The Axial Rigidity of Annular Flange Connections

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1 INTRODUCTION

In order to analyze the vibration of a structure whose elements are connected with bolts, it is necessary to estimate the flexural rigidity and the axial rigidity of bolted connections. Some studies have been reported on annular flange connections (Pavlov, 1976; Jofriet et al., 1981; Sawa et al., 1986). But, in these studies, the bolt axial force and the sealing performance of bolted connections are examined, and the rigidity of bolted connections is not discussed. Moreover, the effect of the dispersiveness of bolt disposition on the rigidity is not sufficiently taken into consideration in the analyses, in which the bolt axial forces are assumed to act uniformly along the pitch circle diameter of flange.

One of the authors has investigated the effect of the dispersiveness of bolt disposition and the effect of the flange-shell (hub) junction on the flexural rigidity of annular flange connections (Kimura et al., 1988).

This paper deals with a calculation method to estimate the axial rigidity of bolted annular flange connections subjected to axial loads, taking account of the dispersiveness of bolt disposition and the rigidity of flange-shell junction. In the analysis, a circular plate model, which is supported/fixed at N points along the outer edge and is connected with shell (hub) along the inner edge, is used for estimating the axial rigidity. For verification, experiments are performed and experimental results are compared with calculated results.

2 THEORETICAL ANALYSIS

Whole axial rigidity K of bolted flange connections can be obtained as follows, assuming that the bolted flange connections are a spring system consisting of bolts and flanges in series.

$$K= 1/(1/K_b + 2/K_f) \tag{1}$$

where K_b is axial rigidity of bolts, K_f axial rigidity of flange. In this paper, we deals with a calculation method to estimate the axial rigidity of flange K_f.

Figure 1 shows two types of bolted annular flange connections in which flanges are fasened by N sets of bolts and nuts. One is the case in which interfaces of flanges are not in contact, the other is in contact. In order to estimate the axial rigidity of these bolted annular flange connections, two models of circular plate, whose outer edge is supported/fixed at N points and the inner edge is connected with a shell, are used as shown in Fig.2.

2.1 The case in which interfaces of flanges are not in contact

In order to analyze the axial rigidity of annular flange connections in which interfaces of flanges are not in contact, the following assumptions are made.

(1) the inner edge of the circular plate is connected with the spring.

(2) the outer edge of the circular plate is supported at N points in order to take account of the dispersiveness of bolt dispositions.

(3) effect of the flange which belongs to the bellows is neglected.

Based on these assumptions, the deflection of a circular plate(Fig.3(a)) symmetrically supported at N points with respect to the center is obtained as follows (Kimura, 1990).

$$w_P = w_0 + \frac{Pa^2}{8\pi D_t}\left\{A_1 + B_1\rho^2 + C_1 \ln \rho + \sum_{m\equiv 0 (\mathrm{mod}\, N)}^{\infty} (A_m\rho^m + B_m\rho^{-m} + C_m\rho^{m+2} + D_m\rho^{-m+2})\cos m\theta\right\} \quad (2)$$

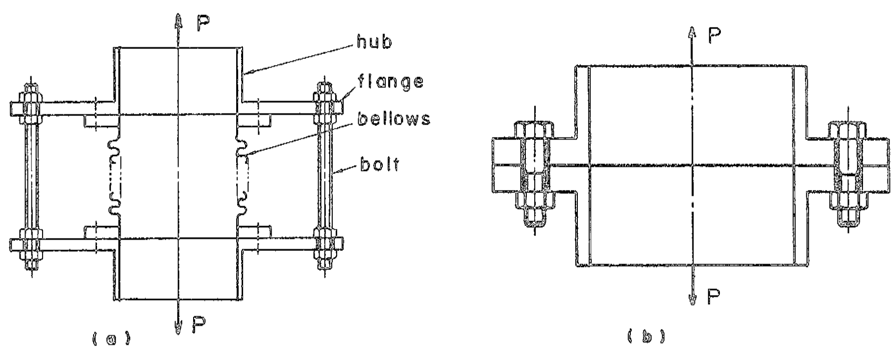


Fig.1 Annular flange connections

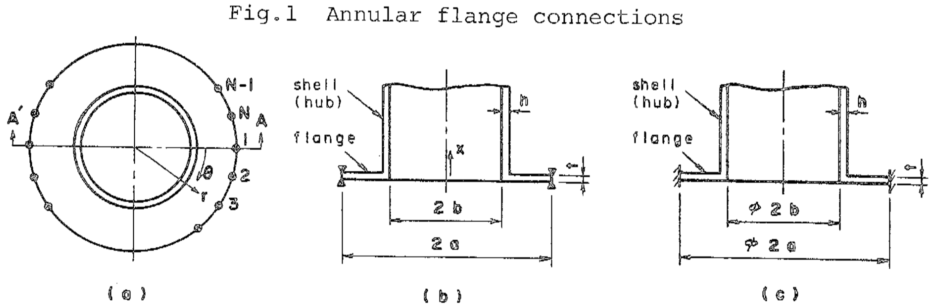


Fig.2 Circular plate of which outer edge is supported/fixed at N points and inner edge is connected with shell

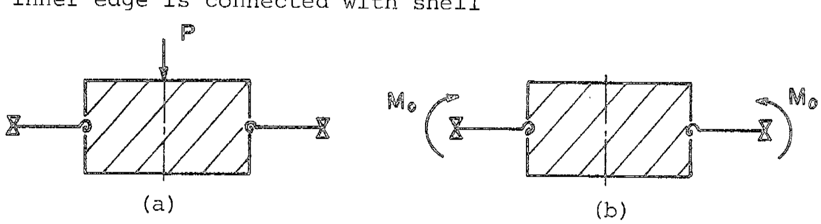


Fig.3 Circular plate of which outer edge is supported at N points and inner edge is connected with the spring

where

$$w_0=\frac{Pa^2}{8\pi D_t}\{(1+H)(1-\rho^2)+(G+\rho^2)\ln\rho\},\quad H=\frac{1-\nu}{2}\,\frac{(1-\nu)(1-\beta^{-2})+2(1+\nu)\ln\beta-x(\beta^{-2}+2\ln\beta-1)}{(1+\nu)(1-\nu)(1-\beta^{-2})-x\{(1-\nu)+(1+\nu)\beta^{-2}\}}$$

$$G=\frac{-2(1+\nu)^2\ln\beta+x\{-2+2(1+\nu)\ln\beta\}}{(1+\nu)(1-\nu)(1-\beta^{-2})-x\{(1-\nu)+(1+\nu)\beta^{-2}\}},\quad x=2\left(1+\frac{kt}{2b}\right)\left(\frac{h}{t}\right)^3 k,\quad k=[3(1-\nu^2)]^{1/4}\sqrt{b/h} \tag{3}$$

$$A_1=-B_1-\sum_{m=0(\mathrm{mod}N)}^{\infty}(A_m+B_m+C_m+D_m),\quad B_1=\frac{1-\nu}{2(1+\nu)}C_1 \qquad D_t=\frac{Et^3}{12(1-\nu^2)}$$

$$C_1=\frac{-2(1+\nu)(1+H)\beta^2+2\beta^2+2(1+\nu)\beta^2\ln\beta-(1-\nu)G+(1+\nu)\beta^2-x\{-2(1+H)\beta^2+2\beta^2\ln\beta+(G+\beta^2)\}}{(1-\nu)(1-\beta^2)+x\left(\frac{1-\nu}{1+\nu}\beta^2+1\right)}$$

$$\Delta A_m=-(m+1)[(1-\nu)m^2\{-(3+\nu)+x\}\beta^{2m+4}+(m-1)\{(1-\nu)m-2(1+\nu)\}\{(1+\nu)-x\}\beta^{2m+2}$$
$$+\{(1-\nu)m+2(1+\nu)\}\{2m-(1+\nu)+x\}\beta^2]$$

$$\Delta B_m=(m-1)[\{(1-\nu)m-2(1+\nu)\}\{2m+(1+\nu)-x\}\beta^{4m+2}+(1-\nu)m^2\{-(3+\nu)+x\}\beta^{2m+4}$$
$$+(m+1)\{(1-\nu)m+2(1+\nu)\}\{(1+\nu)-x\}\beta^{2m+2}]$$

$$\Delta C_m=-m(m-1)[(1-\nu)(m+1)\{(1+\nu)-x\}\beta^{2m+2}+\{(1-\nu)m-2(1+\nu)\}\{(1-\nu)+x\}\beta^{2m}$$
$$+(1-\nu)\{-2m+(1+\nu)\}\{(1-\nu)-x\}\beta^2]$$

$$\Delta D_m=m(m+1)[(1-\nu)\{-(2m+(1+\nu))+x\}\beta^{4m+2}+(1-\nu)(m-1)\{(1+\nu)-x\}\beta^{2m+2}$$
$$+\{(1-\nu)m+2(1+\nu)\}\{(1-\nu)+x\}\beta^{2m}]$$

$$\Delta=\frac{m^2(m+1)(m-1)}{4}[-(1-\nu)(3+\nu)(2m+(1+\nu))\beta^{4m+2}-(1-\nu)^2(3+\nu)m^2\beta^{2m+4}$$
$$+2(1+\nu)(1-\nu)^2(m-1)(m+1)\beta^{2m+2}+(1-\nu)\{(1-\nu)^2m^2+8(1+\nu)\}\beta^{2m}$$
$$+(1-\nu)(3+\nu)\{2m-(1+\nu)\}\beta^2+x\{(1-\nu)(3+\nu)\beta^{4m+2}$$
$$+(1-\nu)^2m^2\beta^{2m+4}-2(1-\nu)^2(m-1)(m+1)\beta^{2m+2}+\{(1-\nu)^2m^2+8(1+\nu)\}\beta^{2m}+(1-\nu)(3+\nu)\beta^2\}] \tag{4}$$

where E is the modulus of elasticity, ν Poisson's ratio, and t the thickness of plate, $\rho=r/a$, $\beta=b/a$, 2a bolt pitch circle diameter, 2b inner diameter of flange.

In the case of interfaces of flanges are not in contact, the axial rigidity K_{f1} of annular flanges can be expressed by Eq. (5).

$$K_{f1}(N,\beta,\kappa)=P/w_P(\rho=\beta) \tag{5}$$

2.2 The case in which interfaces of flanges are in contact

In order to analyze the axial rigidity of annular flange connections in which interfaces of flanges are in contact, the following assumptions are made.

(1) the inner edge of the circular plate is connected with the spring.
(2) the outer edge of the circular plate is fixed at N points in order to take account of the dispersiveness of bolt dispositions. (for simplification, rotation angle at the supporting points be zero.)

Based on these assumptions, an equation for calculating the axial rigidity of annular flange connections is obtained.

The deflection of a circular plate (Fig.2(c)) symmetrically fixed at N points with respect to the center is expressed as follows.

$$w=w_P-\left.\frac{\left(\frac{\partial w_P}{\partial r}\right)}{\left(\frac{\partial w_M}{\partial r}\right)}\right|_{\rho=1,\theta=0}\cdot w_M \tag{6}$$

where w_M is the deflection of a circular plate (Fig.3(b)) subjected to the bending moment M_0 at the supporting points. The general solution of the circular plate subjected to the bending moment M_0 is expressed by Eq.(7).

$$w_M=\frac{M_0 a}{8\pi D_t}\left\{A_0+B_0\rho^2+C_0\ln\rho+D_0\rho^2\ln\rho+\sum_{m=2}^{\infty}(A'_m\rho^m+B'_m\rho^{-m}+C'_m\rho^{m+2}+D'_m\rho^{-m+2})\cos m\theta\right\} \tag{7}$$

Each unknown coefficient A_0 $-D_0$ $,A'_m$ $-D'_m$ can be determined by using following conditions.

(a) equilibrium of the radial moment at the outer boundary

$$M_r(\rho=1)=\frac{M_0}{\pi a}\sum_{i=1}^{N}\left(\frac{1}{2}+\sum_{m=1}^{\infty}\cos m\theta_i\right) \tag{8}$$

where

$$\theta_i=\theta-\gamma_i,\ \gamma_i=2\pi(i-1)/N$$

(b) deflection at the supporting points

$$w_M(\rho=1,\ \theta=\gamma_i)=0 \tag{9}$$

(c) equilibrium of the forces at the outer boundary

$$V_r(\rho=1)=0 \tag{10}$$

(d) equilibrium of the radial moment at the inner boundary

$$M_r(\rho=\beta)=\frac{D_f\kappa}{b}\left(-\frac{\partial w_M}{\partial r}\right)_{\rho=\beta} \tag{11}$$

(e) deflection at the inner boundary

$$\left(\frac{\partial w_M}{\partial \theta}\right)_{\rho=\beta}=0 \tag{12}$$

As shown above, the deflection of a circular plate fixed at N points is obtained by using Eq.(6) and Eq.(7). In the case of interfaces of flanges are in contact, the axial rigidity K_{f2} of annular flanges can be expressed by Eq.(13).

$$K_{f2}(N,\beta,\kappa)=P/w(\rho=\beta) \tag{13}$$

3 EXPERIMENT

In order to clarify the effect of (1) the number of bolts and (2) the rigidity of flange-shell junction on the axial rigidity of annular flange connections, tensile tests are made. Two types of the flange-shell, of which the thickness t of the flange is 10 mm, and the thickness h of the shell are 2 and 4 mm, are prepared (Fig. 4).

After two flanges are fastened by 8 or 4 bolts and nuts, an axial load is applied to the assembly (Fig.5). The deflection of the flange is obtained by subtracting the elongation of shells and bolts from the total deflection which are measured by the experiments.

No	A	B	h	t	h/t	β
1	96	100	2	10	0 2	0576
2	94	102	4	10	0 4	0576

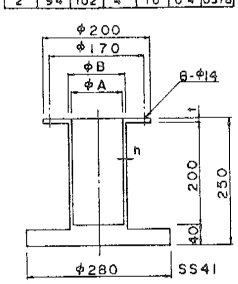


Fig.4 Size of annular flanges used in experiments (mm)

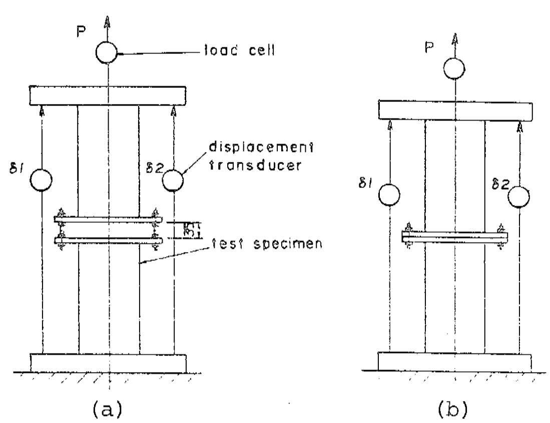


Fig.5 Sketch of experimental apparatus

4 RESULTS

The deflection curves of circular plates obtained by the analyses are shown in Fig.6. $\kappa=\infty$ means that a flange is built in along the inner edge, then $(\partial w/\partial r)_{\rho=\beta}=0$ On the other hand, in the case of $\kappa=0.01$, the restriction of the inner edge of a flange becomes weak, and $(\partial w/\partial r)_{\rho=\beta}$ becomes large; κ represents the rigidity of the flange-shell junction.

Calculated results are shown in Fig.7 and Fig.8 about the effect of number N of bolts and the rigidity κ of flange-shell junction on the axial rigidity of flange. From these figures, above effects are made clear quantitatively on the axial rigidity of flange.

Figure 9 shows one example of the relations between the axial load P and the deflection of flange δ. The axial rigidity K_f of flange is defined by the ratio of P to δ in Fig.9. Figure 10 shows that the effects of thickness ratio h/t on the axial rigidity K_f of flange, in the case of $\beta=b/a=0.576$. It seems that the experimental results are in fairly good agreement with the calculated results. Thus, the axial rigidity of bolted annular flange connections can be estimated by using the calculation method described in this paper.

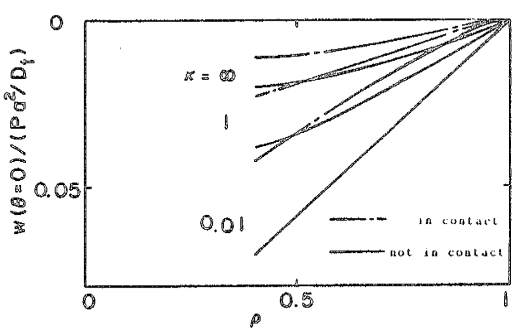


Fig.6 Deflection curve of plate along $\theta=0$ (calculated results, N=4, $\beta=0.4$)

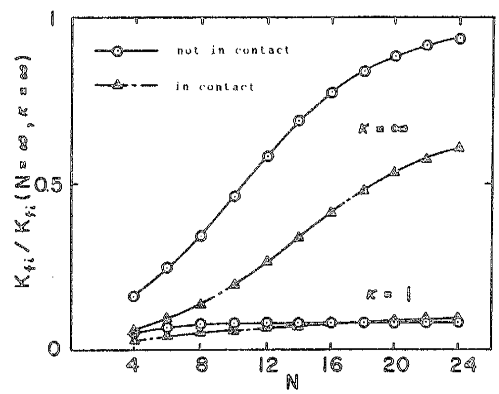


Fig.7 Effects of the number N of bolts on the axial rigidity K_f of flange (calculated results, $\beta=0.8$)

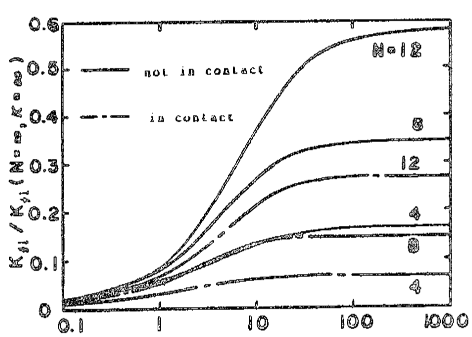


Fig.8 Effects of rigidity κ on the axial rigidity K_f of flange (calculated results, $\beta=0.8$)

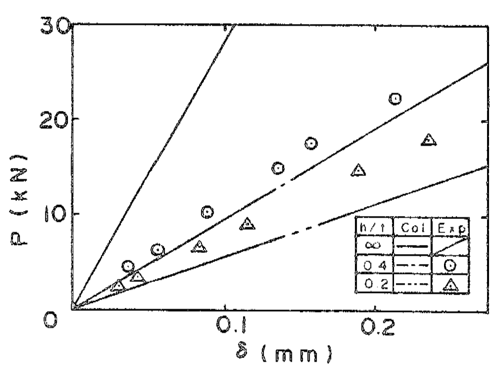
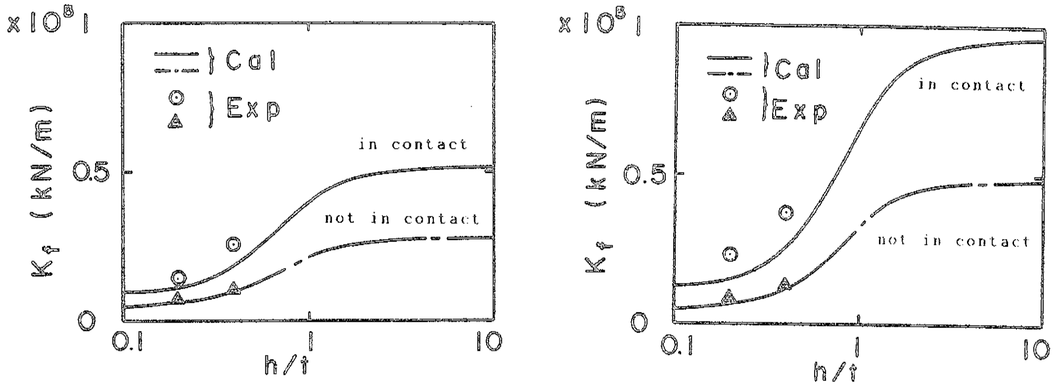


Fig.9 Relation between axial load P and deflection δ (not in contact, N=4)



(a) N=4

(b) N=8

Fig.10 Effects of ratio h/t on the axial rigidity K_f of flange

5 CONCLUSIONS

In order to establish a calculation method for the axial rigidity of bolted annular flange connections, discussions are made from both analyses and experiments. The following results are obtained:

(1) In order to evaluate the axial rigidity of bolted annular flange connections, the deflection of a circular plate subjected to an axial load, which is supported/fixed at N points along the outer edge and is connected with shell (hub) along the inner edge, is obtained analytically. The calculation method for the axial rigidity of bolted annular flange connections is proposed.

(2) Based on (1), the calculations are made about the effects of the number of bolts and the rigidity of flange-shell (hub) junction. And these effects are quantitatively made clear.

(3) Calculated results obtained by (2) are in fairly good agreement with experimental ones.

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