SMiRT-12 / K. Kussmaul (Editor)

L09/2

A CYCLIC CONSTITUTIVE LAW FOR METALS WITH A SEMI-DISCRETE MEMORY VARIABLE FOR DESCRIPTION OF RATCHETING PHENOMENA

S. Andrieux, P. Schoenberger and S. Taheri

Département Mécanique et Modèles Numériques, Etudes et Recherches, Electricité de France, 1 Avenue du Général de Gaulle, F-92141 Clamart Cedex, France

1. INTRODUCTION

The study of cyclic elastoplastic constitutive laws is, at the moment, focused on non proportional loadings, but for uniaxial loadings some problems remain, as for example the ability for a law to describe simultanously ratcheting in non symmetrical load-controlled test, elastic and plastic shakedown in symmetrical and non symmetrical ones. We have proposed in [1] a law with a discrete memory variable which, in addition to previous phenomena, describes the cyclic hardening in a pushpull test, and the cyclic softening after overloading. A modified law has been proposed in [2] to take into account the dependence of cyclic strain stress curve on the history of loading. The extension to 3D situations of this law is proposed in [3]. The discrete nature of the memory leads to discontinuity problems for some loading paths, a modification is then proposed which uses a differential evolution law. For large enough uniaxial cycles, the uniaxial law is nevertheless recovered. In this paper, an incremental form of the implicit evolution problem is given, and we describe the implementation of this model in the *Code Aster* © - a thermomechanical structural software using the f.e.m developed at Electricite de France. Comparison between experiment and numerical results is given for uniaxial ratcheting, non proportional strain controlled test.

2 . THE UNIAXIAL LAW

The proposed law in [1], [2], uses besides the usual plasticity variables, an internal discrete memory variable: namely the plastic strain at the last unloading, denoted by ε^p_n, and the maximal past stress σ_p. It also uses one material parameter named ratcheting stress at which in uniaxial case ratcheting (constant strain increment) appears. Macroscopic variables and also ratcheting stress are deduced from a microscopic analysis. We define now more precisely the macroscopic variables in relation with the microscopic analysis:

- ε^p usual plastic part of the deformation , related to the gliding of dislocations,
- λ cumulated plastic strain, related to the density of dislocations,
- σ_p maximal past absolute value of stress supported by the material in his history, related to the actual mean size of cells, this variable is used partly as $S_r - \sigma_p$,where S_r is the ratcheting stress.
- ε^p_n plastic deformation at the last unloading point, here the significant variable

is the difference $\varepsilon^p - \varepsilon^p_n$, on stabilized cycles, it measures the amplitude of plastic deformation.

The uniaxial constitutive law is now simply described by an elastoplastic model where yield function combines isotropic and kinematic hardening:

$$F(\sigma, \varepsilon^p, \lambda, \sigma_p, \varepsilon^p_n) = |\sigma_D - \mathbf{X}(\varepsilon^p, \lambda, \sigma_p)| - R(\lambda, \sigma_p, |\varepsilon^p_n - \varepsilon^p|)$$

The evolution equations for internal variables σ_p and ε^p_n follow directly from their definition, whereas the usual normality and consistency relations are used for the remaining variables ε^p and λ . Simulation of different cyclic phenomena have been presented for the simple choices of the yield function F, in [1] and [2].

3 . THREE DIMENSIONAL LAW

The extension to three dimensionnal situations of the previous uniaxial law encounters two main difficulties. First, choices have to be made on the variables themselves : Is the small number of internal variables sufficient for 3D situations? To this first question, although some answers can be gained by extra microscopic analysis, we choose here, for the sake of simplicity ,as frequently done, the deviatoric part of the tensors, to keep the uniaxial law's general form. The second class of difficulties arise from extension of the definitions and evolution equations of the memory variables σ_p and ε^p_n. . As a matter of fact, the uniaxial loadings histories are very poor : there is no tangent loadings, cycling is only defined by two extreme values etc..., furthermore in applications incremental time integration is rarely performed because advantage is taken of algebraic relations between extremal states. Intrinsic definitions and more precise evolutions laws are then needed in the 3D case. The variable σ_p is defined as the maximal past deviatoric norm of the stress experienced by the material - the norm is denoted by $|\sigma_D|$ - With initial value σ_p^o of σ_p, the precise definition is :

$$\sigma_p(t) = \left(\underset{u \in [0,t]}{\mathrm{Max}}\right)(\sigma_p^o, |\sigma_D(u)|)$$

and we can rewrite this definition as a new yield function G in the deviatoric stress space :

$$G(\varepsilon^p, \lambda, \sigma_p, \varepsilon^p_n) = \sigma_p - |\mathbf{X}(\varepsilon^p, \lambda, \sigma_p)| - R(\lambda, \sigma_p, |\varepsilon^p_n - \varepsilon^p|)$$

leading to the evolution equation for σ_p (H is the Heavyside function):

$$\dot\sigma_p = H(|\sigma_D| - \sigma_p)\frac{\sigma_D \dot\sigma}{|\sigma_D|}$$

Two questions arise from the definition of the evolution law of ε^p_n . One is common with cyclic memory variables: do the stress-strain curve in a partial elastic unloading and reloading, fit the initial monotonic curve? In other words, can the description of the material behavior admit some undershooting of the monotonic stress-strain curve after an elastic unloading? The answer to this question is not straightforward and can depend on the material and perhaps on the kind of stress, so that we do not address to this problem here. The second question, more important from a physical point of view, is the requirement of continuity of the stress-strain curve with respect to very small unloadings. With full discrete memory, this requirement is generally not fullfiled : any unloading, even as small as possible, leads to an (discontinuous) evolution of the memory variable which induces in turn a discontinuity on the value of the yield function F. This last discontinuity can finally cause the violation of the yield condition $F< 0$.

For 3D loading paths, this problem is of primary importance because *micro-unloadings* can result from changes of direction of the loading path in the stress space.To overcome this last difficulty, we modify the discrete evolution law for ε^p_n to a semi discrete one - the word semi-discrete is used because of the saturation of the memory ensuing from the definition of the evolution -. Starting from the discrete model :

$$\Delta\varepsilon^p_n = \varepsilon^{p+}_n - \varepsilon^{p-}_n = \varepsilon^p - \varepsilon^{p-}_n \qquad \text{if} \qquad F = 0 \qquad (\dot\sigma\frac{\partial F}{\partial\sigma}) \le 0$$

we introduce a scalar differential evolution equation together with a consistency condition ensuring the fullfilment of the yield condition:

$$\dot\varepsilon^p_n = \alpha(\varepsilon^p - \varepsilon^p_n) \qquad \text{if} \qquad F = 0 \qquad (\dot\sigma\frac{\partial F}{\partial\sigma}) \le 0$$

$$\alpha \ge 0 \qquad \alpha F = 0 \qquad F \le 0$$

It can be seen that, with appropriate *generalized hardening conditions* on the yield function F we have:

- the yield condition $(F < 0)$ is never violated,
- the continuity with respect to the chronology parameter is restored,
- the memory shows a *saturation effect*: when, during the unloading, the value of ε^p_n reaches ε^p, then ε^p_n stays at this value and the unloading becomes purely elastic with no internal variable evolution,
- **for uniaxial cycling loadings, the discrete memory is recovered between two successive unloadings, provided the cycle is large enough.**

4. THE IMPLICIT INTEGRATION OF THE 3D CONSTITUTIVES RATES EQUATIONS

The evolution equations of the remaining variables λ and ε^p are deduced from the usual normality and consitency relations. We see that when $|\sigma_D|$ does not reach σ_p the law reduces - in loading evolutions - to the standard plasticity laws. Non standard flow rules are obtained when σ_p varies during loading (but normality for the rate of ε^p still remains).

To write down extensively the proposed constitutive law, we present hereafter it implicit incremental form which will be used in computations. We denote by $\mathbf{e}$ the "state" $(\varepsilon^p, \lambda, \sigma_p, \varepsilon^p_n)$, by **A** the elasticity tensor and by **X** the backstress tensor.

In order to integrate rate equations of the constitutive model over the increment Δt :

- we integrate the flow rule by the backward difference sheme :

$$\Delta\varepsilon^p = \Delta\lambda \left.\frac{\partial F}{\partial\sigma}\right|_{t+\Delta t}$$

- we impose respect of the yield function F when plasticity occurs , at the end of increment : $F_{t+\Delta t} = 0$
- equations of elasticity are written at the end of increment : $\sigma_{t+\Delta t} = \mathbf{A}_{t+\Delta t}\, \varepsilon^e_{t+\Delta t}$

- rate of ε^p_n is integrated by the forward difference scheme in order to respect $\varepsilon^p_{nt+\Delta t} = \varepsilon^p_{nt} + \Delta\varepsilon^p_n = \varepsilon^p_t$ at the end of increment , so we have :

$$\Delta\varepsilon^p_n = \Delta\alpha\ (\ \varepsilon^p - \varepsilon^p_n\ )_t$$

- we impose respect of the *pic function* G when σ_p varies during loading :

$$G_{t+\Delta t} = \sigma_{pt+\Delta t} - |\mathbf{X}|_{t+\Delta t} - R_{t+\Delta t} = 0$$

So , we can exhibit four types of increment for this model , by opposition with the two states, elastic and elasto-plastic, of a standard plasticity model :

- A *Purely Elastic* increment (E), where only the variable σ is incremented,
- A *Pseudo-Elastic* increment (PE),where ε^p_n and σ are actualized,
- An *Elasto-Plastic* increment (EP), where σ , λ and ε^p are actualized,
- And a *Pseudo-Elasto-Plastic* increment (PEP), where σ ,λ ,ε^p and σ_p are actualized.

The integration algorithm first make an *elastic prediction* , in order to decide if the increment is (E)/(PE) or (EP)/(PEP) , and then make a *plastic prediction* to choose beetwen (EP) or (PEP) integration. The algorithm presented hereafter has been implemented in the ***Code Aster*** © , developped at *Electricite de France* [4] . Making hypothesis of a purely elastic increment (H1), we solve :

```
$\sigma_{t+\Delta t} = \mathbf{A}_{t+\Delta t}\,\varepsilon^e_{t+\Delta t}$ , and we compute $F(\sigma_{t+\Delta t}, e_t)$
    if $F(\sigma_{t+\Delta t}, e_t) < 0$ then
        if $F(\sigma_t, e_t) < 0$ then
            the initial state is elastic and then (H1) is true so increment is elastic and integration is over.
        else if $F(\sigma_t, e_t) = 0$ then the initial state is plastic and
            if $\varepsilon^p_{nt} = \varepsilon^p_t$ then
                memory of last plastic deformation $\varepsilon^p_n$ is saturated and (H1) is true so increment is purely elastic and integration is over.
            else if $\varepsilon^p_{nt} \neq \varepsilon^p_t$ then we must actualize $\varepsilon^p_n$ and we solve
                $\Delta\varepsilon^p_n = \Delta\alpha(\varepsilon^p - \varepsilon^p_n)_t$
                $\sigma_{t+\Delta t} = \mathbf{A}_{t+\Delta t}(\varepsilon_{t+\Delta t} - \varepsilon^p_t)$        (S1)
                $F(\sigma_{t+\Delta t}, \varepsilon^p_{nt+\Delta t}, \varepsilon^p_t, \sigma_{pt}, \lambda_t) = 0$
                in order to obtain $\sigma_{t+\Delta t}$ and $\varepsilon^p_{nt+\Delta t}$ (PE increment)
            endif
        endif
    else if $F(\sigma_{t+\Delta t}, e_t) > 0$ then (H1) is false and we have a final EP state.
    We make the hypothesis (H2) of an elasto-plastic increment ( so $\Delta\sigma_p = 0$ )
    and we solve this system , in order to obtain $\sigma_{t+\Delta t}$ $\lambda_{t+\Delta t}$ $\varepsilon^p_{t+\Delta t}$
```

$$\boxed{\begin{array}{ll} \sigma_{t+\Delta t} = \mathbf{A}_{t+\Delta t}\left(\varepsilon_{t+\Delta t} - \varepsilon^p_t - \Delta\lambda \dfrac{\partial F}{\partial \sigma_{t+\Delta t}}\right) & \text{(S2)} \\ F(\sigma_{t+\Delta t}, \varepsilon^p_{nt}, \varepsilon^p_{t+\Delta t}, \sigma_{pt}, \lambda_{t+\Delta t}) = 0 & \end{array}}$$

```
and we compute $G_{t+\Delta t} = G(\varepsilon^p_{nt}, \varepsilon^p_{t+\Delta t}, \sigma_{pt}, \lambda_{t+\Delta t})$ so
    if $G_{t+\Delta t} > 0$ then (H2) is true and integration is over.
    else if $G_{t+\Delta t} < 0$ then(H2) is false and we have a PEP increment
    with $\Delta\sigma_p \neq 0$ , and so we solve :
```

$$\boxed{\begin{array}{ll} \sigma_{t+\Delta t} = \mathbf{A}_{t+\Delta t}\left(\varepsilon_{t+\Delta t} - \varepsilon^p_t - \Delta\lambda \dfrac{\partial F}{\partial \sigma_{t+\Delta t}}\right) & \\ F(\sigma_{t+\Delta t}, \varepsilon^p_{nt}, \varepsilon^p_{t+\Delta t}, \sigma_{pt}, \lambda_{t+\Delta t}) = 0 & \text{(S3)} \\ G(\varepsilon^p_{nt}, \varepsilon^p_{t+\Delta t}, \sigma_{pt+\Delta t}, \lambda_{t+\Delta t}) = 0 & \end{array}}$$

```
    in order to obtain $\sigma_{t+\Delta t}$ , $\lambda_{t+\Delta t}$ , $\sigma_{pt+\Delta t}$ , $\varepsilon^p_{t+\Delta t}$
    endif
endif
```

The three non linear implicit systems of tensorials equations (S1,S2,S3) are solved by a Newton method , preceeded by a dichotomie search for (S1), in order to avoid some numerical problems when $\Delta\alpha$ does reach 1.

5 NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTS

We use the following definition for the function F :

$$R = D(A|\varepsilon_p - \varepsilon^n_p|^\alpha + R_o) \qquad C = C_\infty + C_1 e^{-bp(1-\frac{\sigma_p}{S})} \qquad D = 1 - me^{-bp(1-\frac{\sigma_p}{S})}$$

$$\mathbf{x} = C(S\varepsilon_p - \sigma_p\varepsilon^n_p)$$

Figure 1 shows a progressive deformation phenomena for a 316 stainles steel. In this simulation the amplitude of stress is constant and equal to 400 MPa, while the mean stress-take respectively the values of 20, 40, 60, 80 MPa. The comparison for the maximal strain with the experiment from [5], and also another model from [6], is given (figure 2). It is worth noting that the constants of the model have been obtained on another 316 stainless steel. For the model presented on this paper the values of maximal plastic strain at different stabilized states have been got analytically for fixed maximal and minimal stresses, so are slightly different from the numerical results. Figure 3, for the same constants, shows the simulation of a circular traction shearing strain controled test for $\Delta\varepsilon = 0.4\%$ and $\Delta\gamma/\sqrt{3} = 0.4\%$ (on one element in plane stress). The experimental result of [7]give for equivalent saturated stress a value about 300 MPa, where the equivalent stress is defined by $\sqrt{(\sigma^2+3\tau^2)}$.

REFERENCES

[1] Taheri S., 1989 . Une loi de comportement uniaxial cyclique avec variable a memoire discrete. 9th Con-

gres Francais de mecanique, Metz, France.
[2] Taheri S., 1991, A uniaxial cyclic elasto-plastic constitutive law with a discrete memory variable. SMIRT 10, L22, Tokyo, Japan.

[3] Andrieux S. Taheri S., 1992. A three dimensional cyclic constitutive law for metal with a semi-discrete memory variable. Plasticity 91, Grenoble, France, edited by P. Boehler/Akhtar S. Khan , edition Elsevier.

4] ***Code Aster*** ©*v.2*,(1992). Analyse des Structures et Thermo-mécanique pour des Etudes et des Recherches - Manuel d'Utilisation / Manuel Descriptif Informatique / Manuel de Réference.E.D.F/D.E.R

[5] Goodman A. M., 1983. Development of constitutive equations for computer analysis of stainless steel components. 4th Int. Seminar on inelastic analysis and life prediction in high temoperature environement Chicago, U.S.A.

[6] Chaboche J. L., 1991. On some modification of kinematique hardening to improve the description of ratcheting effects. Int. J. of plasticity Vol 7, 1991.

[7] Murakami S. Kawai M. Ohno, Y., 1989. Effects of amplitude history and temperature history on multiaxial cyclic behaviour of 316 stainless steel. Transaction of ASME, J. of Eng. Mat. and Tech. Vol 111/279.

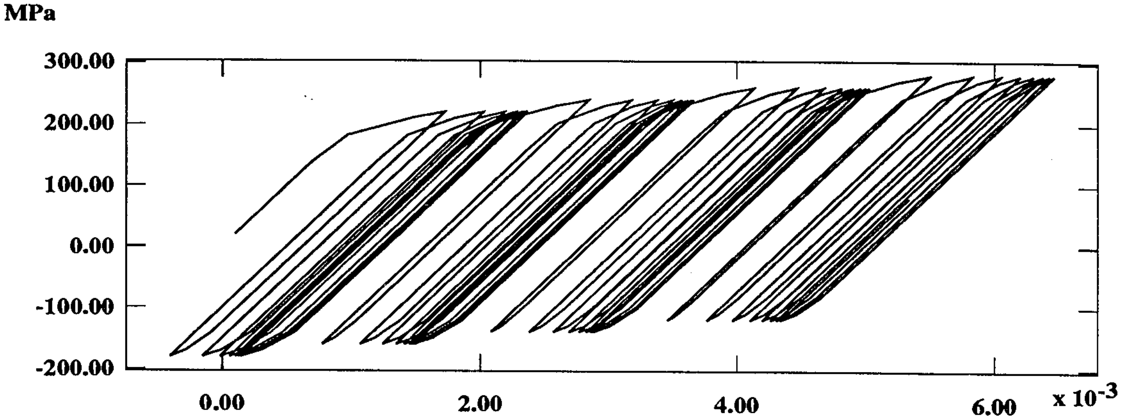

Fig 1 . Progressive deformation for constant stress amplitude and variable mean stress.

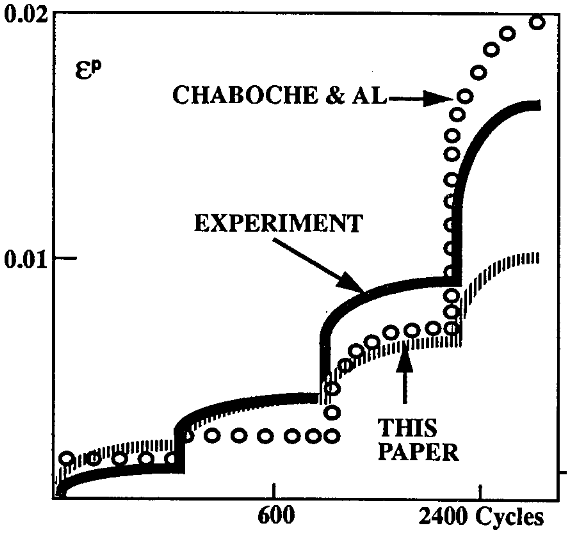

Fig 2 . Comparaison of maximal plastic strain between experiment and different modelisations

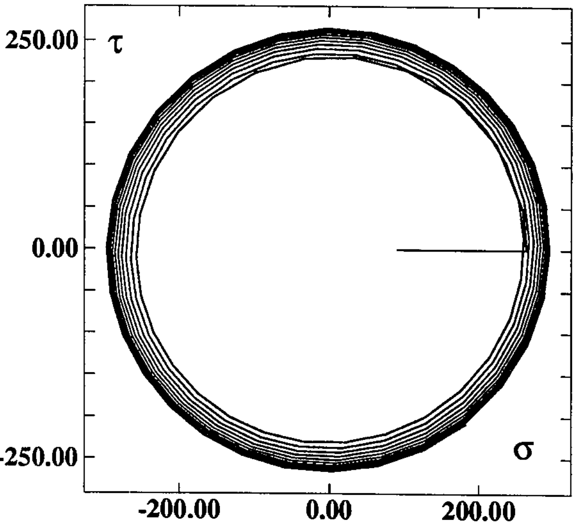

Fig 3 . Stabilized state of stress under circular strain controlled test.