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Probabilistic Analysis of Tokamak Plasma Disruptions

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ABSTRACT

An approximate analytical solution to the heat conduction equations used in modeling component melting and vaporization resulting from plasma disruptions is presented. This solution is then used to propagate uncertainties in the input data characterizing disruptions, namely, energy density and disruption time, to obtain a probabilistic description of the output variables of interest, material melted and vaporized.

1.0 INTRODUCTION

Current tokamak studies [1-2] consider plasma disruptions as an undesirable event that must be accounted for in first wall and in-vessel component designs. The extreme nature of a disruption results in possible loss of material due to vaporization and melting, and, in addition, induces transient and residual thermal stresses, all of which have an impact on first wall and in-vessel component reliability.

One of the most sophisticated models to date, accounting for many physical phenomena believed to occur during a disruption, is that developed in Ref. 3. The authors have considered such phenomena as evaporation, thermal radiation, melting and variable thermo-physical properties, all of which cause the problem to become nonlinear in nature. This has necessitated the development of a computer code to solve the relevant heat conduction equations, which has resulted in a certain lack of insight into the importance that various parameters have in determining the extent of evaporation and melting.

In addition to the complexities involved in solving the relevant nonlinear heat conduction equations, there are uncertainties in the parameters characterizing a disruption. Specifically, there are large uncertainties associated with the length of time over which a disruption occurs and the energy density that impinges on the component of interest. As an example, the heat fluxes considered in [3] resulting from combinations of disruption times and energy densities vary by more than an order of magnitude.

The purpose of this paper is twofold: first, to present an approximate analytical solution of the relevant heat conduction equations used in modeling a plasma disruption in tokamak reactors, and second, using this solution, to assess probability distributions for quantities of interest (e.g. melt layer thickness) resulting from a disruption, which may be required to assess the reliability of in-vessel components.

The analytical solution is based on an approximate method of solving the heat conduction equation known as the heat balance integral. This approach to transient, nonlinear boundary value heat conduction problems has been developed extensively by Goodman [4-6]. However, the problem of two unsteady moving boundaries with the temperature varying on one of them, which is the case in this situation, does not appear to have been analyzed by this approach.

In solving the heat conduction equations via the heat balance integral approach, the desired dependent (output) variables such as melt layer thickness, amount of material vaporized and temperature history are expressed in terms of the independent (input) variables, namely the disruption time period and the disruption energy density. With equations expressing the output variables of interest in terms of the input variables, it is now possible to develop probability distributions for the output variables. This is accomplished by propagating the uncertainties in the input variables, expressed as probability distributions, through the appropriate equations relating output variables to input variables. The probability distributions characterizing the uncertainties in the input variables are assessed through both expert opinion and existing tokamak reactor studies. Since the equations relating the output variables to the input variables are not simple enough to allow the probability distributions for the output variables to be expressed analytically, the propagation of the input uncertainties is carried out via Monte Carlo.

The remainder of this paper is organized as follows: In section 2 the heat balance integral method is summarized. In section 3, the physical model describing the thermal situation present in the material when a disruption occurs is described. In section 4 a discussion of the uncertainties characterizing the energy density and disruption time period is presented for the limiter. Finally, in section 5, a probabilistic description of the melt layer is presented.

2.0 SUMMARY OF THE HEAT BALANCE INTEGRAL METHOD

Solving the problem to be described via the heat balance integral consists of the following steps:

1. Assuming that the temperature distribution depends on the space variable in some particular fashion consistent with the boundary conditions, such as a polynomial for example.

2. Substituting the assumed form of the temperature distribution into the heat conduction equation and integrating with respect to the spatial variable over the appropriate interval to obtain the heat balance integral, which is an ordinary differential equation with time as the independent variable.

3. Solving the heat balance integral to obtain the time dependence of the temperature distribution and whatever moving boundaries are present as applicable.

3.0 PHYSICAL MODEL OF MATERIAL THERMAL RESPONSE

The problem at hand can be modeled as having four separate phases as shown in Figure 1. It is assumed that the extreme temperature effects are limited to a region near the front surface of the material, allowing the material to be modeled as semi-infinite in nature. It is also assumed that the geometry of the situation allows the component of interest to be adequately modeled as a slab. Finally, the thermophysical properties in the solid and liquid phases are modeled as constants in each of the four phases.

Initially, in phase 1, for times between $t=0$ and $t=t_m$, where t_m denotes the time at which melting commences, the material is exposed to an extremely high heat flux which causes the surface temperature to rise rapidly from its initial value of T_o. Although some heat transfer via thermal radiation, and vaporization to some degree can occur, simple calculations show that the radiative heat flux is negligible compared to the heat flux load from the plasma during the disruption, therefore radiation will be neglected during phase 1. Vaporization is also minimal during this phase and will be neglected. Note that the conduction process during this phase is expected to penetrate to some depth $\delta(t)$, beyond which no heat transfer is assumed to occur.

During the second phase, for times between t_m and t_d, where t_d denotes the length of the plasma disruption, the slab begins to melt and vaporization becomes substantial. This, in turn creates two moving boundaries: a vaporization line denoted by $s(t)$ and a melt line denoted by $m(t)$. While the temperature at the melt line is constant, the temperature at the vaporizing surface is increasing, thus causing an increased amount of heat transfer via radiation at the surface. The model for the second phase is based on the assumption that the radiative heat transfer can be neglected throughout the second phase. It can be shown that this assumption is indeed satisfied for the disruption parameters found in the references. Finally, it can be seen that $\delta(t)$ continues to grow.

At the end of phase two, the vaporization line has progressed to $s(t_d)$ and the melt line to $m(t_d)$. The heat flux is assumed to drop instantly to zero, with an attendant decrease in surface temperature. Thus, as the surface temperature drops, the melted portion of the slab begins to resolidify. This resolidification process consists of two parts: loss of sensible heat and loss of latent heat. In this model it is assumed that all of the sensible heat is given up at a constant melt layer thickness first, followed by the loss of latent heat and resolidification of the melt layer. Once again radiative processes at the surface can be neglected since they are negligible when compared to the conduction processes taking place, which implies an insulated boundary at the surface. In addition, it is assumed that no further vaporization occurs after t_d.

The fourth phase is essentially a continuation of the third phase with the surface temperature continuing to decrease as the energy near the surface diffuses into the solid. The surface temperature and energy content in the solid at the end of phase three serve as the initial conditions for phase four. Radiative heat transfer during this phase is also neglected since the time scale for radiative cooling is large compared to that of conduction.

4.0 DISCUSSION OF INPUT UNCERTAINTIES

Uncertainties in E_d and t_d, the energy density and disruption time period respectively, are due to the fact that no fusion reactors have yet been built that are similar enough to those being planned and analyzed to allow for confident extrapolation of observed parameter values. t_d can only be estimated at the current time and there are several sources of uncertainty in E_d [1].Uncertainty in E_d includes the following:

1. Total amount of plasma and magnetic energy that appears as heat in the first wall or limiter.
2. Fraction of total heat flux to the wall which appears as X-rays.
3. Location of the impact area of the plasma energy.
4. Peaking factors of the plasma heat flux.

An analysis assessing possible values of disruption parameters was performed in [3].

Since the heat flux H , is given by E_d/t_d a range of possible heat fluxes results from uncertainties in E_d and t_d. Thus, if the next step in assessing the uncertainty characterizing E_d and t_d is taken, namely assigning probabilities to the variables E_d and t_d, then H, along with other quantities of interest such as the maximum melt thickness and the amount of material vaporized become probabilistic in nature. Table 1 shows the 5th, 50th and 95th percentiles of the probability distributions assigned to the variables E_d and t_d for this study. These values are the result of using a lognormal distribution with the 5th percentile assigned to the "optimistic" value of E_d and the 95th percentile assigned to the "pessimistic" value of E_d, for the high load to limiter case in Ref. [1]. The 5th and 95th percentile values for t_d are the result of using a lognormal distribution with the 95th percentile assigned to the 20 ms reference case, and the 5th percentile assigned to the 5 ms alternate case in Ref [1]. The material considered for these calculations is beryllium, a likely surface coating for the limiter [1-2].

5.0 PROBABILISTIC DESCRIPTION OF THE MELT LAYER

Figure 2 shows the results of the Monte Carlo in histogram format. The relative frequency of the different melt thicknesses resulting from the input probability distributions shown in Table 1, is plotted on the ordinate axis. The range of the melt thickness is shown on the abscissa, and varies between 0 and 250 microns. A melt thickness of 0 results when the combination of E_d and t_d is such that no melting occurs. The maximum melt thickness is limited by the fact that a steady situation within the melt layer develops.

As can be seen in the plot, 85 percent of the melt thicknesses fall between 100 and 200 microns. The average melt thickness is 148 microns, and the 5th and 95th percentiles are approximately 90 and 210 microns respectively. Thus, the spread between the 5th and 95th percentiles in both energy density and disruption time probability distributions, is reduced in the resultant melt layer distribution.

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REFERENCES

1. INTOR, Chapter 7, "Impurity Control and First Wall Engineering," FED-INTOR/ICFW/82-17 (1982)

2. STARFIRE - A Commercial Tokamak Fusion Power Plant Study, Argonne Report ANL/FPP-80-1 (1980)

3. A. Hassenein, G. Kulcinski, W. Wolfer, Surface Melting and Evaporation During Plasma Disruptions in Magnetic Fusion Reactors, University of Wisconsin Fusion Engineering Program Report, UWFDM-494, September 1984

4. T. Goodman, "Application of Integral Methods to Transient Nonlinear Heat Transfer," Advances in Heat Transfer, 1, 51-122, (1964)

5. T. Goodman, "The Heat Balance Integral and Its Application to Problems Involving a Change of Phase," ASME Transactions, 80, 335-342, (1958)

6. T. Goodman, "The Melting of Finite Slabs," ASME Transactions, 82, 16-24, (1960)

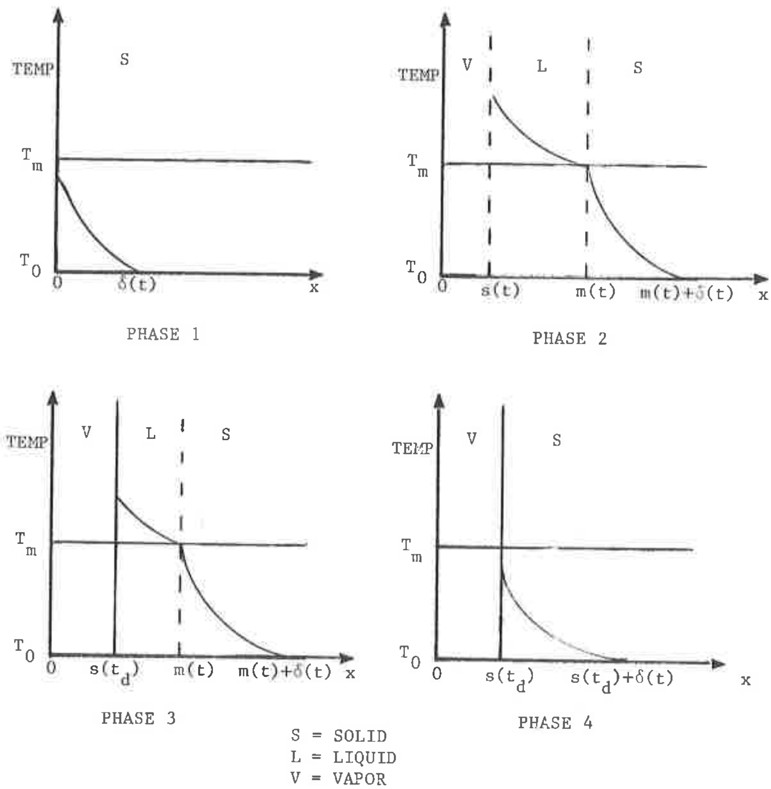

FIGURE 1 - THE FOUR PHASES OF MELTING AND VAPORIZATION

TABLE I - PROBABILITY DISTRIBUTIONS FOR E_d, t_d

PARAMETER	5th PERCENTILE	50th PERCENTILE	95th PERCENTILE
$E_d (J/cm^2)$	267.5	535	1070
t_d (msecs)	5	10	20

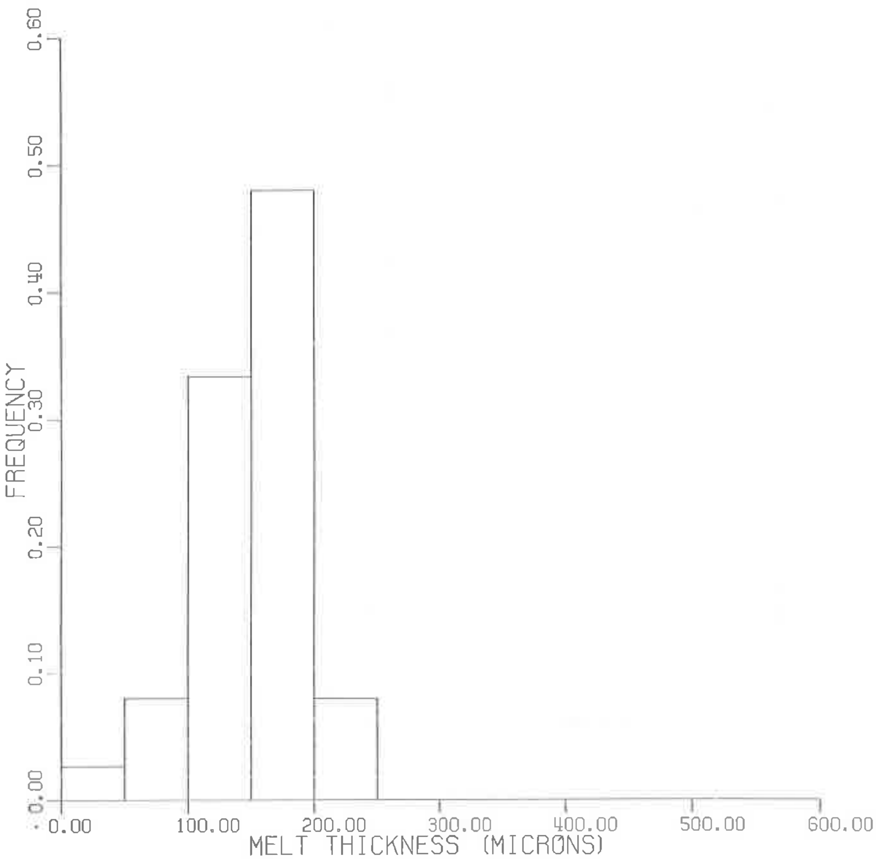

FIGURE 2 - MELT THICKNESS HISTOGRAM