ANALYSIS OF THE OVERALL STRUCTURAL BEHAVIOR DUE TO THE IMPACT OF DEFORMABLE MISSILES

M. M. ETTOUNEY, R. R. RADINI, P. S. HSUEH
Burns & Roe, Inc.,
185 Crossways Park Drive, Woodbury, New York 11797, U.S.A.

Abstract

This paper presents a method of analysis to evaluate the overall behavior of reinforced concrete structures subjected to impact from deformable missiles. This method approaches the analysis in a very simple and practical way. The analysis is based on approximating the structure-missile system by a two-degree of freedom model. The two degrees of freedom model represents the missile and the structure, respectively. The hysteretic damping effects are considered implicitly through the nonlinearity of the two springs. Empirical formulas are presented for the evaluation of the dynamic properties of the nonlinear spring representing the concrete structure. The impact is simulated by applying an impulse on the two degrees of freedom system, then by the method of step by step numerical time integration (central difference formula is used) the time histories of the displacements and velocities of both the missile and structure are obtained. The numerical procedure is simple enough to be programmed by a hand or desk calculator which makes the method handy for most engineers and analysts.

Application of this method to practical cases of missile impact were made. The results show some differences between this proposed method and the conventional and less accurate method. It is believed that this new method of analysis represents a simple, more accurate and powerful tool for this type of problem.

1. Introduction

Different structural elements of nuclear power plants are often required to withstand the effects of missile impact. Missiles could be from externally generated sources such as tornadoes, airplanes, or from internally generated sources such as high pressure systems, fan blades, turbine blades, valve stems and others. Many authors studied the analysis of structures due to the different impacting missiles [1, 2 and 3]. However, the methods available for the design of targets subjected to the impact of deformable missiles [1 and 3], introduce many assumptions and limitations. The mass of missile to mass of target ratio has to be within certain limits, the rebounding of the missile is not considered and evaluation of the contact pressure between the missile and the target assume that the target is rigid. This paper introduces a method of analysis of the overall behavior of structure subjected to the impact of deformable missiles. This method accounts for dynamic coupling between the missile and the target and relieves all the above mentioned assumptions and limitations.

2. Coupled Method of Analysis

The suggested method of analysis is based on modeling the deformable missile-target system as a nonlinear dynamic two-degree of freedom system. A set of initial conditions is applied to this system to determine the required system responses. The dynamic properties of the system are dependent on the different parameters of the problem. A brief description of the main elements of the problem will follow.

2.1 Properties of the Two Degree of Freedom System

Figure 1 shows the two degree of freedom model. The top mass (m_1) represents a part of the missile mass (m_i). The bottom mass (m_2) is the sum of the remainder of the missile mass and a percentage of the target mass (m_t), such that:

$$
\begin{aligned}
m_2 &= \overset{m}{\acute{n}}_{2i} + \alpha m_t \\
m_1 &= m_{1i} \\
&= \beta m_i \\
m_i &= m_{1i} + j_{2i}
\end{aligned}
\qquad (1)
$$

The equivalent target mass is determined by (αm_t). There are many expressions for evaluating α, [1, 2 and 3]. However, the expressions in [3] will be used as a lower bound of the equivalent target mass. The factor ranges between 0.0 and 1.0. The actual value of β is undetermined because it is problem dependent. However, for practical design problems, several values of β should be considered to determine the most conservative system response.

The top spring of the system, with a stiffness of $K_1(\Delta U)$, is a nonlinear spring. The force deformation relation of this spring is shown in Figure 2. The loading part a-b-c consists of two zones, the linear part a-b, and a curvilinear part b-c. The unloading zones c-d is assumed to be linear, with a stiffness equal to that of the elastic zone [4]. After the force in the rebounding spring becomes zero, the spring is deactivated and the missile is assumed to have rebounded away from the target. The bottom spring represents the target with a nonlinear stiffness of $K_2(U_2)$. Modeling a nonlinear structural system with a single degree of freedom is a well-known practice [5, 6 and 7]. Although many other curves could be used a bilinear force-deformation relation will be used for the purpose of this paper [7], Figure 3.

The damping of the system consists of two parts, hysteretic and viscous [8]. The hysteretic damping of the system is automatically accounted for by the nonlinearity in the springs. The viscous damping of the system is of lesser importance for this class of problem where the strains are very large [8]. However, the damping matrix will be taken as a linear combination of both mass and stiffness matrices, [9].

2.2 Elastic Stiffness of the Reinforced Concrete Targets

Many tests [10 and 11], showed that the ultimate moment capacity and the bending rigidity of the reinforced concrete slabs subject to dynamic loadings is a function of many parameters, especially the rate of loading. Based on these reported results, the dynamic bending rigidity could be expressed as

$$(EI)_d = 1.58\ (EI)_s \tag{2}$$

where

$(EI)_s$ = static bending rigidity

The dynamic ultimate moment capacity could also be expressed as:

$$(M_u)_d = 1.28\ (M_u)_s \tag{3}$$

where

$(M_u)_s$ = static ultimate moment capacity.

Equations (2) and (3) will be used to evaluate the dynamic properties of the reinforced concrete targets.

2.3 Dynamic Equations of Motion

The dynamic properties of the two degree of freedom system could be assembled to form the matrix equation

$$\utilde{M}\,\utilde{U} + \utilde{C}\,\utilde{U} + \utilde{K}\,\utilde{U} = \utilde{0} \tag{4}$$

where

$$\underset{\sim}{U}(t) = \left[U_1 \,\vdots\, U_2\right]^T$$

and $\underset{\sim}{M}$, $\underset{\sim}{C}$ and $\underset{\sim}{K}$ are the mass, damping and stiffness matrices, respectively. Equation (4) is a coupled second order differential equation with initial conditions

$$\begin{aligned} \underset{\sim}{U}\ (t=o) &= \underset{\sim}{o} \\ \underset{\sim}{\dot{U}}\ (t=o) &= \underset{\sim}{\bar{\dot{U}}} \end{aligned} \qquad (5)$$

The initial velocity of the system U could be expressed as

$$\underset{\sim}{\bar{\dot{U}}} = \left[\bar{\dot{U}}_1 \,\vdots\, \bar{\dot{U}}_2\right]^T$$

where

$$\begin{aligned} \bar{\dot{U}}_1 &= \text{velocity of impact} \\ \bar{\dot{U}}_2 &= \bar{\dot{U}}_1\sqrt{\frac{m_{2i}}{m_2}} \end{aligned} \qquad (6)$$

This distribution of initial velocities preserves the initial kinetic energy of the missile.

The dynamic equation of motion with these initial conditions is then solved using numerical integration techniques. It is recommended to use the central difference method [12], due to the fact that the nonlinear systems are very sensitive, and the central difference method guarantees the convergence of the problem. The step-by-step solution generates the maximum structural ductility, contact pressure time history and the missile rebounding, if any.

2.4 Advantages of the Coupled Method of Analysis

There are many advantages to the use of this method of analysis:

i - It does not make any assumption for evaluating the contact pressure time history which is evaluated naturally based on the velocity of impact.

ii - It is valid for any mass of missile to mass of target ratio.

iii - It accounts for the possibility of rebounding missiles.

iv - It accounts for the possible dynamic coupling of the missiles and the targets.

3. Example Problem

The coupled method of analysis is compared with the method described in [1]. A 207 pound, eight-inch schedule 40 hollow pipe missile, impacting a 5000 psi concrete barrier at 320 feet per second is considered.

Since the method described in [1] does not account for the deformability of the target, and the coupled method of analysis does, some practical properties of the target had to be assumed. A six feet long square reinforced concrete slab was used, with ultimate load capacity of 800.0 kips, thickness of 12.0 inches and elastic stiffness of 7180.0 kips/inch. It was assumed that there will be no punching shear failure in the target. The effective mass of the target was found to be 3.5 slugs. The missile was assumed to be 30.0 inches long. Two crushing strengths of the missile were considered, 40.0 ksi and 60.0 ksi. Figure 4 shows a comparison between the contact pressure time histories as predicted by the two methods. The predicted contact pressures by the coupled method of analysis is shorter than that of the conventional method. The reason is that the coupled method predicted rebounding of the missile, whereas the conventional method cannot predict any rebounding. This resulted in a longer contact pressure time history, which seems to be unnecessarily conservative.

5. Conclusions and Recommendations

A coupled method of analyzing the overly structural behavior due to the impacting of deformable missiles. The method relieves many of the assumptions which are present in the conventional method of analysis. A comparison between the two methods showed large differences in the predicted results. More studies are needed to get a better understanding of the behavior of the coupled missile-structure system.

6. Acknowledgment

The help of Dr. B. Bedrosian of Burns and Roe, Inc. of Woodbury, New York is appreciated. This work was made possible by the use of the facilities of the Civil Engineering Department of Burns and Roe, Inc. Mrs. W. Smith helped in typing this paper.

7. References

[1] Kennedy, R. P., "A Review of Procedures for the Analysis and Design of Concrete Structures to Resist Missile Impact Effects," Nuclear Sciences Group, Holmes & Narver, Inc., Anaheim, California, September 1975.

[2] Long, A. E., "A Two-Phase Approach to the Prediction of the Punching Strength of Slabs," American Concrete Institute Journal, February 1975.

[3] Linderman, R. B., et al, "Design of Structures for Missile Impact," BC-TOP-9A, Rev. 2, Bechtel Power Corporation, San Francisco, September 1974.

[4] Wu, R. H., and Witmer, E. A., "Finite Element Analysis of Large Transient Elastic-Plastic Deformations ... ," M.I.T. Dept. of Aeronautics and Astronautics, Report ASRL TR 154-4, January 1972.

[5] Jennings, P. C., "Periodic Response of a General Yielding Structure," Journal Engineering Mechanics Division, ASCE, Vol. 90, EM2, 1964.

[6] Mark, K., "Nonlinear Dynamic Response of R. C. Frames," Ph.D. Thesis, M. I. T. Dept. of Divil Engineering, 1974.

[7] Ang, A., and Newmark, N., "Development of a Transmitting Boundary for Numerical Wave Motion Calculations," University of Illinois, Dept. of Civil Engineering, 1972.

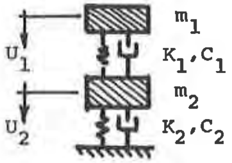
[8] McClintock, F. A., and Argon, A. S., Mechanical Behavior of Materials, Addison-Wesley, 1966.

[9] Gosh, S., and Wilson, E., "Dynamic Stress Analysis of Axisymmetric Structures under Arbitrary Loading," University of California at Berkeley, Earthquake Engineering Research Center, 69-10, September 1969.

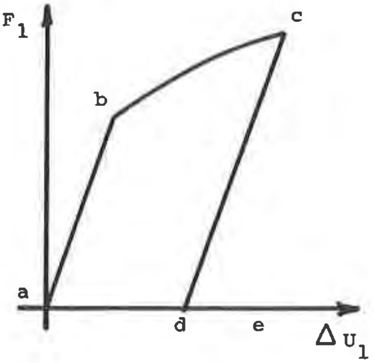
[10] Ghali, A., et al, "Punching of Flat Plates Under Static and Dynamic Horizontal Forces," American Concrete Institute Journal, October 1976, pp. 566-572.

[11] Dilger, W., et al, "Flat Plates with Special Shear Reinforcement Subjected to Static Dynamic Moment Transfer," American Concrete Institute Journal, October 1976, pp. 543-549.

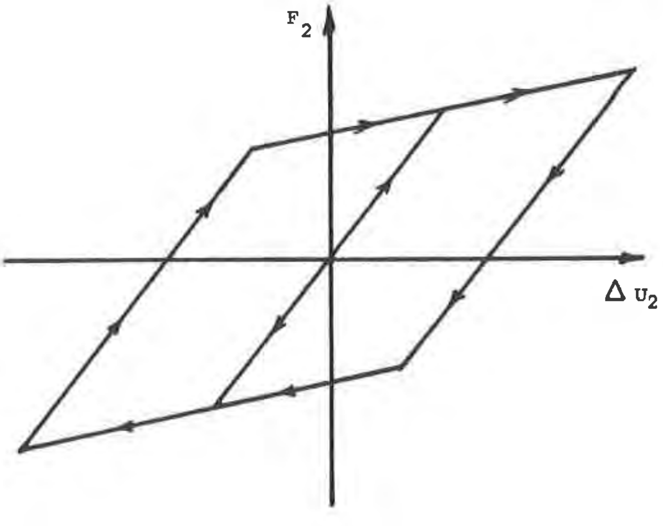
[12] Ettouney, M. M., "Nonlinear Finite Element Dynamic Analysis of a Two-Dimensional Continuum," Sc.D. Thesis, Civil Engineering Department, M. I. T., 1976.



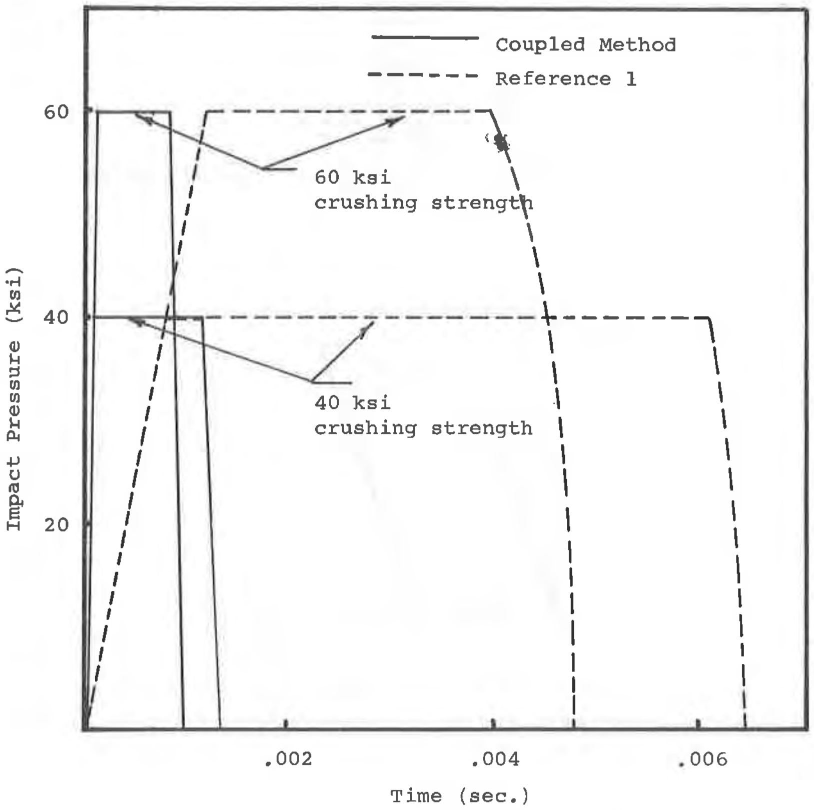
1 Two Degrees of Freedom Coupled System



2 Force-Deformation Relation of the Missile



3 Force-Deformation Relation of the Target



4 Contact Pressure Time History