A Parametric Study on J-integral under Pressurized Thermal Shock by Using Statically Indeterminate Fracture Mechanics

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ABSTRACT

The author proposed one application of elastic-plastic fracture mechanics for a statically indeterminate structure to a two-dimensional crack problem in the pressure vessel of pressurized water reactor (PWR) under pressurized thermal shock (PTS). This method makes us to evaluate the J-integral in an explicit form and is efficient in clarifying the mechanical characteristics of the PTS event. This paper describes a parametric study of the J-integral under PTS using this technique.

1 INTRODUCTION

The J-integral is valid for elastic-plastic fracture mechanics analysis, especially in ductile fracture analysis. But the original J-integral proposed (Rice 1968) is not used for fracture mechanics analysis under thermal loading, because the J-integral does not keep its path independence in this case. Special path independent integrals have been proposed for this purpose, for example, the J-integral proposed (Aoki et al. 1981).These special integrals can only be evaluated by using a numerical analysis method, finite element method, etc. So the integrals are not generally familiar to engineers because of their difficulty and complexity. The author has proposed statically indeterminate elastic-plastic fracture mechanics (SIEPFM) and confirmed its validity for an application to a PTS problem in the CSNI Project FALSIRE. It can overcome the defect of the special path independent integrals, simplify evaluating the J-integral under PTS and clarify the fracture mechanics characteristics of the PTS event. This paper describes parametric study of the J-integral under PTS based on the application of SIEPFM to a PTS problem.

2 EVALUATION OF THE J-INTEGRAL

The hoop stress conferred by PTS is divided into three components, membrane stress, bending stress and stress with non-linear distribution balanced in the direction of wall thickness.

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SIEPFM is applied to evaluation of the J-integral due to membrane and bending stresses, those with linear distribution. It is assumed that circumferential force P and bending moment M are acting on the end surface of a cylindrical shell, which has a slit and combines with a cracked member with zero length (see Fig.2). The displacement u and rotation θ of the slit cylindrical shell must be respectively equal to the displacement Δu and rotation $\Delta\theta$ due to the crack. The following equation can be obtained;

$$u = \Delta u \quad , \quad \theta = \Delta\theta \tag{1}$$

Once we evaluate the deformation of the shell and the cracked member as functions of P and M, we can conversely obtain P and M through equation (1). These are used to obtain the J-integral value. P and M are determined by the following procedure. The simultaneous equations of P and M are given by:

$$\left.\begin{aligned} f(P,M) &= (C_P + \Delta C_P)P + (C_{PM} + \Delta C_{PM})M \\ &\quad + \frac{\partial \Omega_{cp}}{\partial P} - C_P P_0 - C_{PM} M_0 + \Delta u_n = 0 \\ g(P,M) &= (C_{PM} + \Delta C_{PM})P + (C_M + \Delta C_M)M \\ &\quad + \frac{\partial \Omega_{cp}}{\partial M} - C_{PM} P_0 - C_M M_0 + \Delta \theta_n = 0 \end{aligned}\right\} \tag{2}$$

where Ω_{cp} is the complimentary energy (Parks et al. 1982). The equations are solved by the Newton-Raphson method:

$$\begin{bmatrix} f_P(P_i,M_i) & f_M(P_i,M_i) \\ g_P(P_i,M_i) & g_M(P_i,M_i) \end{bmatrix} \begin{Bmatrix} \delta P \\ \delta M \end{Bmatrix} = \begin{Bmatrix} -f(P_i,M_i) \\ -g(P_i,M_i) \end{Bmatrix} \tag{3}$$

$$\begin{aligned} \begin{Bmatrix} \delta P \\ \delta M \end{Bmatrix} &= \begin{bmatrix} f_P(P_i,M_i) & f_M(P_i,M_i) \\ g_P(P_i,M_i) & g_M(P_i,M_i) \end{bmatrix}^{-1} \begin{Bmatrix} -f(P_i,M_i) \\ -g(P_i,M_i) \end{Bmatrix} \\ &= \frac{1}{f_P \cdot g_M - f_M \cdot g_P} \begin{bmatrix} g_M & -f_M \\ -g_P & f_P \end{bmatrix} \begin{Bmatrix} -f(P_i,M_i) \\ -g(P_i,M_i) \end{Bmatrix} \end{aligned} \tag{4}$$

where $f_P = \frac{\partial f}{\partial P}$, $f_M = \frac{\partial f}{\partial M}$, $g_P = \frac{\partial g}{\partial P}$ and $g_M = \frac{\partial g}{\partial M}$.

Appropriate values, P_i and M_i are substituted into equation (4). If δP and δM are extremely small compared with P_i and M_i, calculation may be stopped. If no, recalculation must be made using new P_i $(= P_i + \delta P)$ and M_i $(= M_i + \delta M)$ until the above condition can be satisfied.

The J-integral can be obtained from the following equation by using the P and M values:

$$\begin{aligned} J &= J_{elastic} + J_{plastic} \\ &= \frac{1-\nu^2}{E} K^2 + \frac{1}{2}\alpha\epsilon_0\sigma_0 W^2 \left(\frac{AP^2 + 2BPM/W + CM^2/W^2}{P_{ref}^2} \right)^{\frac{n-1}{2}} \\ &\quad \left\{ \frac{1}{P_{ref}^2} \left(\frac{\partial A}{\partial a} P^2 + 2\frac{\partial B}{\partial a}\frac{PM}{W} + \frac{\partial C}{\partial a}\frac{M^2}{W^2} \right) - \frac{2}{P_{ref}^2}\frac{dP_{ref}}{da} \left(AP^2 + \frac{2BPM}{W} + \frac{CM^2}{W^2} \right) \right\} \end{aligned} \tag{5}$$

where σ_0 and ϵ_0 are the reference stress and strain, n is the hardening exponent, α is the coefficient in the Ramberg-Osgood law. A, B and C are derived from the fully plastic solution (Shih and Needleman 1984). The reference load P_{ref} is defined as:

$$\left.\begin{aligned} &P_{ref} = 1.455\sigma_0(W-a)\eta \\ &\eta = \left\{1+\left(\frac{a}{W-a}\right)^2\right\}^{\frac{1}{2}} - \frac{a}{W-a} \end{aligned}\right\} \qquad (6)$$

3 RESULTS AND DISCUSSIONS

Fig.3 shows an effect of the constitutive law, the material constants in the Ramberg-Osgood law, on the J-integral under PTS. As evident from the figure, the J-integral under PTS is not significantly influenced by the constitutive law.

Fig.4 shows an effect of the thermal conductivity on the J-integral under PTS. The J-integral under PTS develops a tendency to small increase with a decrease of the thermal conductivity.

Fig.5 shows an effect of the heat transfer coefficient on the J-integral under PTS. The J-integral under PTS develops a tendency to increase with an increase of the heat transfer coefficient.

4 CONCLUSIONS

A parametric study was made on the J-integral under PTS by using SIEPFM. The following points were made clear;

(1)The J-integral under PTS is not significantly influenced by the constitutive law.

(2)The J-integral under PTS increases with a decrease of the thermal conductivity.

(3)The J-integral under PTS increases with an increase of the heat transfer coefficient.

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Table 1. Conditions considered in analysis.

Mean radius	(m)	2.0
Wall thickness	(m)	0.2
Initial temperature	(K)	580
Final temperature	(K)	300
Internal pressure	(MPa)	15
Young's modulus	(GPa)	193 — 206
Poisson's ratio		0.3
Constants of Ramberg-Osgood's law		
Coefficient		0.0208-1.12
Power's number		9.71 — 11.5
Reference stress	(MPa)	375 — 413.7
Reference strain		0.002
Coefficient of thermal expansion	(1/K)	1.5×10^{-5}
Specific heat	(kJ/(kgf·K))	0.5
Specific weight	(kgf/m³)	7800
Thermal conductivity	(W/(m·K))	30 — 50
Coefficient of heat transfer	(W/(m²·K))	2500 - 7500
Cooling rate	(1/min)	∞

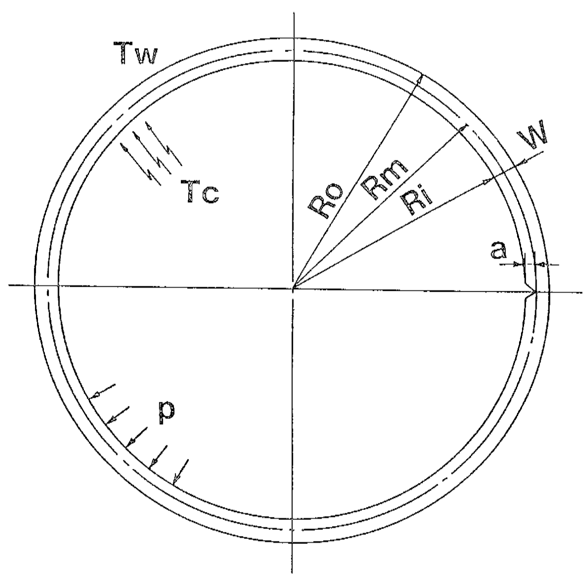

Fig. 1. Analysis model.

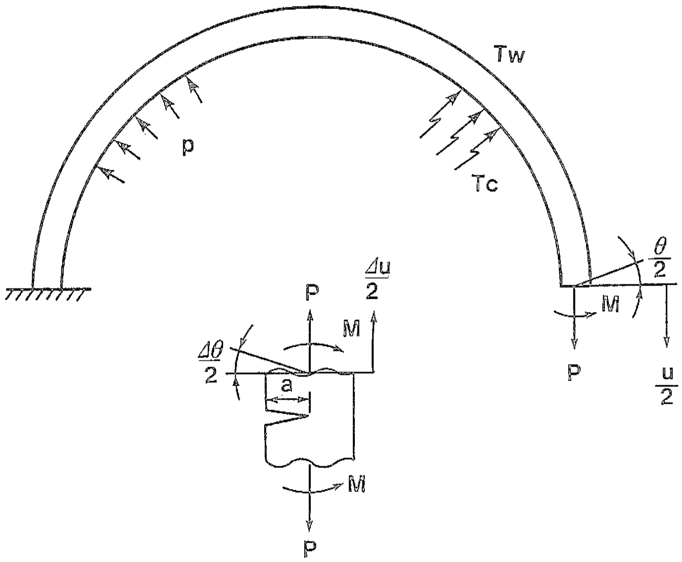

Fig. 2. Statically indeterminate fracture mechanics model.

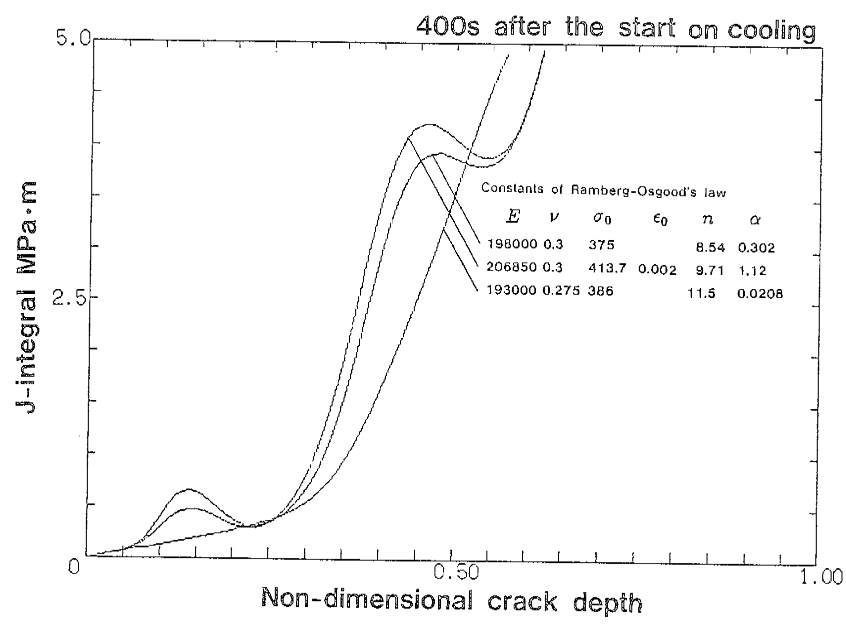

Fig. 3. Effect of the constitutive law of the pressure vessel's steel on the J-integral under PTS.

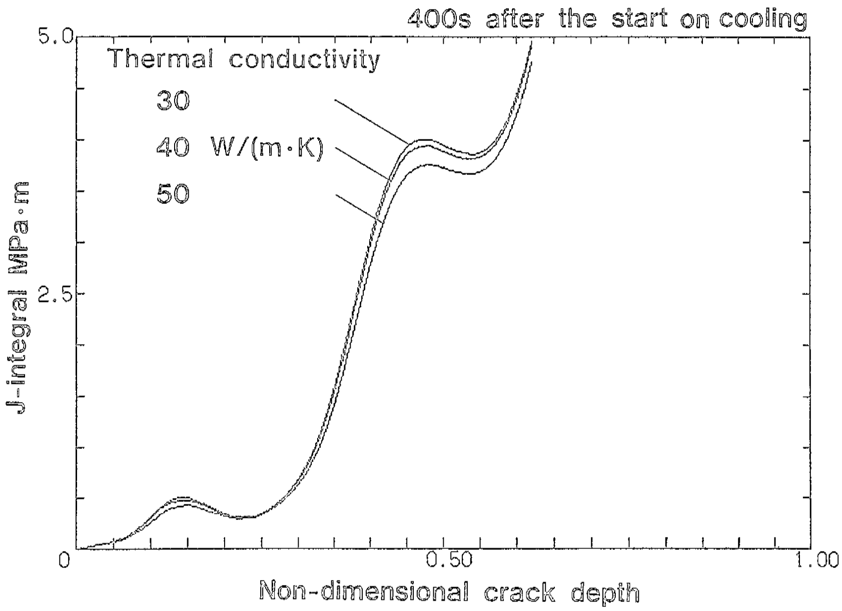

Fig. 4. Effect of the thermal conductivity of the pressure vessel's steel on the J-integral under PTS.

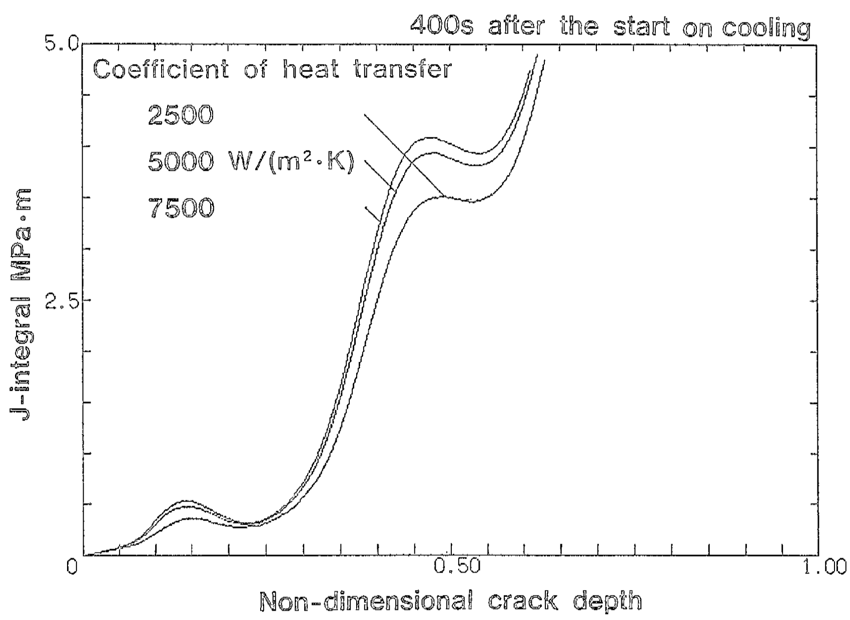

Fig. 5. Effect of the heat transfer coefficient on the J-integral under PTS.