064.6	-5220.61
17	17	57179	56414.3	58076.1	-897.06
18	18	69828	69123.2	70784.9	-956.89
19	19	51406	54131.9	55793.6	-4387.61
20	20	57787	57776.8	59438.6	-1651.57
21	21	75230	77720.0	79381.7	-4151.72
22	22	49471	51869.2	53530.9	-4059.93
23	23	34128	39740.5	41402.2	-7274.22
24	24	26332	24997.6	26659.3	-327.33

APPENDIX 5

Correlograms and Normality Plots for the Log-Transformed Smoothing Models in Table 5



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.49	2.38	6.40	8	-0.06	-0.23	10.66	15	-0.12	-0.37	29.32	22	-0.00	-0.00	29.83
2	0.35	1.43	9.97	9	0.03	0.13	10.71	16	-0.04	-0.14	29.47	23	-0.00	-0.01	29.84
3	0.08	0.28	10.14	10	-0.12	-0.45	11.37	17	0.01	0.04	29.49				
4	0.03	0.09	10.16	11	-0.18	-0.64	12.86	18	0.01	0.04	29.50				
5	-0.09	-0.34	10.44	12	-0.44	-1.57	22.77	19	0.03	0.09	29.62				
6	-0.04	-0.15	10.50	13	-0.23	-0.75	25.73	20	0.03	0.08	29.73				
7	-0.01	-0.02	10.50	14	-0.21	-0.66	28.36	21	-0.02	-0.07	29.83				

Figure 40: Correlogram of DC1-Sku1 Log Seasonal Exponential Smoothing Residuals

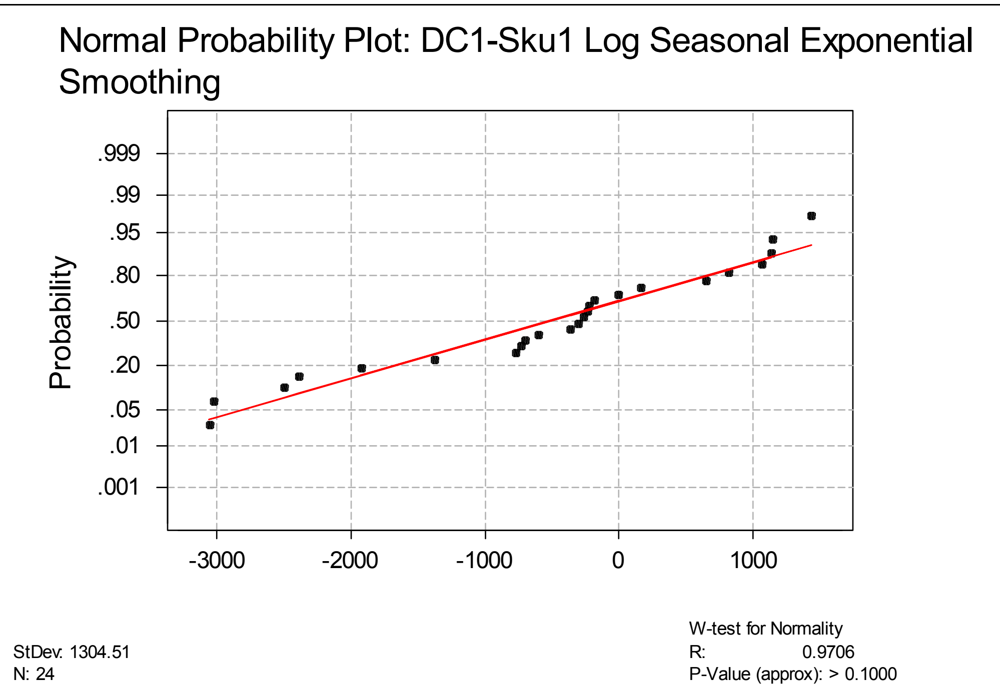
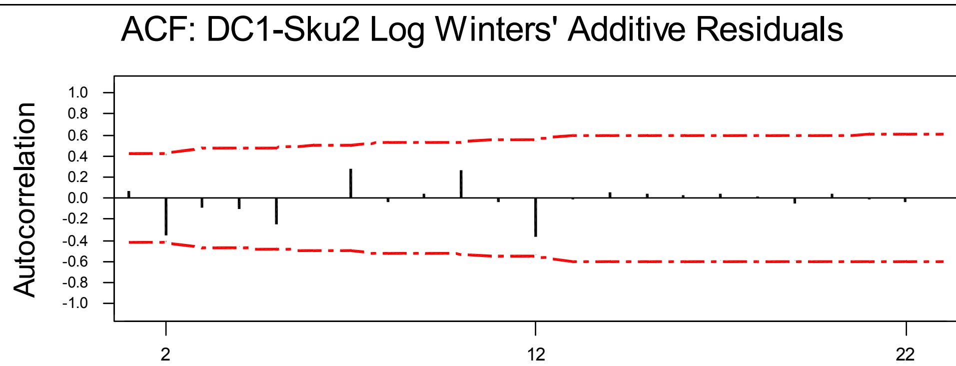


Figure 41: Normal Plot of DC1-Sku1 Log Seasonal Exponential Smoothing Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.06	0.30	0.10	8	-0.04	-0.15	9.23	15	0.04	0.13	20.05	22	-0.03	-0.12	21.21
2	-0.36	-1.75	3.75	9	0.04	0.15	9.29	16	0.03	0.09	20.11	23	-0.01	-0.02	21.22
3	-0.09	-0.39	3.98	10	0.27	1.05	12.55	17	0.04	0.12	20.22				
4	-0.10	-0.43	4.29	11	-0.04	-0.15	12.63	18	0.01	0.04	20.24				
5	-0.25	-1.08	6.38	12	-0.37	-1.38	19.76	19	-0.05	-0.16	20.52				
6	-0.00	-0.01	6.38	13	-0.01	-0.03	19.76	20	0.04	0.15	20.84				
7	0.28	1.13	9.17	14	0.06	0.19	19.96	21	-0.01	-0.03	20.85				

Figure 42: Correlogram of DC1-Sku2 Log Winters Additive Residuals

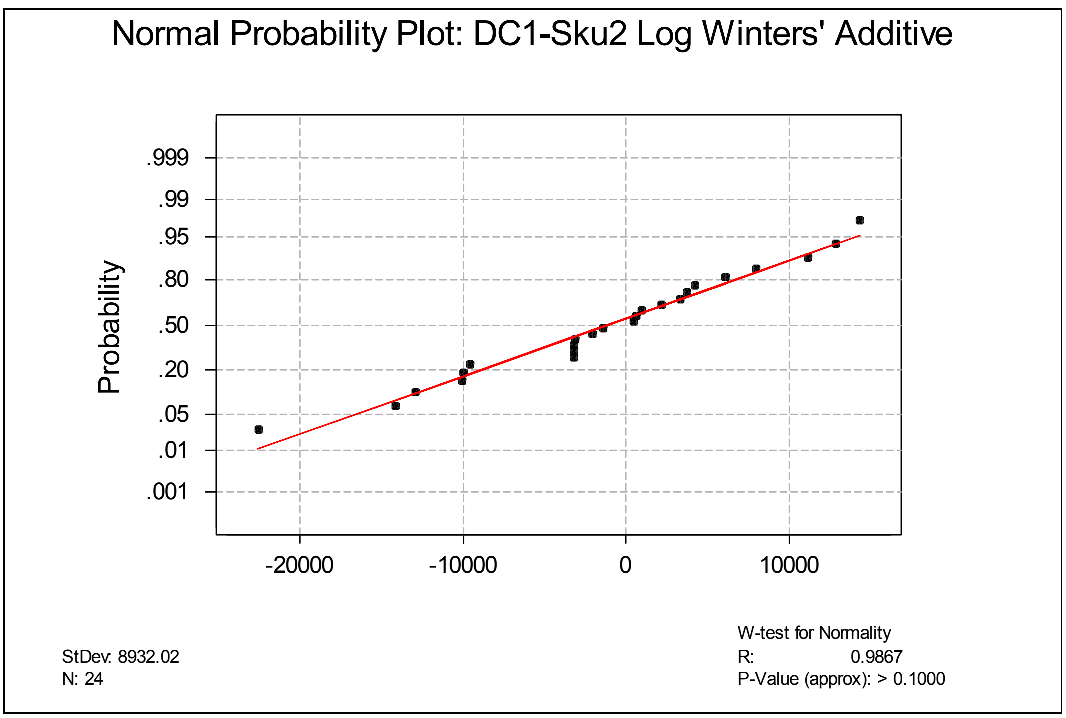
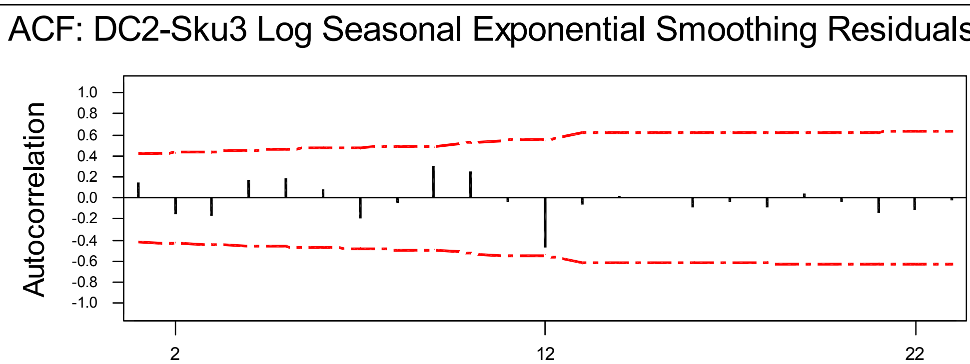


Figure 43: Normal Plot of DC1-Sku2 Log Winters Additive Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.15	0.73	0.61	8	-0.05	-0.20	5.91	15	-0.00	-0.01	24.92	22	-0.12	-0.38	35.45
2	-0.16	-0.75	1.30	9	0.31	1.30	9.91	16	-0.09	-0.30	25.57	23	-0.02	-0.07	35.77
3	-0.17	-0.81	2.18	10	0.26	1.01	12.87	17	-0.05	-0.15	25.76				
4	0.17	0.77	3.08	11	-0.04	-0.16	12.96	18	-0.10	-0.32	26.75				
5	0.19	0.83	4.21	12	-0.47	-1.78	24.65	19	0.04	0.14	26.96				
6	0.08	0.36	4.45	13	-0.07	-0.23	24.91	20	-0.04	-0.13	27.21				
7	-0.19	-0.83	5.82	14	0.01	0.04	24.92	21	-0.14	-0.46	31.29				

Figure 44: Correlogram of DC2-Sku3 Log Seasonal Exponential Smoothing Residuals

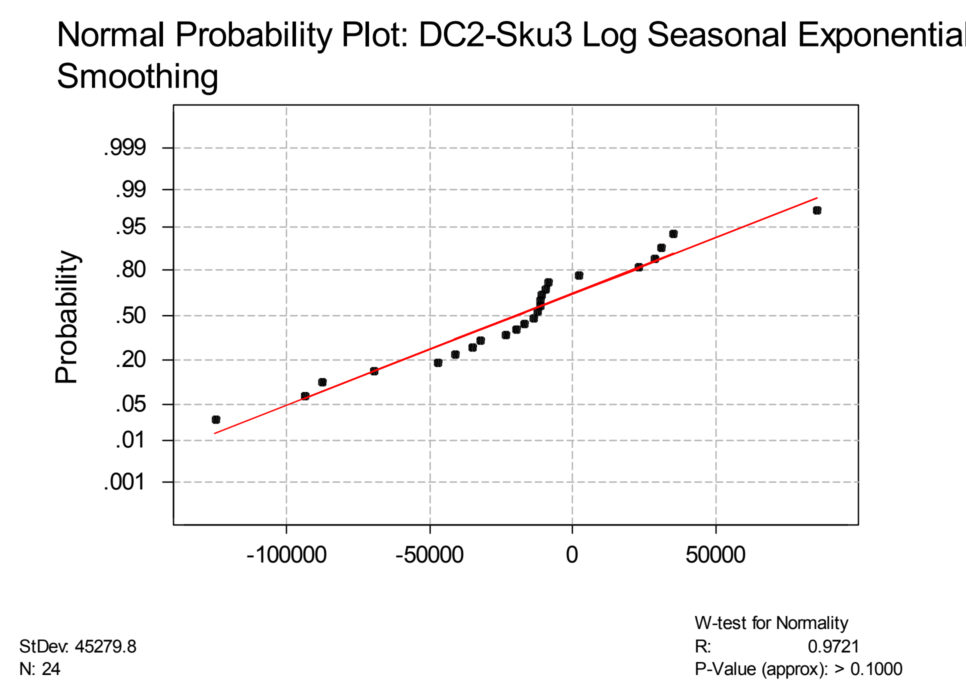
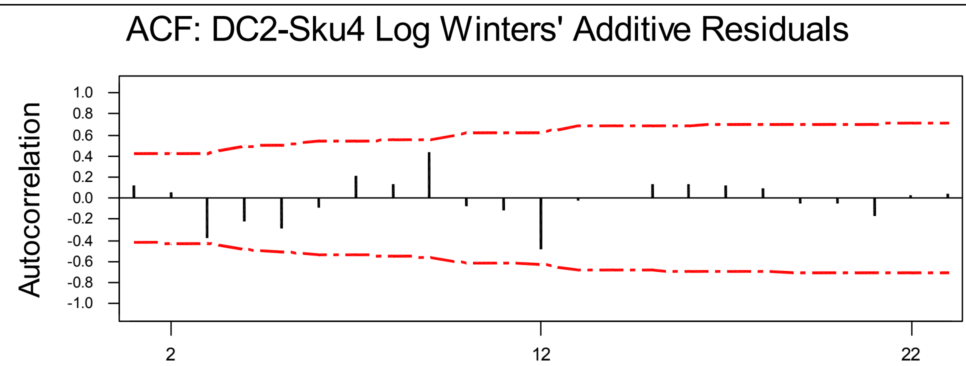
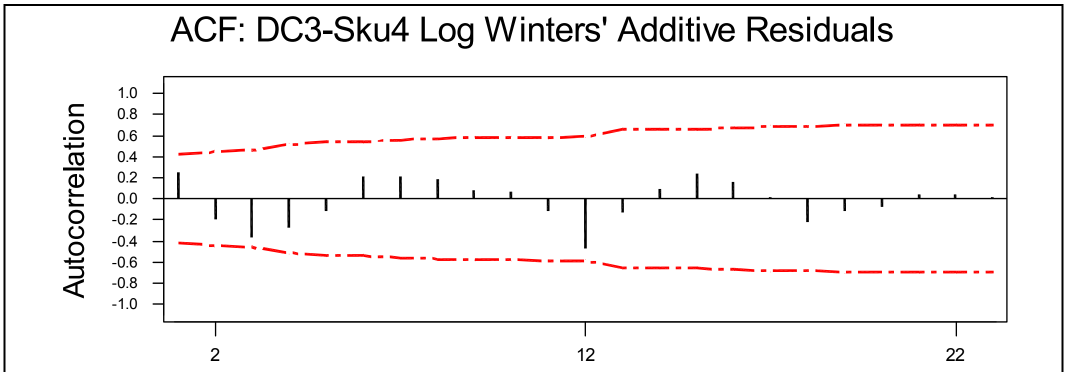


Figure 45: Normal Plot of DC2-Sku3 Log Seasonal Exponential Smoothing Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.12	0.58	0.38	8	0.13	0.49	12.06	15	0.13	0.39	34.70	22	0.02	0.06	45.59
2	0.06	0.28	0.48	9	0.44	1.63	20.17	16	0.14	0.41	36.19	23	0.04	0.11	46.54
3	-0.39	-1.88	5.02	10	-0.08	-0.26	20.43	17	0.12	0.35	37.43				
4	-0.22	-0.95	6.58	11	-0.12	-0.41	21.17	18	0.09	0.27	38.28				
5	-0.29	-1.20	9.41	12	-0.49	-1.61	33.46	19	-0.05	-0.15	38.60				
6	-0.09	-0.36	9.71	13	-0.03	-0.09	33.51	20	-0.05	-0.16	39.04				
7	0.22	0.83	11.40	14	-0.00	-0.01	33.51	21	-0.18	-0.51	45.45				

Figure 46: Correlogram of DC2-Sku4 Log Winters Additive Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.25	1.20	1.64	8	0.18	0.65	14.09	15	0.23	0.73	32.40	22	0.04	0.12	43.18
2	-0.20	-0.95	2.82	9	0.08	0.29	14.37	16	0.16	0.49	34.41	23	0.02	0.06	43.41
3	-0.37	-1.64	6.86	10	0.07	0.25	14.58	17	0.01	0.03	34.42				
4	-0.27	-1.10	9.17	11	-0.12	-0.43	15.29	18	-0.22	-0.68	39.63				
5	-0.12	-0.45	9.61	12	-0.48	-1.67	27.08	19	-0.12	-0.34	41.29				
6	0.22	0.83	11.26	13	-0.13	-0.41	28.03	20	-0.08	-0.23	42.27				
7	0.21	0.77	12.83	14	0.10	0.31	28.63	21	0.04	0.13	42.69				

Figure 47: Correlogram of DC3-Sku4 Log Winters Additive Residuals

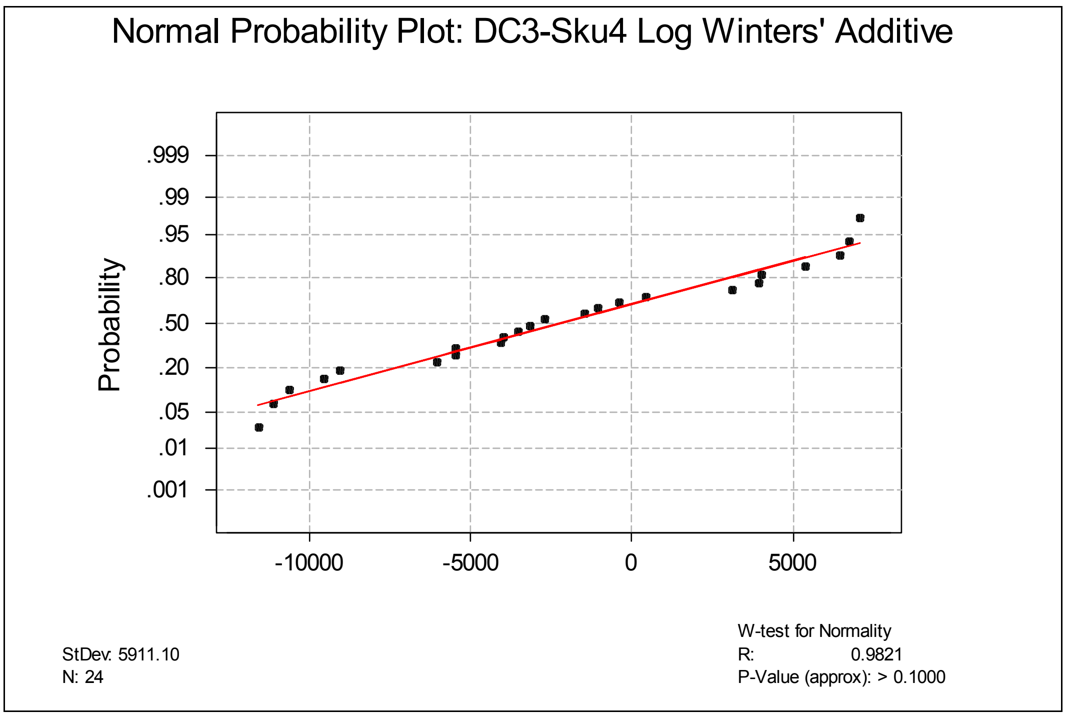
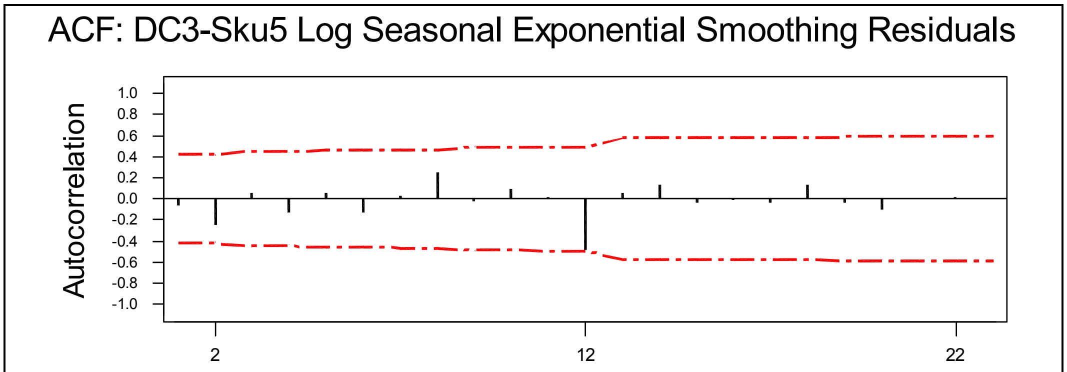


Figure 48: Normal Plot of DC3-Sku4 Log Winters Additive Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	-0.06	-0.31	0.11	8	0.25	1.12	5.81	15	-0.04	-0.13	20.31	22	0.02	0.07	24.75
2	-0.25	-1.23	1.91	9	-0.02	-0.10	5.84	16	-0.01	-0.05	20.33	23	0.01	0.02	24.77
3	0.05	0.23	1.98	10	0.10	0.40	6.24	17	-0.04	-0.14	20.47				
4	-0.13	-0.61	2.53	11	0.02	0.06	6.25	18	0.14	0.49	22.48				
5	0.05	0.23	2.62	12	-0.49	-2.07	18.92	19	-0.04	-0.15	22.69				
6	-0.14	-0.63	3.29	13	0.05	0.18	19.06	20	-0.11	-0.39	24.63				
7	0.03	0.12	3.31	14	0.14	0.49	20.23	21	0.01	0.02	24.64				

Figure 49: Correlogram of DC3-Sku5 Log Seasonal Exponential Smoothing Residuals

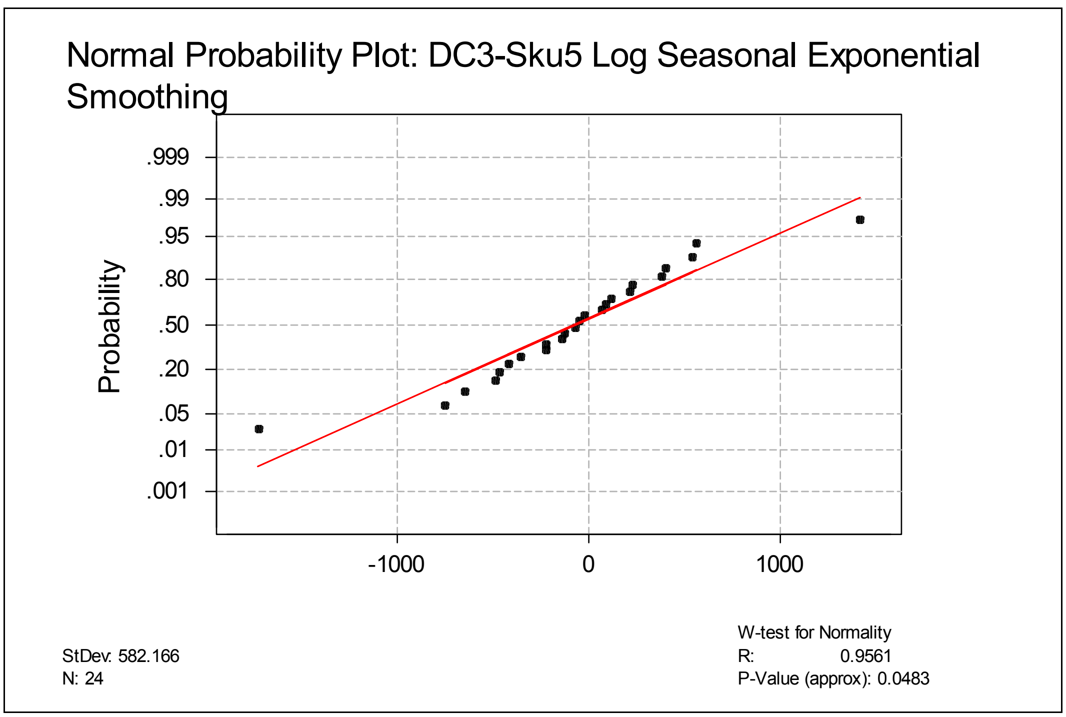
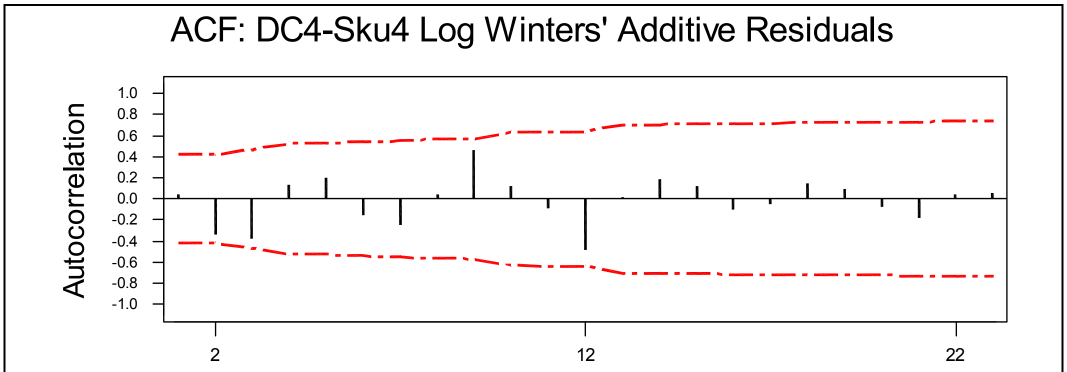


Figure 50: Normal Plot of DC3-Sku5 Log Seasonal Exponential Smoothing Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.04	0.18	0.04	8	0.04	0.15	12.90	15	0.11	0.33	38.60	22	0.04	0.10	51.89
2	-0.34	-1.66	3.29	9	0.46	1.68	21.80	16	-0.10	-0.29	39.40	23	0.05	0.13	53.27
3	-0.39	-1.71	7.76	10	0.12	0.38	22.41	17	-0.06	-0.16	39.67				
4	0.13	0.53	8.33	11	-0.09	-0.30	22.83	18	0.15	0.43	42.04				
5	0.21	0.80	9.72	12	-0.49	-1.59	35.52	19	0.10	0.27	43.18				
6	-0.15	-0.59	10.55	13	0.01	0.04	35.53	20	-0.08	-0.22	44.12				
7	-0.25	-0.94	12.84	14	0.19	0.55	37.72	21	-0.19	-0.53	51.47				

Figure 51: Correlogram of DC4-Sku4 Log Winters Additive Residuals

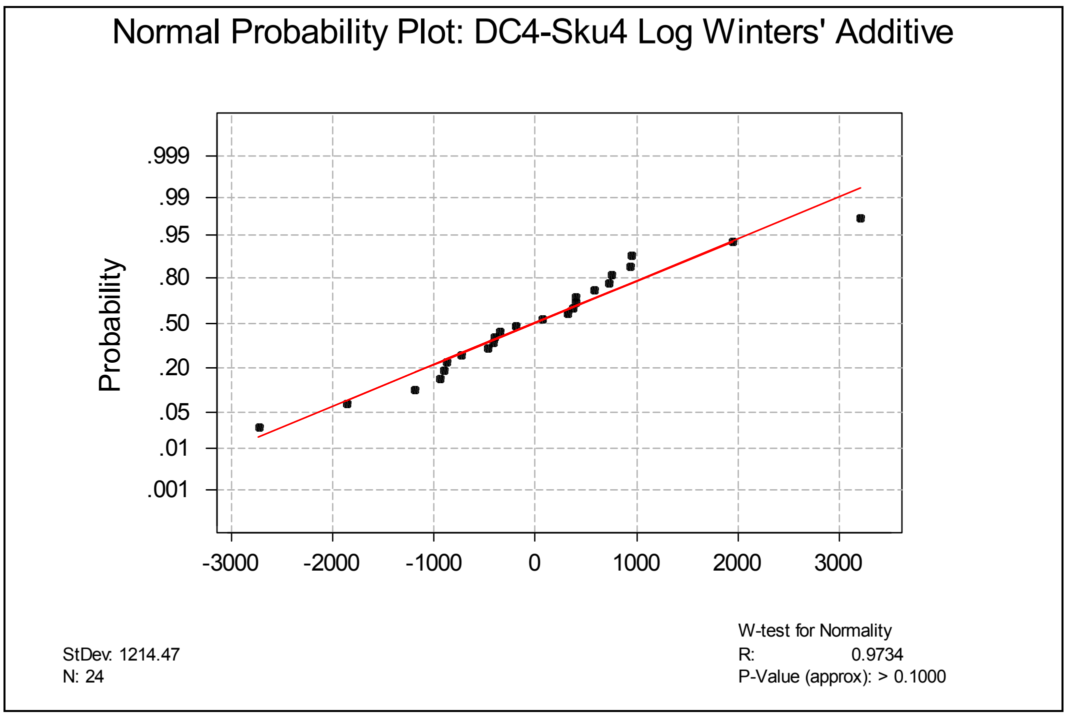
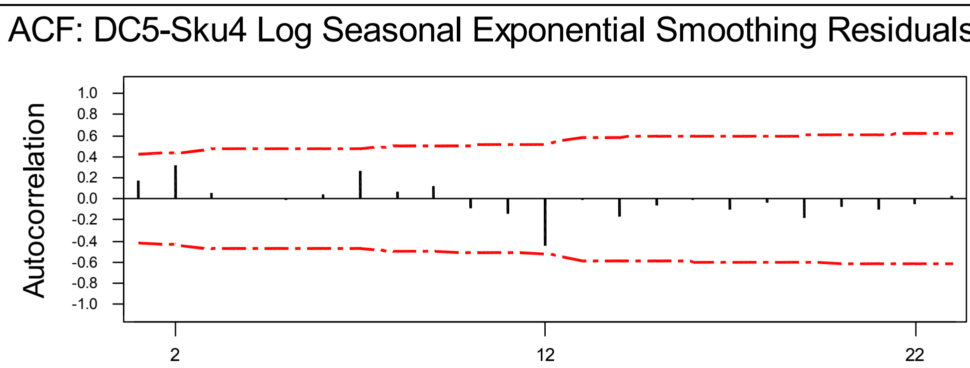


Figure 52: Normal Plot of DC4-Sku4 Log Winters Additive Residuals



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.18	0.86	0.84	8	0.06	0.25	6.62	15	-0.07	-0.25	21.76	22	-0.05	-0.18	31.58
2	0.32	1.54	3.79	9	0.12	0.49	7.22	16	-0.01	-0.03	21.77	23	0.03	0.09	32.07
3	0.06	0.25	3.90	10	-0.09	-0.36	7.58	17	-0.11	-0.37	22.76				
4	-0.01	-0.03	3.90	11	-0.15	-0.60	8.64	18	-0.04	-0.14	22.93				
5	-0.01	-0.06	3.90	12	-0.45	-1.81	19.40	19	-0.18	-0.63	27.12				
6	0.04	0.15	3.95	13	-0.02	-0.06	19.41	20	-0.09	-0.29	28.25				
7	0.26	1.14	6.47	14	-0.18	-0.63	21.39	21	-0.11	-0.36	30.66				

Figure 53: Correlogram of DC5-Sku4 Log Seasonal Exponential Smoothing Residuals

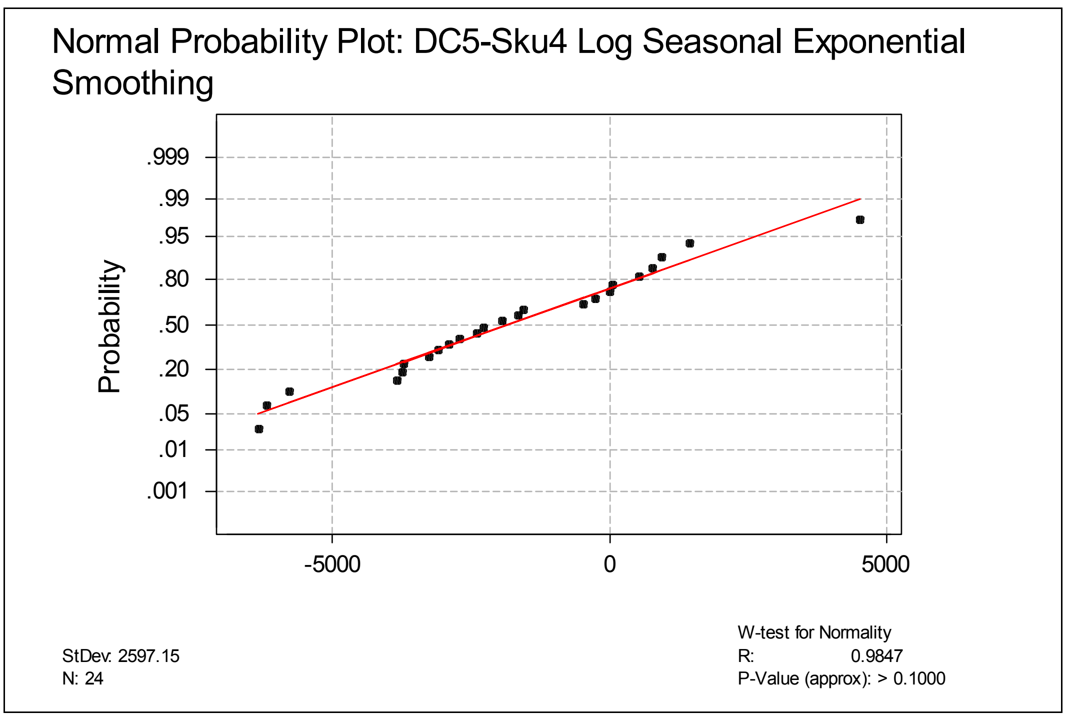


Figure 54: Normal Plot of DC5-Sku5 Log Seasonal Exponential Smoothing Residuals

APPENDIX 6

Normality Plots for the Non-Transformed Smoothing Models in Table 5[13]

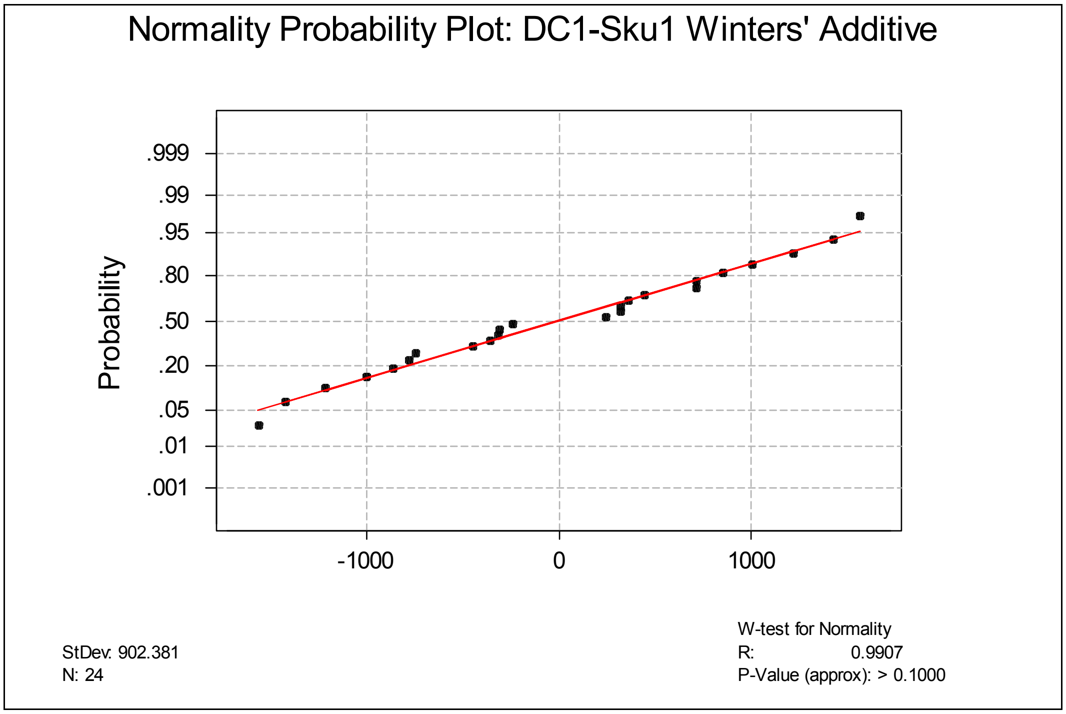


Figure 55: Normal Plot of DC1-Sku1 Winters Additive Residuals

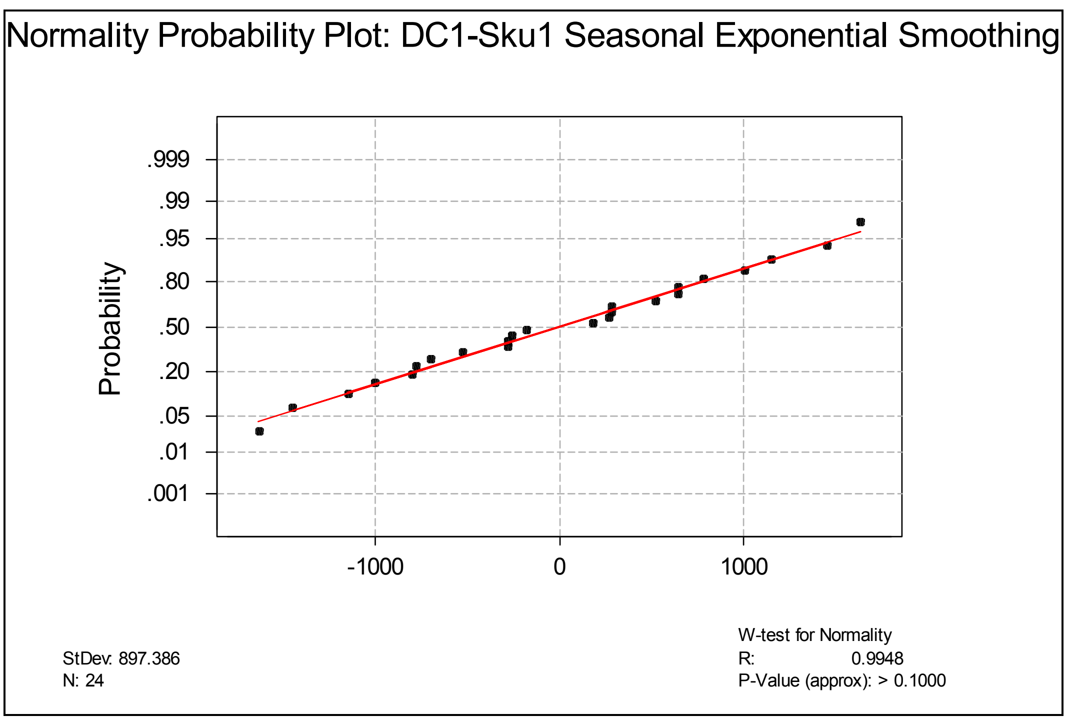


Figure 56: Normal Plot of DC1-Sku1 Seasonal Exponential Smoothing Residuals

[13] These models were found to be independent before the normality tests were performed.

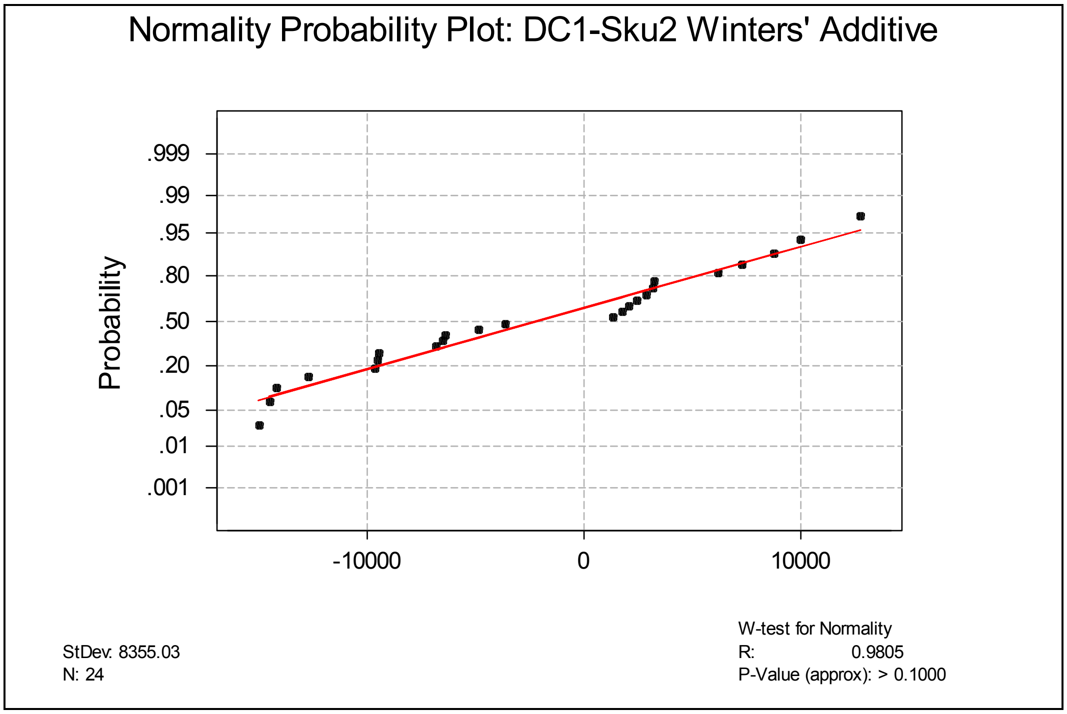


Figure 57: Normal Plot of DC1-Sku2 Winters Additive Residuals

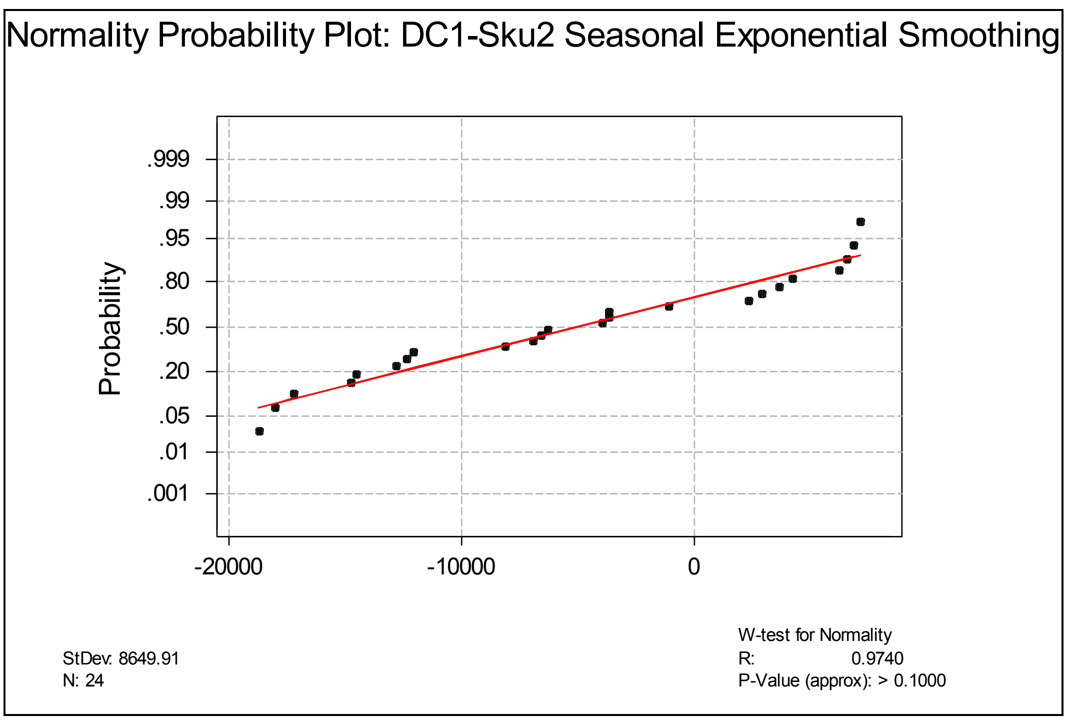


Figure 58: Normal Plot of DC1-Sku2 Seasonal Exponential Smoothing Residuals

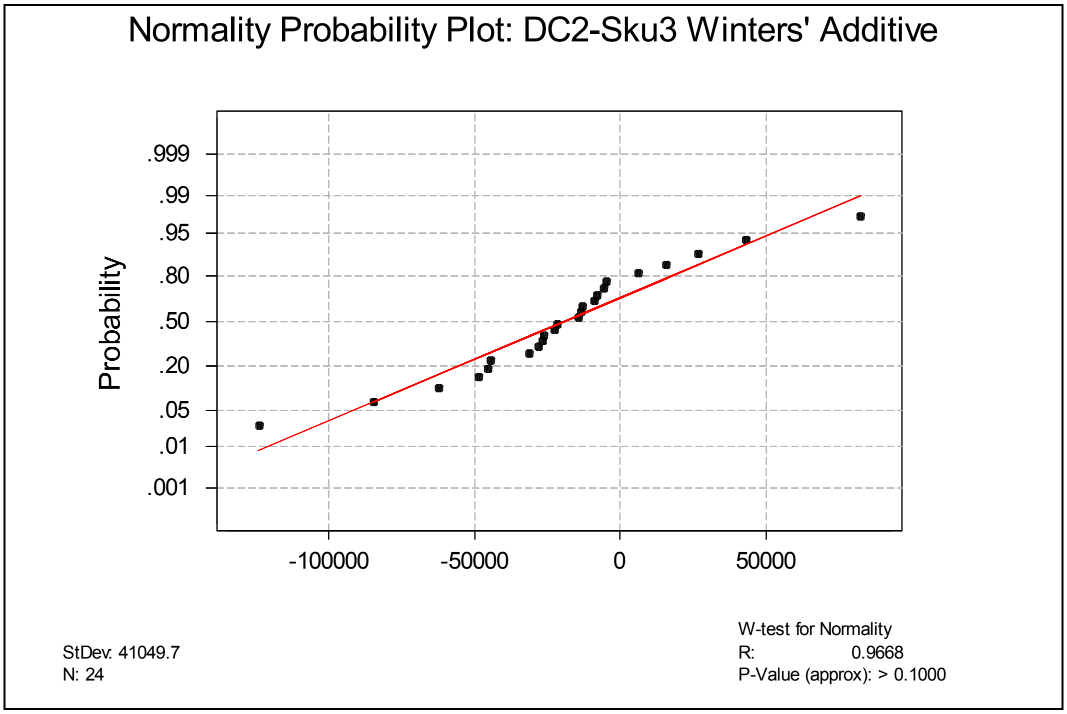


Figure 59: Normal Plot of DC2-Sku3 Winters Additive Residuals

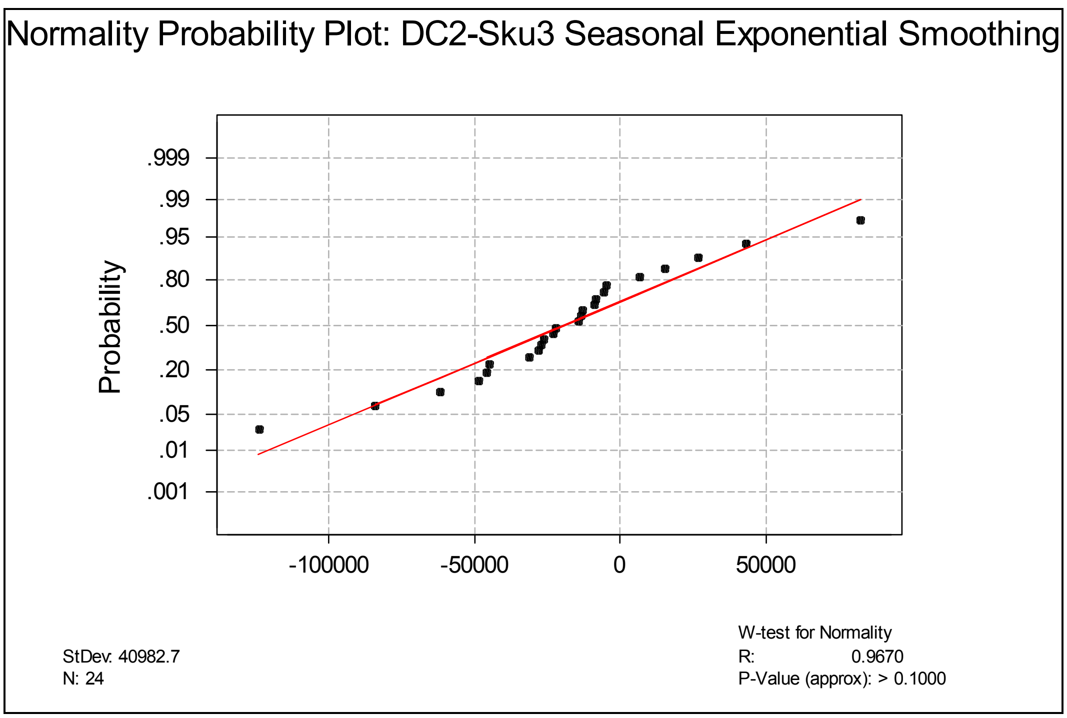


Figure 60: Normal Plot of DC2-Sku3 Seasonal Exponential Smoothing Residuals

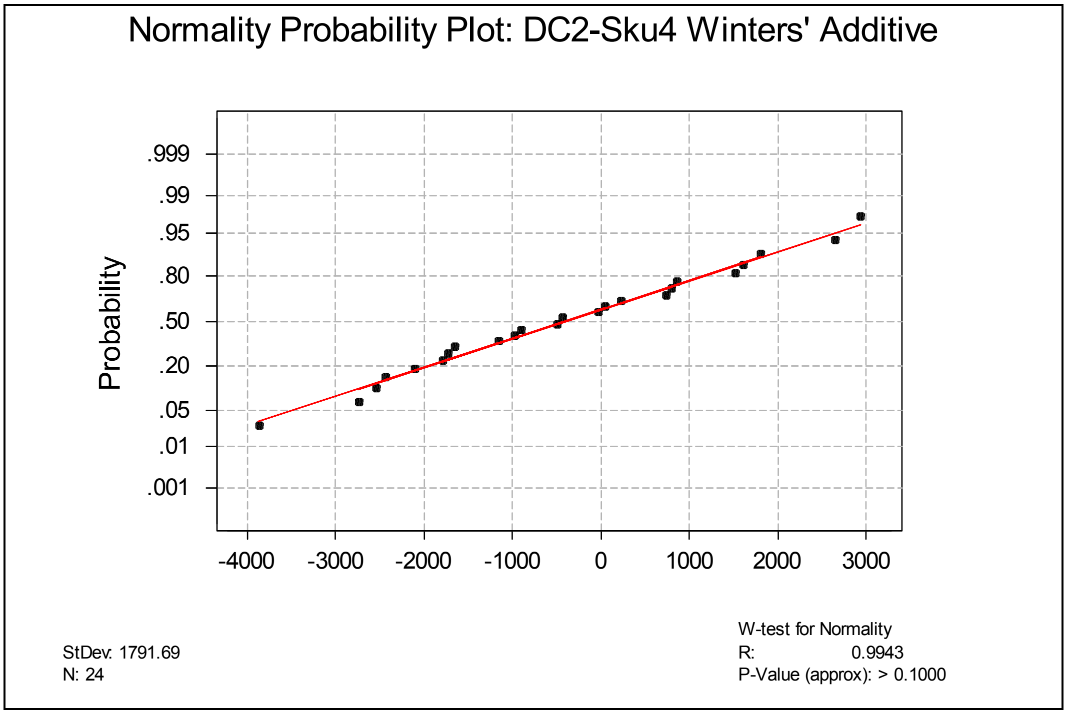


Figure 61: Normal Plot of DC2-Sku4 Winters Additive Residuals

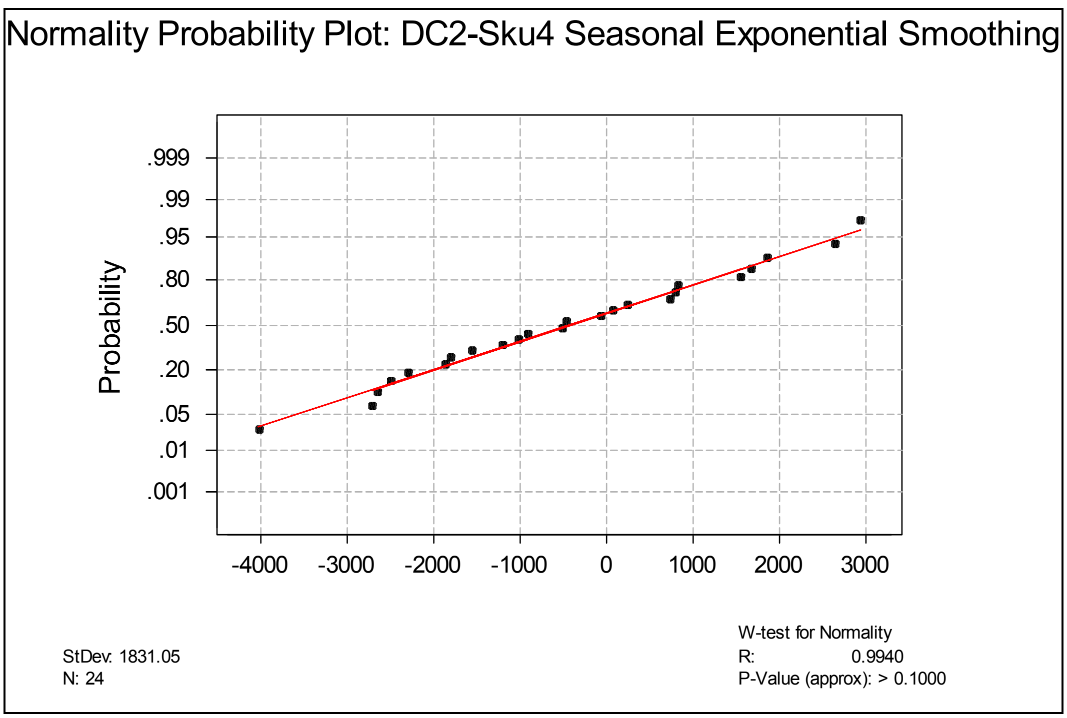


Figure 62: Normal Plot of DC2-Sku4 Seasonal Exponential Smoothing Residuals

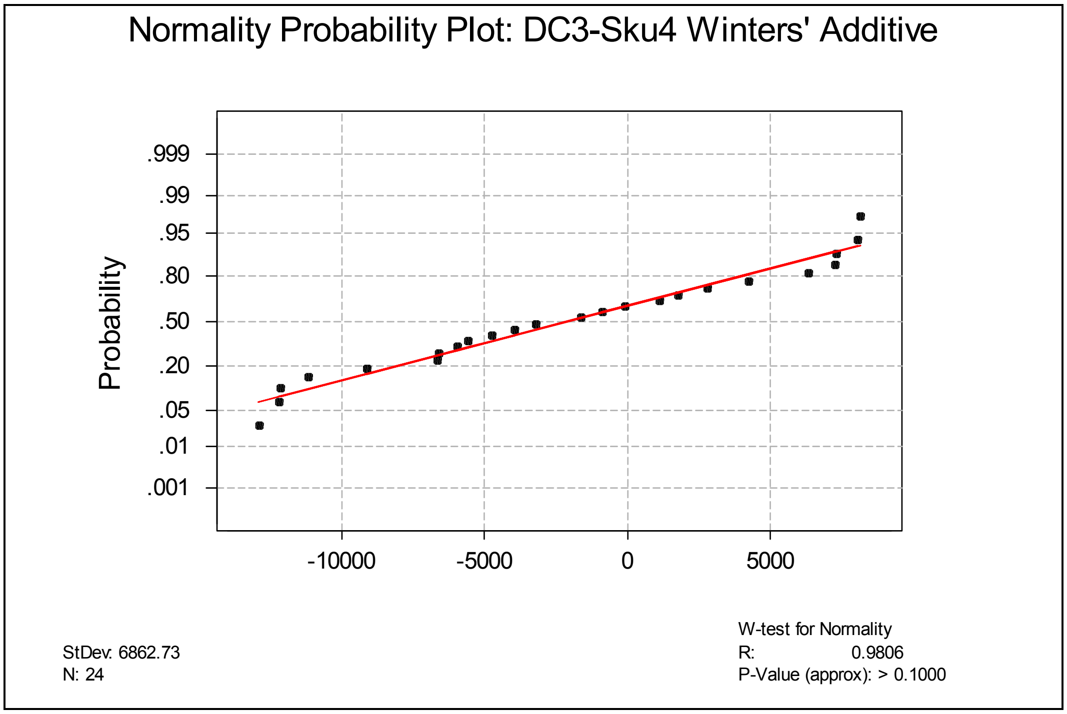


Figure 63: Normal Plot of DC3-Sku4 Winters Additive Residuals

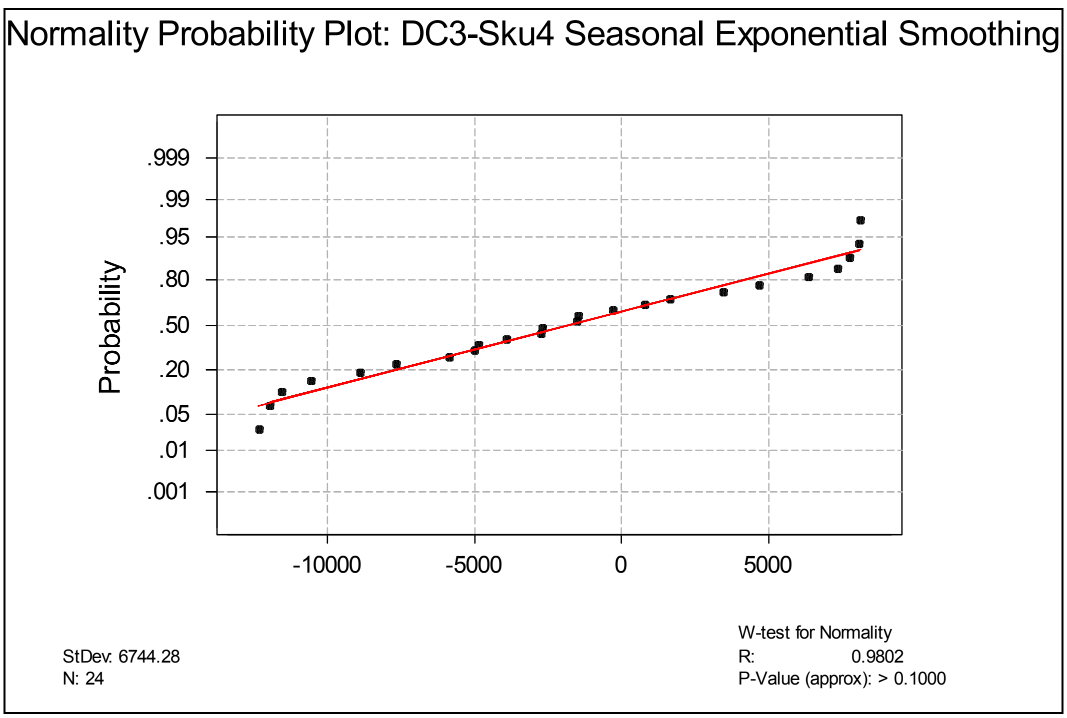


Figure 64: Normal Plot of DC3-Sku4 Seasonal Exponential Smoothing Residuals

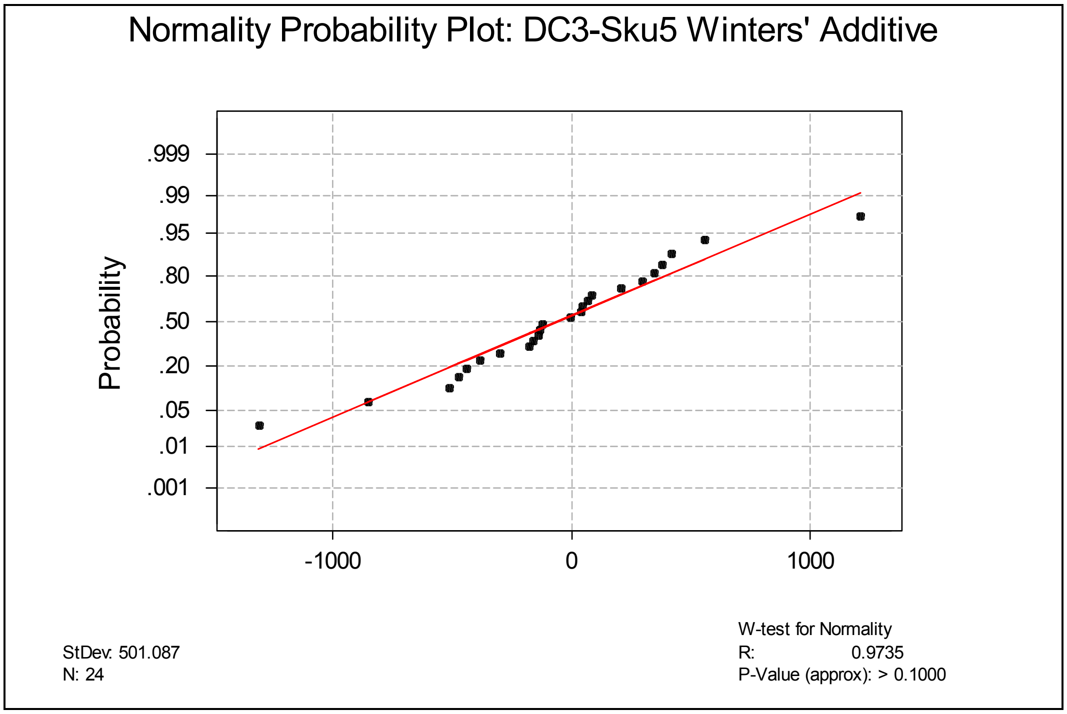


Figure 65: Normal Plot of DC3-Sku5 Winters Additive Residuals

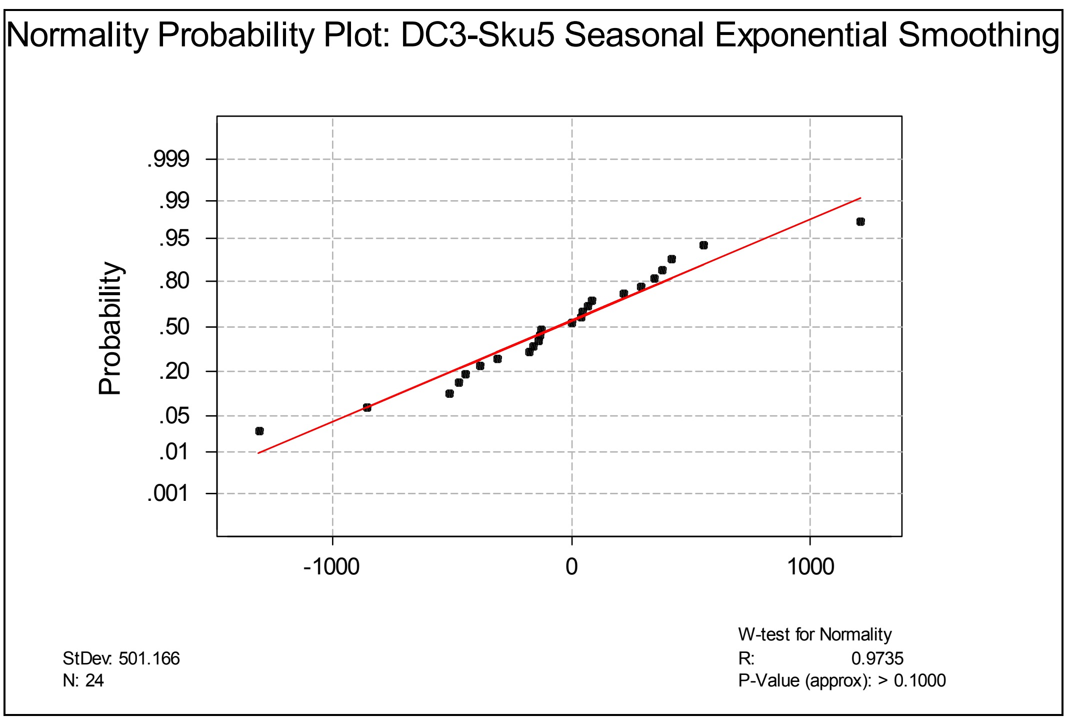


Figure 66: Normal Plot of DC3-Sku5 Seasonal Exponential Smoothing Residuals

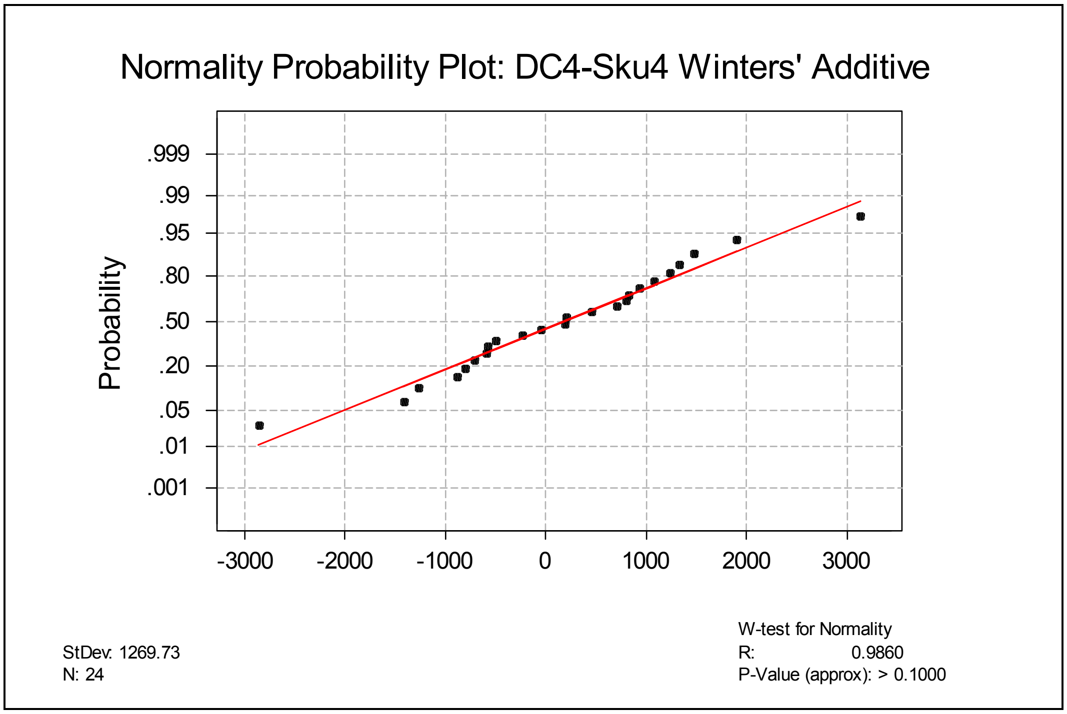


Figure 67: Normal Plot of DC4-Sku4 Winters Additive Residuals

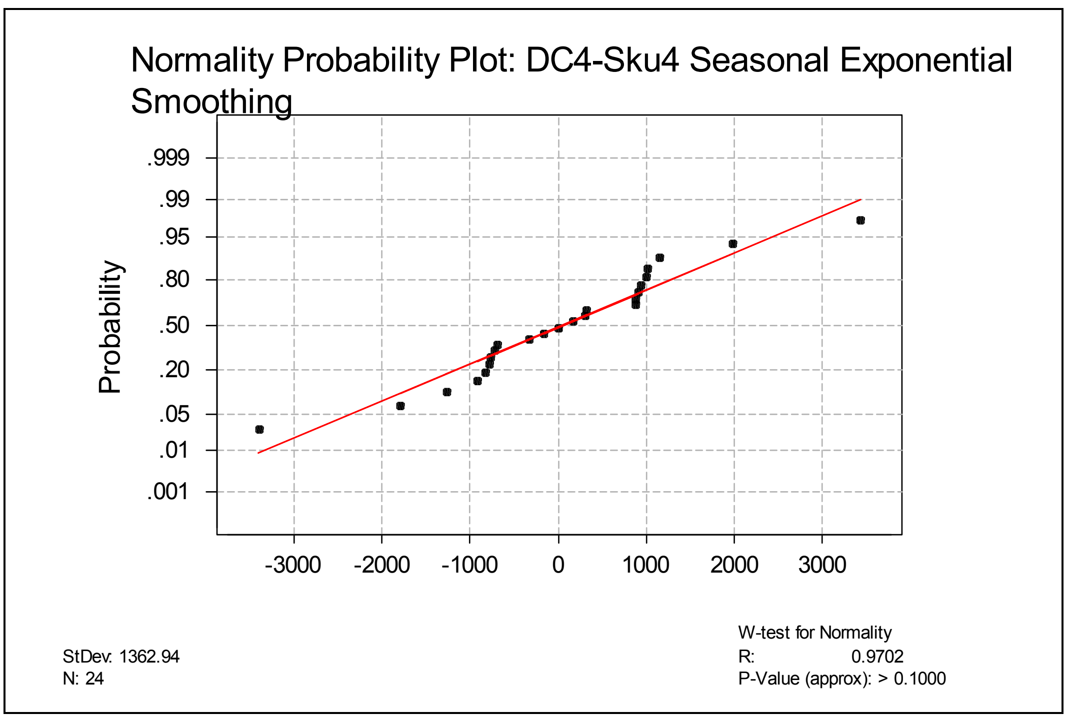


Figure 68: Normal Plot of DC4-Sku4 Seasonal Exponential Smoothing Residuals

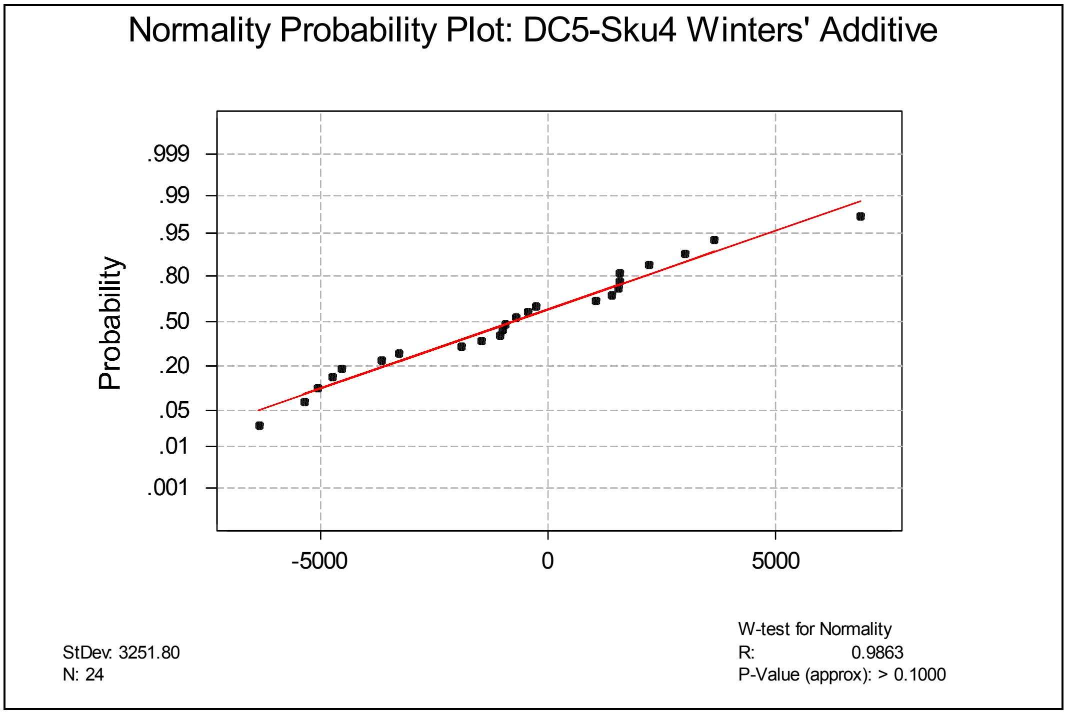


Figure 69: Normal Plot of DC5-Sku4 Winters Additive Residuals

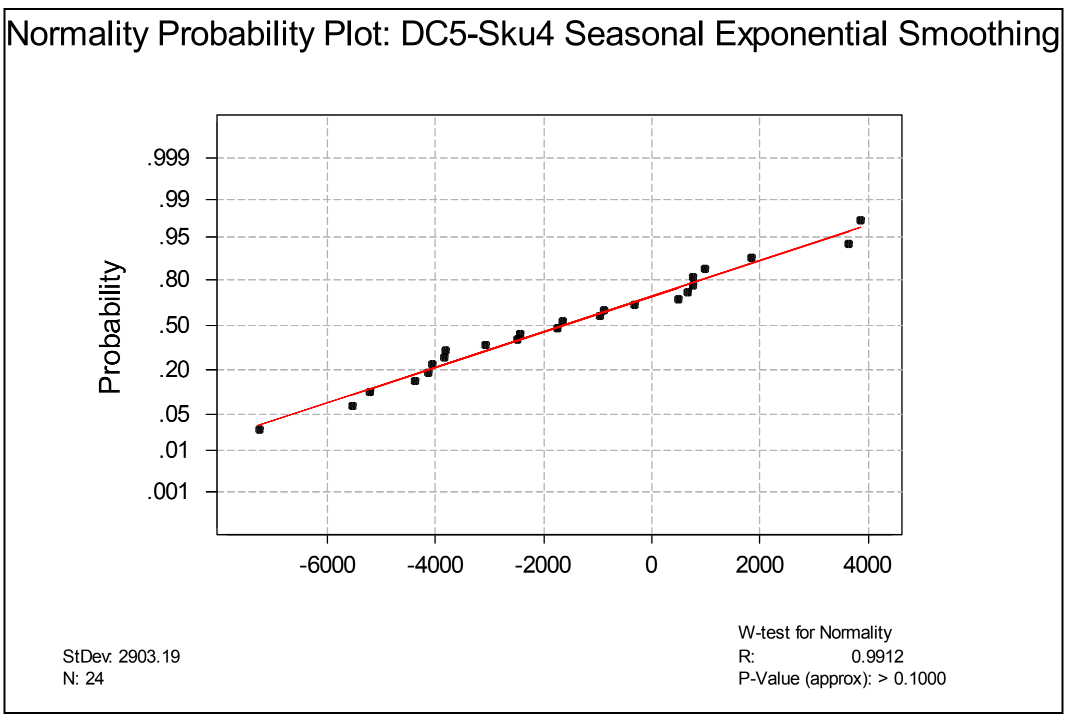


Figure 70: Normal Plot of DC5-Sku4 Seasonal Exponential Smoothing Residuals

APPENDIX 7

Average Temperature Time Series Plots.

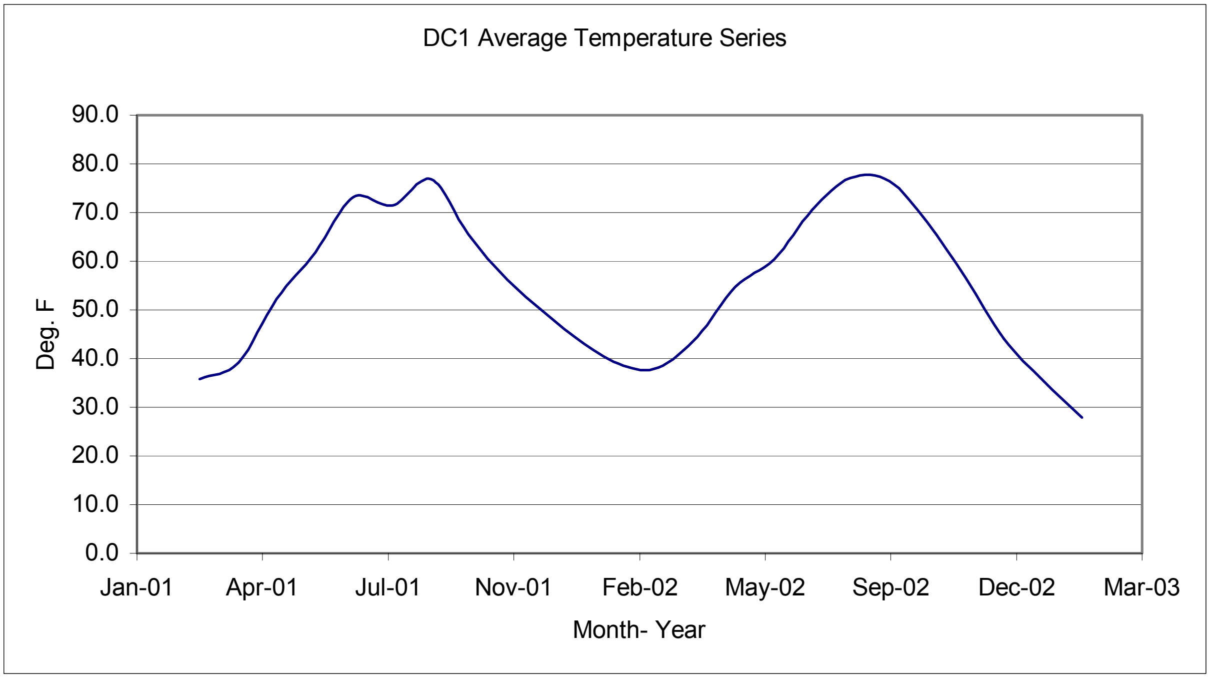


Figure 71: Average Temperature Time Series Plot for the DC1 region.

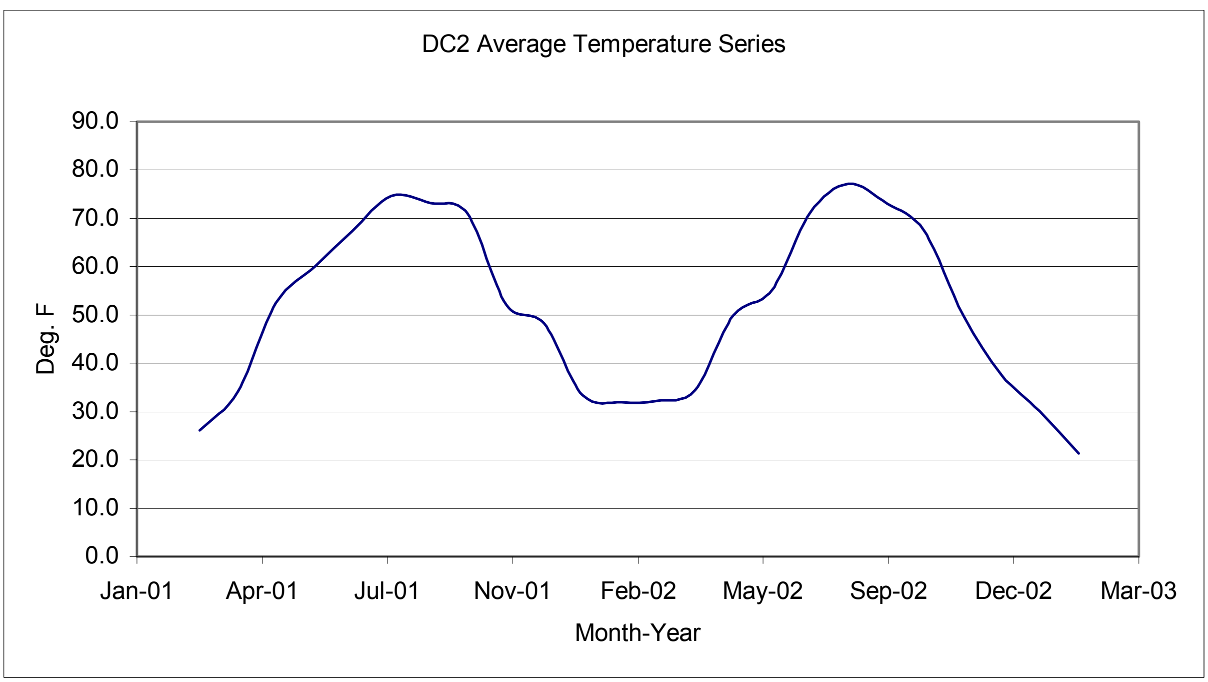


Figure 72: Average Temperature Time Series Plot for the DC2 region.

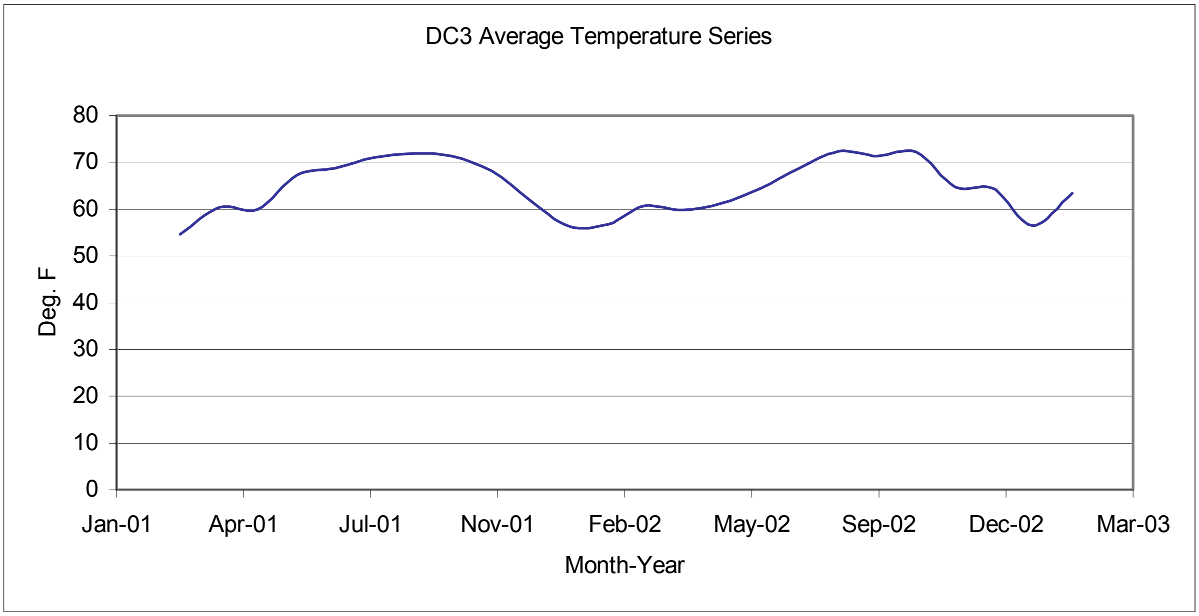


Figure 73: Average Temperature Time Series Plot for the DC3 region.

APPENDIX 8

Minitab Output of Step-wise regression of Housing Starts, Average Monthly Temperature and Price on POS.

Stepwise Regression: DC1-SKU1 versus Temp, Price, Housing Starts

```
Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

Response is DC1-SKU1 POS on 3 predictors


    Step          1         2         3
Constant    24010.5   21457.3    -592.1

Temp            352       304       250
T-Value        1.81      3.43      3.00
P-Value       0.113     0.009     0.015

Price        -39782    -37745
T-Value       -1.32     -1.38
P-Value       0.227     0.206

Housing        -283
T-Value       -0.28
P-Value       0.789

S              4397      4135      4337
R-Sq          60.06     59.62     50.05
R-Sq(adj)     42.94     49.53     44.50
C-p             4.0       2.1       1.8
PRESS      3.61E+08  2.62E+08  2.60E+08
R-Sq(pred)     0.00     22.63     23.23
```

Stepwise Regression: DC1-SKU2 versus Temperature, Price, Housing Starts

```
Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

Response is DC1-SKU2 POS on 3 predictors

    Step          1        2        3
Constant     788543   750465   710458

Temp          -3129    -3209    -3803
T-Value       -1.47    -1.66    -3.89
P-Value       0.185    0.135    0.004

Price        -18838
T-Value       -0.15
P-Value       0.887

Housing       -2438    -2360
T-Value       -0.35    -0.36
P-Value       0.736    0.726

S             66061    61889    58830
R-Sq          63.46    63.34    62.74
R-Sq(adj)     47.79    54.18    58.60
C-p             4.0      2.0      0.1
PRESS      9.21E+10 7.13E+10 4.55E+10
R-Sq(pred)     0.00    14.67    45.58
```

Stepwise Regression: DC2-SKU3 versus Temperature Price, Housing Starts

```
Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

Response is DC2-SKU3 POS on 3 predictors


    Step           1        2        3
Constant      788543   750465   710458

Temp           -3129    -3209    -3803
T-Value        -1.47    -1.66    -3.89
P-Value        0.185    0.135    0.004

Price         -18838
T-Value        -0.15
P-Value        0.887

Housing        -2438    -2360
T-Value        -0.35    -0.36
P-Value        0.736    0.726

S              66061    61889    58830
R-Sq           63.46    63.34    62.74
R-Sq(adj)      47.79    54.18    58.60
C-p              4.0      2.0      0.1
PRESS       9.21E+10 7.13E+10 4.55E+10
R-Sq(pred)      0.00    14.67    45.58
```

Stepwise Regression: DC2-SKU4 versus Temperature, Price, Housing Starts

```
Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

Response is DC2-SKU4 on 3 predictors


    Step          1        2        3
Constant   -49448.9 -60309.5    804.7

Temp             86
T-Value        0.40
P-Value       0.702

Price          6170     7095
T-Value        1.23     1.69
P-Value       0.257    0.129

Housing         872     1114     1140
T-Value        1.26     3.44     3.21
P-Value       0.249    0.009    0.011

S              6122     5791     6362
R-Sq          66.42    65.65    53.37
R-Sq(adj)     52.03    57.07    48.19
C-p             4.0      2.2      2.7
PRESS      7.22E+08 4.29E+08 4.82E+08
R-Sq(pred)     7.54    45.03    38.31
```

Stepwise Regression: DC3-SKU4 versus Temperature, Price, Housing Starts

```
Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

Response is DC3-SKU4 on 3 predictors



    Step          1        2        3        4
Constant     -33178   -15420    32118   112690

Temp            764      878
T-Value        0.56     0.89
P-Value       0.596    0.404

Price          2956
T-Value        0.13
P-Value       0.898

Housing       1914     1951     2239
T-Value        1.17     1.31     1.56
P-Value       0.285    0.232    0.158

S             16131    14956    14758    15884
R-Sq          31.24    31.04    23.27     0.00
R-Sq(adj)      0.00    11.33    13.68     0.00
C-p             4.0      2.0      0.7      0.7
PRESS      4.60E+09 3.54E+09 2.94E+09 2.80E+09
R-Sq(pred)     0.00     0.00     0.00     0.00
```

Stepwise Regression: DC3-SKU5 versus Temperature, Price Housing Starts

```
 Alpha-to-Enter: 0.1 Alpha-to-Remove: 0.1

 Response is DC3-SKU5 POS on 3 predictors


    Step         1        2
Constant      5586     7316

Temp            54
T-Value       0.87
P-Value      0.416

Price        -1104     -983
T-Value      -2.39    -2.27
P-Value      0.054    0.057

Housing        160      177
T-Value       1.80     2.08
P-Value      0.122    0.076

S              887      872
R-Sq         61.86    57.01
R-Sq(adj)    42.79    44.73
C-p            4.0      2.8
PRESS      9485493  8428663
R-Sq(pred)   23.42    31.95
```