An Effective Modified Ritz Vector Direct Superposition Method

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Abstract

In this paper, an effective modified Ritz Vector Direct Superposition Method (MRVDS method) is presented. With this method, only one set of Ritz Vector are required in conducting dynamic response analysis of structure in the case of two or three directions of loading, instead of two or three sets of Ritz vector with original Ritz Vector Direct Superposition Method(RVDS method) [Wilson E.L. et al. ,1]. For complex structures under earthquake loading, its convergence is much faster than those of the conventional Mode Superposition Method(MS method) and the RVDS method. In addition, examples show that for evaluating the same number of Ritz vectors or mode shapes, itrequires only about one fourth even to several tenth of the run-time in comparison to that of the improved subspace iteration method [Wilson E.L. et al. ,2]. It is emphasized that close relationship exists between mode shapes and Ritz vectors produced by this method.

1. Introduction

The RVDS method has attracted wide attentions since it was presented in 1982 by prof. E.L.-Wilson et al. because of its faster convergence rate than that of the MS method in the case of one direction of loading only. Yet still there are some questions which have to be answered before it can be applied in practice. In using the RVDS method, for instance, one set of Ritz vector has to be evaluated in each direction of loading, and totally three sets of Ritz vector are required in the case of three directions of loading. So, the users would be concerned about whether or not the advantage of the RVDS method is still existing in the case of more than one direction of loading.

In answering the questions mentioned above, the MRVDS method is developed and presented.

2. A Heuristic Example

What is the problem with the RVDS method in the case of more than one direction of loading? A heuristic example will be given in this section.

In the first paper about the RVDS method [E.L.Wilson et al. ,1], for the case of more than one direction of loading, the expression of loading is given as follows,

$r(s,t)=\sum F_j(s)\, g_j(t)$. So,for the type of earthquake loading

$r(s,t)=\sum_{j=1}^{n_k} M_j(s)\, g_j(t)$, where $n_k(\leq 3)$ is the number of earthquake components.

$g_j(t)$ earthquake acceleration in j-direction

$M_j(s)$ spatial distribution vector of structure mass in j-direction

The expressions of transposition of $M_j(s)$ are as follows

$M_1^T=M_x^T=(m_1\ 0\ 0\ m_2\ 0\ 0\ ---\ m_n\ 0\ 0)$ $M_2^T=M_y^T=(0\ m_1\ 0\ 0\ m_2\ 0\ ---\ 0\ m_n\ 0)$

$M_3^T=M_y^T=(0\ 0\ m_1\ 0\ 0\ m_2\ ---\ 0\ 0\ m_n)$

So,there is a different set of Ritz vector corresponding to different direction of earthquake input.

To evaluate the total response of structure, the response of each Ritz vector in each direction of input has to be caculated first. Then the superposition of responses of all Ritz vectors in different direction has to be conducted separately. So, the total response of structure is equal to a certain combination of responses of structure in different directions.

The example, an unsymmytrical 2D-frame with x and y directions of earthquake input illustrated in Fig.1 and Table 2, shows that for the RVDS method, more than 20 Ritz vectors are required(i.e. more than 10 Ritz vectors in each direction of input) while only 10 mode shapes are needed in total if the MS method is used with the max. error $\Delta_m < 4\%$ in both cases. So, it is obvious that the convergence rate of the RVDS method is not always faster than that of the MS method for the case of more than one direction of input.

Fortunately, the results in Table 2 show that there is a way to improve the RVDS method in the multi-direction input case. Let's contrast two sets of approximate eigenvalue corresponding to two sets of Ritz vector in X and Y directions with exact frequencies of the system. The first 10 pairs of eigenvalue in X and Y directions are almost the same, and only last two eigenvalues are different in different directions. But both are approximate frequences of the system, and they are complementary with each other. This fact suggest us that there must be a certain relationship between Ritz vectors and mode shapes. If (M_x+M_y) is used as a particular spatial distribution vector of loading for this example, Ritz vectors and their corresponding eigenvalues produced by (M_x+M_y) may possess both characteristics which are belonging to those two different sets of Ritz vector and corresponding eigenvalue in X and Y directions, and play the simular role of mode shapes and frequencies of system in the MS method.

That is the idea where the MRVDS method come from.

3. The MRVDS Method and Characteristics of Its Ritz Vectors

Unlike the RVDS method, only one set of Ritz vector is produced and used by the MRVDS method for conducting responses superposition of Ritz vectors in any direction of input, and is just like the role of only set of mode shape in the MS method. The procedure of produring such a set of Ritz vector is basically the same as the RVDS method, but with special form of spatial distribution vector of loading which is shown as follows

$F(s)=F_x+F_y+F_z$

where F_x, F_y, F_z are spatial distribution vectors of loading in X,Y and Z directions separately. For earthquake loading,

$F(s)^T=M_1^T+M_2^T+M_3^T=M_x^T+M_y^T+M_z^T=(m_1\ m_1\ m_1\ m_2\ m_2\ m_2 --- m_n\ m_n\ m_n)$

coresponding to the case of three directions of input.

or $F(s)^T=M_1^T+M_2^T=M_x^T+M_y^T=(m_1\ m_1\ 0\ m_2\ m_2\ 0\ ---\ m_n\ m_n\ 0\ )$

corresponding to the case of X and Y directions of input.

F(s) here is called all-direction spatial distribution vector of loading.

Now, Let's look into the characteristics of Ritz vectors produced by this method and compare its convergence rate with those of the RVDS method and the MS method. Three examples (Fig.1, 2 and 3) are selected for this purpose. Example 1 is an unsymmytrical 2D-frame(14 d.o.f.) under two directions of earthquake input, example 2 a 3D symmytrical cantilever(15 d.o.f.) under three directions of earthquake input, and example 3 a symmytrical 4-story 3D-frame building (8 d.o.f.) under two directions of transversal earthquake input. Frequencies, eigenvalues corresponding to Ritz vectors and max. errors Δ_m of those systems for different dimensions n_v of Ritz vector subspace are evaluated and shown in Tables 1-3 and Figures 1-3. Because of the limitation of space, not all results are listed in Tables.

Tables 1, 2 and 3 show that Ritz vectors and corresponding eigenvalues produced by the MRVDS method have three important characteristics as follows,

1) At the extreme case, when the dimension n_v of Ritz vector subspace is equal to n, the d.o.f. of the system, all Ritz vectors and their corresponding eigenvalues are mode shapes and frequencies of the system. This also can be proved mathematically as follows,

Suppose X_i, ω_i are the final Ritz vectors and corresponding eigenvalues, $X_i = XZ_i \quad i=1,2,---,n$ where $X=(x_1,x_2,---,x_n)$, x_i are initial Ritz vectors and satisfy M-orthogonalization and M-normalization conditions, i.e. $X^T M X = I$; and Z_i, ω_i are the solution of the following standard eigenvalue problem,

$$(K^* - \omega_i^2 I)Z_i = 0 \qquad i=1,2,---,n \qquad \text{where } K^* = X^T K X$$

$$\therefore \ (K^* - \omega_i^2 I)Z_i = X^T KXZ_i - \omega_i^2 X^T MXZ_i = X^T(KXZ_i - \omega_i^2 MXZ_i) = X^T(K^\circ X_i - \omega_i^2 M^\circ X_i) = 0$$

$$\because \ X^T M X = I \Rightarrow (X^T)^{-1} = MX, \quad \text{so, X is a reversible n*n matrix}$$

It follows that $K^\circ X_i - \omega_i^{2\circ} X_i = 0$ is the only solution according to the Existance and Unique Theorem of solution of linear algebraic equations, which means that ${}^\circ X_i$, ω_i are mode shapes and frequencies of the system as it is concluded above.

However, when $n_v < n$, generally, only part of Ritz vectors approximate some lower mode shapes of the system, and big differences may exist between the rests of them. The number of Ritz vectors and corresponding eigenvalues approximating mode shapes and frequencies of the system increases with the increment of n_v (Table 1).

2) With the increment of n_v, some Ritz vectors corresponding to higher frequency components but with larger contributions h_i to the spatial distribution vector of loading may appear earlier than those corresponding to lower frequency components but with smaller h_i (Table 1) while some intermediate components may not appear. This is an essential difference between the MRVDS method and the MS method. For the latter, lower frequency components always appears earlier than higer frequency components in conducting response superposition of mode shapes. This may be the main reason why the MRVDS method has faster convergence rate than that of the MS method.

3) All mode shapes and frequencies of the system can be produced by the MRVDS method in which the all-direction spatial distribution vector of loading is used. But, in general, with single direction of spatial distribution vector of loading in the RVDS method, only part of mode components which can be excited by that direction of loading can be produced. All of mode components of the system consists of the total set of mode components produced by all possible single directions of spatial direction vector of loading (Table 3). For unsymmytrical systems, a certain degree of coupling exists among different sets of Ritz vector in different directions(Table 2), but no coupling at all for symmytrical system (Table 3). This may be the reason why the MRVDS method has faster convergence rate than that of the RVDS method for complex structures.

To this point, engineers may expect that Ritz vectors produced by the MRVDS method play the simular role of mode shapes of system and play better.

4. Advantages of The MRVDS Method Over The MS Method and The RVDS Method

In this section, the advantages of the MRVDS method will be shown through three examples.

Example 1 is unsymmytrical, and components with larger contributions h_i are scattered. Example 3 is symmytrical, and components with larger h_i are concentrated on the first two. So the former represents the most complex structures, the latter the simplest and example 2 in between.

Now,two clear trends can be seen in Figures 1-3 and Table 4.

1) In all three examples, within the range of frequencies interested in engineering practice,the convergence rates of the MRVDS method are faster than those of the MS method, at least at the same lever (for the simplest structures as example 3). The advantage of the MRVDS method over the MS method is also shown in Table 4. For evaluating the same number of frequencies or eigenvalues corresponding to Ritz vectors, it requires only about one fourth even to one several tenth of the run-time in comparison to that of the improved subspace iteration method.

2) For complex structures with mode coupling, the MRVDS method also has much faster convergence rate than that of the RVDS method while requirements of memory are basically the same. For instance, in example 1, when $\Delta_m < 5\%$ is asked for, then 16 Ritz vectors (8 in each direction) are needed by the RVDS method but only 8 Ritz vectors in total are required by the MRVDS method.

5. Conclusion

For conducting dynamic response analysis of complex structures with multi-direction of earthquake input, the MRVDS method has obvious advantages over both the MS method and the RVDS method. Although still some other questions may have to be answered, for instance, "How effective it is for other types of loading?" etc. , it is hopeful that the MRVDS method becomes an ideal alternative at least in the field of earthquake engineering in the near future.

References

/1/ Wilson E. L., Ming-Wu Yuan et al., " Dynamic Analysis by Direct Superposition of Ritz Vectors" EERC report, UCB/EERC-82/04, June, 1982.

/2/ Wilson E. L. and Tatsuji Itoh, " AN Eigensolution Strategy for large Systems", EERC report, UCB/EERC-82/04, June, 1982.

TABLE I : Comparison Between Frequencies And Eigenvalues Corresponding To Ritz Vectors Produced By The MRVDS Method

i	ω_i	$n_v^* = 14$		$n_v = 12$		$n_v = 8$		$n_v = 4$		$n_v = 2$	
		$\bar{\omega}_i$	h_i	$\bar{\omega}_i$	h_i	$\bar{\omega}_i$	h_i	$\bar{\omega}_i$	h_i	$\bar{\omega}_i$	h_i
1	.271010	.271010	.33	.271010	.33	.271011	.33	.271011	.33	.271016	.33
2	.881931	.881931	.01	.881931	.01	.881931	.01	.883434	.01		
3	1.21799	1.21799	.16	1.21799	.16	1.21799	.16	1.24121	.18	1.30001	.34
4	1.66680	1.66680	.03	1.66680	.03	1.66695	.03				
5	1.88346	1.88346	.01	1.88346	.01	1.95601	.01				
6	2.02387	2.02386	.01	2.02386	.01						
7	2.40993	2.40993	.24	2.40993	.24	2.42500	.30	2.41991	.43		
8	2.55551	2.55551	.09	2.55551	.09						
9	2.83713	2.83713	.09	2.83713	.09	2.82046	.11				
10	5.40500	5.40502	.02	5.40502	.03	5.78771	.05				
14	9.69962	9.69951	.00								

* n_v -----= The dimension of Ritz vector subspace

** h_i ----- The contribution of i^{th} Ritz vector to the all-direction spatial distribution vector of loading

TABLE II : Approximate Frequencies of 2D-Frame Produced by Three Different Methods

I	Improved Subspace Iteration Method	MRVDS method ($n_v = 14$)		RVDS method ($n_v = 12$)			
	ω_i	ω_i	h_i	ω_{xi}	h_{xi}	ω_{yi}	h_{yi}
1	.271010	.271010	.33	.271010	.69	.271010	.00
2	.881931	.881931	.01	.881927	.00	.881931	.01
3	1.21799	1.21799	.16	1.21798	.28	1.21799	.00
4	1.66680	1.66680	.03	1.66680	.00	1.66679	.09
5	1.88346	1.88346	.01	1.88342	.01	1.88347	.00
6	2.02387	2.02386	.01	2.02375	.01	2.02386	.04
7	2.40993	2.40993	.24	2.40993	.01	2.40992	.38
8	2.55551	2.55551	.09	2.55654	.00	2.55551	.18
9	2.83713	2.83713	.09	2.83714	.00	2.83713	.21
10	5.40500	5.40502	.02	5.40995	.00	5.40500	.06
11	6.18950	6.18953	.01	6.18866	.00		
12	6.59121	6.59118	.02			6.59122	.03
13	9.62862	9.62868	.00	9.47813	.00		
14	9.69962	9.69951	.00			9.69309	.00

TABLE III : Approximate Frequencies of 4-story Frame Building Produced By Three Different Methods

i	Improved Subspace Iteration Method	MRVDS method ($n_v = 8$)		RVDS method ($n_v = 4$)			
	ω_i	ω_i	h_i	ω_{xi}	h_{xi}	ω_{yi}	h_{yi}
1	4.22308	4.22309	.45	4.22308	.89		
2	6.97554	6.97554	.45			6.97554	.89
3	12.1544	12.1544	.04	12.1544	.08		
4	18.6089	18.6090	.01	18.6089	.02		
5	20.0763	20.0764	.04			20.0763	.08
6	22.8133	22.8141	.002	22.8133	.004		
7	30.7376	30.7378	.01			30.7376	.02
8	37.6822	37.6821	.002			37.6822	.004

TABLE IV: Time Consumption Comparison Between Two Methods

Name of method used \ NO. of EigenValues Caculated	2	4	6	8	10	12
Improved Subspace Interation Method	2'54"	4'34"	8'30"	10'	10'40"	11'5"
MRVDS method	≤ 5"	≤ 15"	≤ 40"	≤ 1'10"	≤ 1'50"	≤ 2'40"

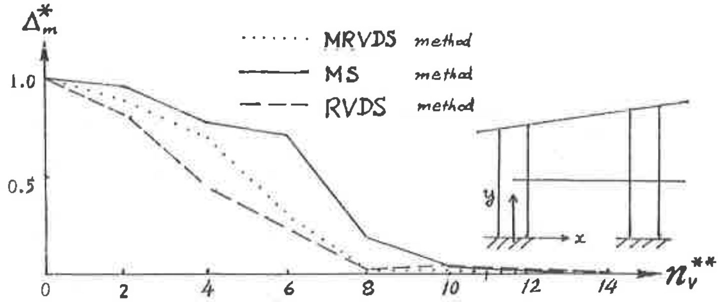

Fig. 1 Covergence Rate Comparison Among Three Methods With Example 1 - 2D Frame Under X & Y Directions of Earthquake Loading

* $\Delta_m = \max(|\delta_s|, |\delta_m|)$, δ_s, δ_m are relative errors of max. base shear and max. base moment.

** n_v--The total number of Ritz vectors or mode shapes used for the MRVDS method and the MS method, but the number of Ritz vectors used in each direction of loading for the RVDS method.

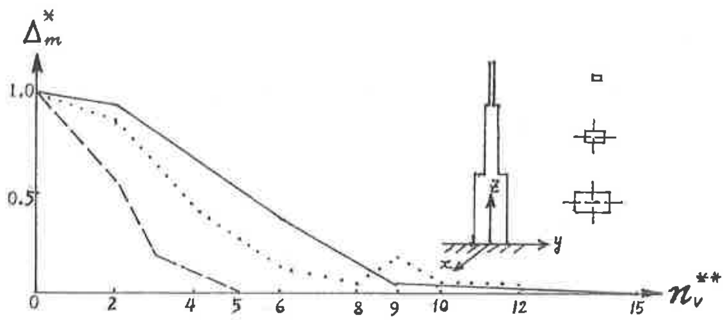

Fig. 2 Convergence Rate Comparison Among Three Methods With Example 2 --- 3D Cantilever Under 3 Directions of Earthquake Loading

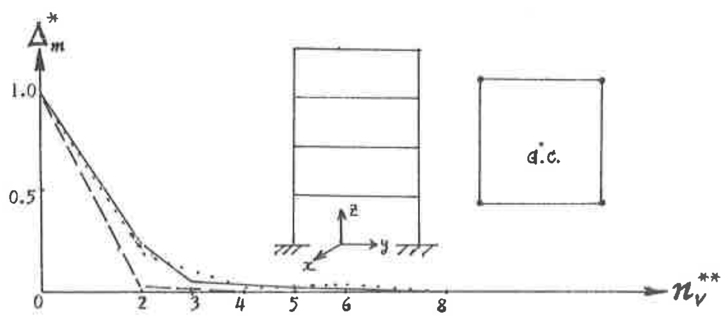

Fig. 3 Convergence Rate Comparison Among Three Methods With Example 3 --- 4-Story 3D Frame Building Under X and Y Directions of Earthquake Loading