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QPPS: A New Efficient 4 Node Shell Element with Physical Stabilisation

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Abstract:

A lot of 4 nodes finite element shells have been presented in the litterature in the past years. For the implicit computations the MIT4 element proposed by KJ Bathe or the Q4γ24 element proposed by JL Batoz are considered as the best available elements. But these elements are very time consuming if they are used for dynamic explicit transient analysis, because they require an integration with 4 gauss points in elasticity and with four times at least 5 points in case of elastoplasticity. A lot of reduced integration schemes have also been presented: all these elements use one integration point in the mid plane, or at least 5 points through the thickness in case of plastic behavior. The most widely used element is based on the Belytchko-Tsai formulation; but this formulation requires an user input parameter to control the hourglass modes, and this parameter has to be tuned if one wants to pass with a good precision all the standard elastic or plastic test-cases. This paper gives the formulation of a new 4 node general shell element with one integration point. This element is based on an assumed strains formulation, as the Belytchko-Leviathan element. This type of formulation has the advantage that no user parameter is needed to control the hourglass modes. But the previous finite element has been observed to be too stiff when they undergo large membrane plastic strains. The QPPS finite element proposed in this paper is such that the antihourglass control internal forces take into account the plastic state of the material via the tangeant modulus. This simple idea gives the element a very good behavior as well in the elastic as in the elastoplastic or large plastic strain cases. A number of impact problems are given to illustrate this good behavior.

1 Introduction

A lot of work has been produced these last twenty years to give a "good" 4 node shell element. This paper is devoted only to 4 nodes nonlinear shell elements. It is aimed to give the formulation and validation of an efficient element for highly nonlinear geometrical as well as material behavior, mainly for impact and fast transient simulations. The finite element presented in this paper is fully described in reference [1]. The paper summaries the main results and the main advantages of this type of formulation.
In standard 4-node shell element, both full integration (FI) and reduced integration (RI) are employed. The FI scheme is often used in implicit dynamic or satic finite element tools. It uses 4 integration points on the mid surface. This scheme has the advantage to have no stability problem, but sometimes involves locking, and the computations are expensive, if this formulation is used in an explicit integrator. The RI scheme with one integration point on the mean surface, is widely used in the explicit computer codes for fast transients. These formulations significantly decrease the computing times, and are very efficient provided that the spurious modes (often called hourglass modes) are "well" stabilised. All the stabilisation techniques rely on the following concept: one adds a "control" energy associated with the

hourglass modes to the strain energy. One immediately observes that this "control" energy is artificial and has to remain small if one wants to have a "good" prediction of the behavior of the shell. The elements mainly differ by the way of introducing this control energy.

2 Usual formulations for reduced integration.

The most widely used elements with RI integration are based the Belytschko Tsai element [2]. For this element the control energy is computed on a flat shell which is the projection of the shell geometry on the plane passing by the 4 points on the middle of each side of the quadrilateral element (it can be easily proved that these 4 points are coplanar). The material parameters used for the evaluation of control energy are constant for the whole computation; it remains the same wenether the structure is elastic or elastoplastic. These choices permit to have very simple and efficient computations of hourglass control forces. But his technique has the drawback to produce unsufficient control when the element is distorted and a different prediction when it is elastic or plastic. For instance this element does not pass the twisted beam test with the standard choice of the material hourglass parameter. This element has considerably been improved by Belytschko and Leviathan [3] by using the concept of "assumed strains". This type of stabilisation technique can be called physical stabilisation because it is based on physical concepts. This technique is very powerfull and interesting, but there is nearly no constraint for the interpolation of assumed strains within the element. This element has been studied with an elastic behavior and prooves to have a superb behavior; in particular there is no need to choose an adequate material constant for the evaluation of the stabilisation forces. Neththeless we found that this formulation is poor when the shell undergoes elastoplastic strains. The aim of the paper is to present an improvement of this element which performs also well in the range of elastoplastic material behavior.

3 The new QPPS element

The developpement of this element is based on the successfull formulation of the FI 4 node shell element MIT4 developped by Dvorkin-Bathe [4] as well as the $Q4\gamma24$ developped by Batoz and Dhatt [5]. Their main feature is that classical displacement method is used to interpolate the in plane strains (membrane and bending) and a mixed collocation method is used to interpolate the out-of plane strains (transverse shear). These elements are based on a Reissner-Mindlin model; the in plane strains are linear across the thickness, the out of plane strains are constant. The in-plane rate of deformation tensor D is given by:

(1) $$D = \frac{1}{2}(L + L')$$

The velocity gradient L is then linearised across the thickness z, and the strains expressed in the local tangent reference frame. The out of plane rate of deformation uses an assumed strain evaluation. The out of plane strain rates are interpolated from the values of the 4 mid-side points A1,A2,B1,B2; we have:

(2) $$\begin{cases} \gamma_\xi = \dfrac{1-\eta}{2}\gamma_\xi^{A1} + \dfrac{1+\eta}{2}\gamma_\xi^{A2} \\ \gamma_\eta = \dfrac{1-\xi}{2}\gamma_\xi^{B1} + \dfrac{1+\xi}{2}\gamma_\eta^{B2} \end{cases}$$

The velocities are interpolated within the element using the nodal tangential frame are called t_i and the velocity within the element is computed by the following equation:

$$(3) \qquad v_i = N_I\left[v_i^I + z\left(-\delta_{i2}\overline{\omega}_1^I + \delta_{i1}\overline{\omega}_2^I\right)\right]$$

In this equation N_I is the shape function for node I, v_i is the velocity componant in the local frame direction i, v_i^I is the velocity in the local frame direction i at node I, $\overline{\omega}_i^I$ is a modified rotational velocity around tangent direction i. The modified rotational velocity is given by:

$$(4) \qquad \overline{\omega}^I = \omega^I - \left(\omega^I . n^I\right) n^I = P(n^I)\,\omega^I$$

In quation (4) n^I is the normal to the shell at node I. Equation (4) shows that this modified rotational velocity is the projection of the velocity on the tangent plane at node I. This allows one one hand to eliminate the drilling rotation and on the other hand to introduce the warping of the element even with one gauss point. This is in fact one the most essential features of the Belytschko-leviathan element.

We shall first present the method to obtain the in plane strain rates at point z across the thickness. The strain rate D can be expressed by:

$$(5) \qquad \begin{cases} D = D^m + zD^b = \left[B_I\right] v^I \\ \left[B_I\right] = \left[B_I^m\right] + z\left[B_I^b\right] \\ v^I = \left\langle v_1^I, v_2^I, v_3^I, \overline{\omega}_1^I, \overline{\omega}_2^I \right\rangle \end{cases}$$

The B matrix can be separated in a constant part B^0 which does not vary on the mean surface of the shell and a non constant part B^H which is variable on the mean surface. For instance the membrane B matrix is written as:

$$(6) \quad \left[B_I^m\right] = \left[\left(B_I^m\right)^0\right] + \left[\left(B_I^m\right)^H\right]$$

The constant parts are then compted as usual and their detailed expressions can be found in [1]. The non-constant part is computed using the approximation that the element is flat. With this hypothesis it can be observed that the membrane strain rate does not vanish when a warped element undergoes a rigid body rotation; the membrane strain matrix is modified is modified in order to have a zero strain rate in this case. The formulation is here different of that of Belytschko-Leviathan element which introduces a coupling between rotational degree of freedom and membrane strains. Hence one can fear membrane locking, for Leviathan element, when a distorted element undergoes a constant bending moment.

For the out of plane strain rates a closed form can be found which is given by equation:

$$(7) \qquad \begin{Bmatrix} D_{13} \\ D_{23} \end{Bmatrix} = \left[B_{Ic}\right] \begin{Bmatrix} v_3^I \\ \overline{\omega}_1^I \\ \overline{\omega}_2^I \end{Bmatrix}$$

With these expressions of the strain rates one can easily compute the elastic stresses and hence the internal force vector.

(8) $\{f_I^{\text{int}}\} = \int_{V^{element}} [B_I]^t [C][B_J]\{v^J\} dV = \int_{V^{element}} [B_I]^t \{\sigma\} dV$

Where C is the Hooke's matrix . Taking into account all the hypothesis one can show that the internal forces can be given by:

(9) $$\begin{cases} \{f_I^{\text{int}}\} = \{(f_I^{\text{int}})^0\} + \{(f_I^{\text{int}})^H\} \\ \{(f_I^{\text{int}})^0\} \text{ is the constant part} \\ \{(f_I^{\text{int}})^H\} \text{ is the hourglass part} \end{cases}$$

The constant part is integrated traditionnaly with one gauss point.
The non-constant part is integrated efficiently by introducing the hourglass mode rates in the expressions. In case of elastoplastic behavior the usual technique consists of using the elastic stress rate to compute the hourglass part of the internal forces. This technique implies a too stiff behavior of the element when it undergoes significant plastic strains. The QPPS element uses the following equations for hourglass stress rates at a point z across the thickness:

(10) $$\{\dot{\sigma}\}^H = \frac{E_t(z)}{E}[C]\{D\}^H$$

In equation (10) E is the Young's modulus, $E_t(z)$ is the tangent modulus at point z and $\{\dot{\sigma}\}^H$ is the hourglass stress rate. As one can observe, from equation (10), that the hourglass forces are different when the element undergoes elastic or plastic deformation: one can say that the element hourglass control forces decrease when the element plastifies. In some sense the element automatically adapts to the elestic or plastic cases. This of real interest for practical applications where it is impossible to predict a priori what shall be the plastic state of an element. The following examples will show the interest of this way of computing the control forces.

4 Numerical examples

41 elastic cases

This element has been tested on usual elastic test-cases: the element passes uccessfully all classical elastic reference validation cases: it behaves exactly as the Belytschko-Leviathan element except for the pinched hemispherical shell case described on figure 1.

Number of elements	QPPS	Leviathan	Q4γ24
3	.77	.28	.78
12	.745	.73	.83
48	.9	.9	.92
192	.98	.98	.99

Table 1 Convergence for hemispherical elastic shell

Table 1 shows the compared convergence of the QPPS element, the Belytschko-Leviathan and the $Q4\gamma24$. The computed radial displacement at point A is normalised by the analytical displacement of 0.0924. It can be observed from that table that the convergence of the QPPS is faster than the Belytschko-Leviathan case which exhibits a rather stiff behavior for very crude mesh. This might be due to membrane locking when the element is mainly sollicated in bending.

42 Plastic cases

When the element is sollicitated in plasticity the difference with the other elements is larger. We have developped a plastic test for the hemispherical shell whose geometry is the same as that of figure 1 except the thickness which is increased to 0.12. The material is elastoplastic with perfect plasticity and a yield stress of 300MPa. The convergence study on the radial displacement of the node A is given on table 2.

Number of elements	QPPS	Leviathan	Q4γ24
12	.00537	.00531	.00677
48	.0114	.0109	.012
192	.01265	.01213	.01287
678	.0132	.0128	.0135

Table 2 Plastic hemisphere convergence study

From table 2 it is clear that the QPPS element converges slightly faster than the Leviathan element.

A second typical case is the cylindrical pannel under impulsive load; this case is not very discriminatory and all elements behave well for that case.

A third case is the S beam crushing test. The geometry is defined on figure 2; on this figure all the datas are given in mm. The thickness is chosen to be 1.5 mm.The Young's modulus is chosen to be 199355MPa the poisson's ratio 0.3, the density 7.8Kg/m^3. The stress strain curve is discribed by the following usual equation:

(11) $$\sigma = \sigma_y^0 + b(\varepsilon_p)^n$$

Whe have chosen the parameters as σ_y^0=185.4MPa, b=540MPa, n=0.32.

An added mass of 500Kg is put on the moving face and an initial velocity V of 5000mm/sec is given to the same points. The results are compared on figure 3 for the deformed shape at 18.ms (on this figure QPPS_Ce means the Levaithan element), and on figure 4 for the axial displacement crushing force relation. This test is extremely interesting and shows that the the new QPPS element has a very similar behavior than the $Q4\gamma24$ element (the buckling mode is the same, and the dynamic load-displacement curve is very similar). The Leviathan element is too stiff and exhibits mainly an elastic buckling behavior (as can be observed from the final deformed shape) but also has a residual force at the end which is twice that of the $Q4\gamma24$ element which is considered as the reference case.

An other test called the Box beam test has been used for the validation of the element because there is a comparison with an experimental result. The detailed results can be seen in [1]. Here again the element has shown a much better behavior than the Leviathan element.

5 Conclusion

The new QPPS element presented here is a very efficient one point quadrature 4 node shell element. It has the main advantage to be self adaptative for the whole range of application elastic or plastic and does not need any input of hourglass control constant. It has been proved to pass successfully all the elastic and plastic test cases. The computation efficiency is also rather good; it is only 30% slower than the standard Belytcschko Tsai element which is thought to be the fastest 4 node element available today. The detailed formulation is a bit complex but the basic ideas are rather simple and well based physically. This type of element should be the basic four node shell elements for the dynamic explicit codes and its use in an implicit context should be worth examining in terms of precision and of efficiency.

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Q4γ24

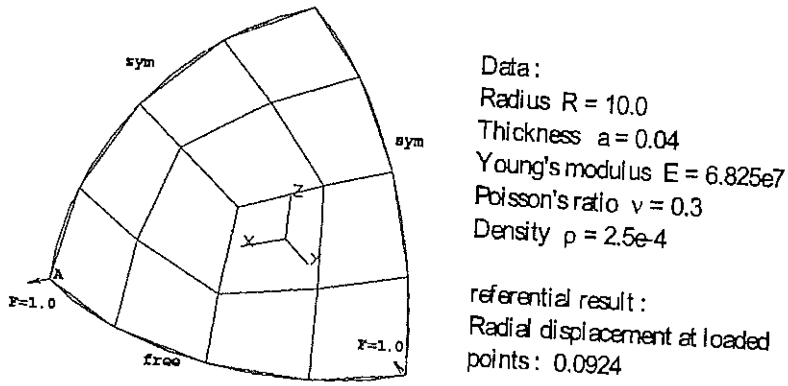

Figure 1: geometry of the Hemispherical shell test case.

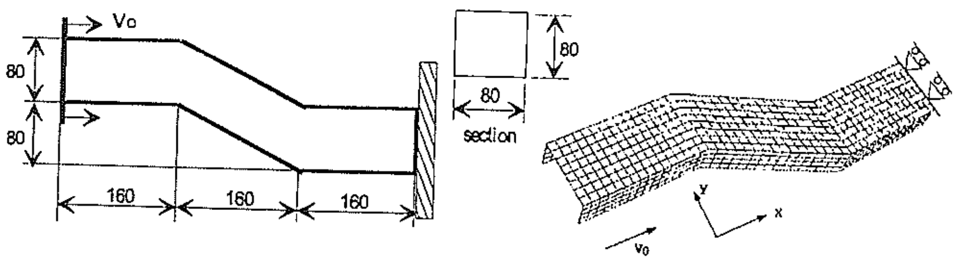

Figure 2 geometry of the S beam case

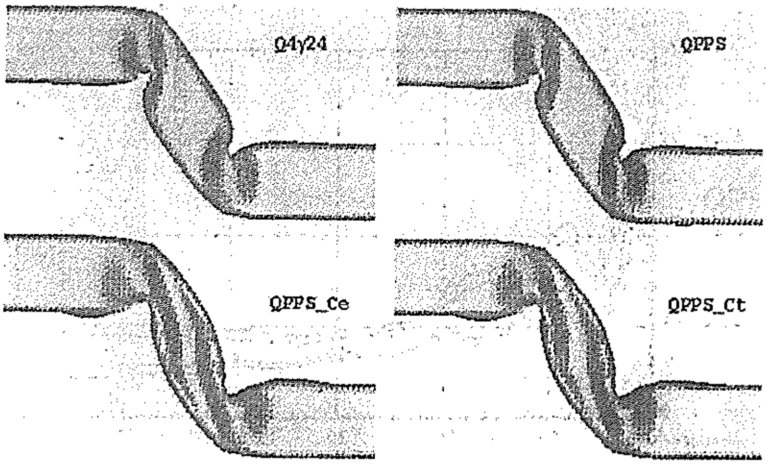

Figure 3 deformed Sbeam at 18.ms

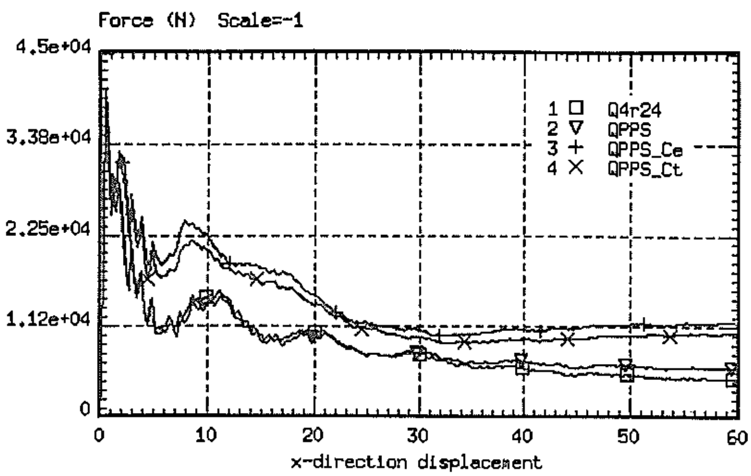

Figure 4 Axial displacement crushing force curve for S beam test