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OPTIMUM TRUSS DESIGN BY STOCHASTIC SIMULATED ANNEALING

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ABSTRACT

Plane truss optimization problems are solved using stochastic simulated anealing (SSA). The SSA, a branch of combinatorial optimization techniques, establishes a probabilistic criterion for acceptance or rejection of current design obtained by random disturbance of design variables and iteratively improves its design to arrive at an optimal point. Using SSA, tested are simple truss optimization problems whose constrains include combinations of yielding and buckling strengths of truss members, multiple loading conditions, and linking members in group. Satisfactory results are obtained and some discussions are given for the behavior of SSA on the tested truss structures.

1. INTRODUCTION

Among many, analytical or numerical optimization techniques-mainly based on either linear or nonlinear programming techniques-have been widely employed in structural optimization problems[1-5]. These techniques, however, generally have deficiencies of being dependent upon the choice of initial values and thus reaching at a local minimum in design space.

A combinatorial optimization technique, a recent stochastic approach to optimization problem (possibly with large interacting degrees of freedom) is formulized as $C:R \rightarrow \langle R \rangle$ i,e., finite or possibly countable infinite set of configurations(R in (R,C)) with corresponding cost(C) assigns a real number to each configuration. Stochastic simulated annealing (SSA), a branch of combinatorial techniques finds a nealy global minimum of an objective function by combining gradient descent with a random process[6,7]. The method allows, under certain conditions, choice of a set of design variables which actually increase the value of objective function, thus providing SSA with a mechanism for escaping local minima. A brief review on theoretical background on SSA is given in the following.

2. REVIEW: STOCHASTIC SIMULATED ANNEALING (SSA)

An ensemble in thermodynamics is defined as a mental collection of a very large number, η, of systems, each contains N molecules and constructed to be a replica on a thermodynamis(macroscopic) level of the actual thermodynamic system. A "canonical ensemble" is defined if a system is closed and isothermal. In a canonical ensemble, the probabiliy(P_j) observing a given quantum state Ej is equal to:

$$Pj = \frac{1}{\eta} \cdot \frac{\Sigma\ \Omega_t . n_j}{\Sigma\ \Omega_t} \quad (1)$$

where η = total number of systems in the supersystem;
n_j = number of systems with energy state of E_j;
Ω_t = possible no. of states in supersystem consistent with distribution(n1,.., nk).

If $\eta \to \infty$, the most probable distribution(to which the largest Ω_t belongs) completely dominates in average computation and thus Eq. 1 becomes:

$$Pj = \frac{e^{-Ej/kT}}{\sum_i e^{-Ei/kT}}, \quad j=1,2,... \quad (2)$$

where E_j = energy state of Ej (for a given number of molecules(N) and a volume(V));
K = Boltsman constant; and
T = tenperature.

Using Eq. 2, the average energy in the system of canonical ensemble is given as:

$$\langle E \rangle = \int E(x) \cdot P(x)dx$$

$$= \int \frac{E(x) \cdot e^{-E(x)/kT}}{\int e^{-E(x)/kT}\, dx} dx \quad (3)$$

where E,P = energy and probability in enery of E, respectively ; and
X = configurational variables
= (X1, X2,.., Xn).

The difficulties in evaluating P(X) in Eq. 3 and the proportionality of P(x) to the exp(-E/kT) lead to the use of importance sampling technique forming a MarKov chain [6]. The MarKov chain constructed below by Metropolis et.al.[6] has unique limit distribution π $(=c \cdot e^{-E/kT})$ and does not depend on the configurational integration:

$$\begin{aligned} Pij &= Pij^* & \pi_j \geq \pi_i \quad j \neq i \\ & Pij^* . \pi_j / \pi_i & \pi_j < \pi_i \quad j \neq i \\ Pii &= 1 - \sum_{j \neq i} Pij \end{aligned} \quad (4)$$

where $Pij = Pr[j, t+1/ i, t]$; (5)
$\pi_j(t) = \Sigma \pi_i(t-1)Pij$; and
Pij^*=transition matrix of the underlying MarKov chain.

Schematic SSA algorithm realizing the above equations(Eq.1 through Eq.5) is summarized below and based on which optimal solutions of trusses having different constrints are sought. C language is used for its implementation.

SSA Algorithm:

```
T = initial temperature;
while (T > Tfreezing)
    do until (equilibrium is reached)
            purturb cross sectional areas;
            obtain new volume;
            if (new volume < old volume)
                   accept purturbated cross sectional areas;
            else
                   r = random number;
                   if (exp (-(new volume-old volume)/T) > r)
                        accept purturbated areas;
                   else
                        reject;
   T is lowered
```

3. FORMULATION ON TRUSS OPTIMIZATION PROBLEMS

Design variables of interest in this study are chosen to be member cross sectional areas. Constraints being considered are tensile and compressive yielding strengths, buckling strengths and some practical considerations given by linking members in group and including multiple loading case. The problem can thus be formulated as:

$$\text{minimize } z = \sum_{k=1}^{nlc} \cdot \sum_{j=1}^{ng} \cdot \sum_{i=1}^{nm(j)} \cdot Amn(i,j).Lmn(i,j) \tag{6}$$

$$\text{subject to } g_{n,m}(A_l) \leq 0,\quad l=1,2,..,m$$
$$m=1,2,..,nlc$$
$$n=1,2,..,m$$
$$A_l \geq A_{lower\text{-}limit}$$

where nlc = number of different loading conditions;
ng = number of groups, each group with same sectional areas;
m = total number of members;
nm(j) = number of members in group j;
mn(i,j) = member no belong to group j;
Amn(i,j) = cross sectional area of i-th member in group j (mn(i,j)); and
Lmn(i,j) = length of i-th member in group j (mn(i,j)).

The temrs $g_{n,m}$ in the above equation includes the following conditions:

$$\max.(\sigma_{yc}, -\pi^2 EKA_j/L_j{}^2) \leq f_{j,k}/A_j \leq \sigma_{yt} \tag{7}$$

where
$f_{j,k}$ = internal force of member j under the k-th loading condition;
$-\pi^2 EKA_j/L_j{}^2$ = Euiler buckling strength;
E = modulus of elasticity;
K = proportional constant for radius of gyration
= 0.2 (as assumed in this study); and
σ_{yc}, σ_{yt} = yield strengths in compression and tension, respectively.

The above constrained optimization problem can be transformed into the unconstrained one by including penalty function terms[8]:

$$\text{minimize } z = \sum_{k=1}^{nlc} \cdot \sum_{j=1}^{ng} \cdot \sum_{i=1}^{nm(j)} \cdot (\, Amn(i,j) \cdot Lmn(i,j) + R \cdot \langle g_{i,j,k} \rangle^2 \,) \tag{8}$$

where R = penalty parameter; and

$$\langle g_{i,j,k} \rangle = \begin{cases} 0 & \text{if } g_{i,j,k} \le 0 \\ g_{i,j,k} & \text{if } g_{i,j,k} > 0 \end{cases}$$

4. NUMERICAL EXAMPLES

4.1 Three bar truss with a limit on yielding strengths(Fig.1)

SSA is applied to a rather simple three-bar truss which is subjected to the load P=20K with yield strengths (σ_{yc} = -15 Ksi and σ_{yt} = 20 Ksi). The exact volume of 263.9in^3 at A1=0.788in^2 and A2=0.410 in^2 is given by Schmit [10]. For the same truss the results from SSA is presented in Fig. 2. Minimum volume attained from SSA is 264.8in^3 at A1=0.76in^2 and A2=0.5in^2.

4.2 Ten bar truss with different yielding strengths for different members (Fig.3)

Another example is given for the same truss under the same loading condition except that $|\sigma_i| \le 25$, $(i=1,2,\cdots,9)$ and $|\sigma_{10}| \le 50$. The cross-sectional area at true optimum (column (2), Table 1), the areas obtained from SSA starting from initial areas of $A^o{}_i$=5 and their corresponding stresses are given in columns (3),(4) and (5) in Table 1, respectively. It can be seen from this table that except members 8 and 10, all others are of fully or almost fully stressed state. Fig. 4 shows the trends of convergence of SSA in this particular problem. The column (6) in Table 1 shows the results obtained by assigning relatively small initial areas ($A^o{}_i$=1) to each member for ten bar truss. The result reveals that regardless of the choice of initial areas, SSA allows the design variables to converge to a global minimum. It is worth mentioning that since the SSA decreases acceptance probability steadily to zero as the temperature gets lowered this makes SSA itself enable to break out of local minimum and converge upon the global minimum.

4.3 Ten bar truss with considerations on different yielding strengths, buckling strengths, multiple loading condition, and grouping between members(Fig.5):

Members are linked into 5 different groups as the following relations: $A_1=A_5$; $A_2=A_4$; $A_3=A_8$; $A_6=A_7$; and $A_9=A_{10}$. Material properties are assigned as E=300Ksi, σ_{yc} = -36 Ksi and σ_{yt} = 42 Ksi. Results from SSA are presented in Table 2 and two identical results are obtained for two different initial areas given by 1.3 in^2 and 0.4 in^2 (see Figs. 6a and 6b). At the first loading case, members 3 and 10 and at the second loading case, members 1,2 and 7 are observed to reach their buckling strengths, respectively (Table 2).

5. CONCLUSIONS

Applications of SSA to the plane truss optimization were successfully performed in obtaining optimum sectional areas of trusses. SSA has a capability of escaping local minima by accepting uphill moves during the cooling process. This characteristic of SSA was observed in this study by the independency of the SSA on the choices of initial values in reaching at the global minimum. In the future study, more realistic constraints may be introduced and some refinement of the algorithm for the efficiencies in terms of computing time is needed.

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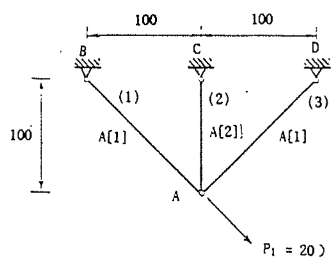


Fig. 1 Three bar truss

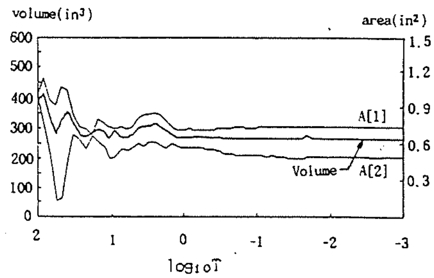


Fig. 2 Convergence to a minimum volume during the annealing process for the three bar truss. (σ_{yc} = -20 Ksi, σ_{yt} = 15 Ksi).

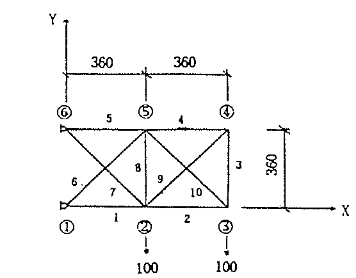


Fig. 3. Ten bar truss

Table 1. Results from SSA for ten bar truss. (σ_y^i = -25 Ksi for i=1,··,9 and σ_y^{10} = 50 Ksi for tension and compression).

(1) member no.	(2) true optimum	(3) sectional area (A_i^0=5)	(4) member force (A_i^0=5)	(5) stress (A_i^0=5)	(6) sectional area (A_i^0=1)
1	8.06	8.12	-203.1	-25	8.11
2	3.94	3.89	-96.8	-25	3.98
3	0.10	0.13	3.21	24.7	0.106
4	0.10	0.13	3.21	24.7	0.105
5	7.94	7.9	196.9	25	7.94
6	5.57	5.53	-136.9	-24.8	5.51
7	5.74	5.82	145.9	23.8	5.85
8	0.10	0.11	0.0744	0.676	0.105
9	0.1	0.18	-4.54	25	0.15
10	5.57	3.62	136.9	37.8	3.67
total volume	14976	15020			15052

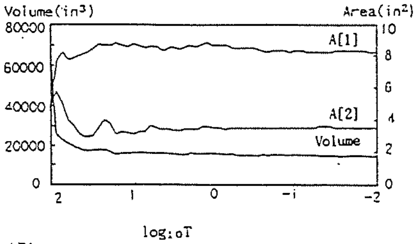
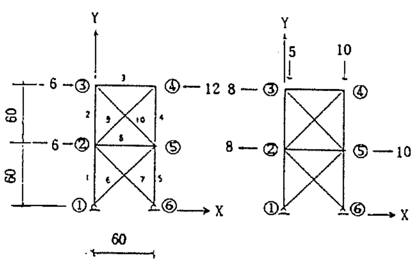
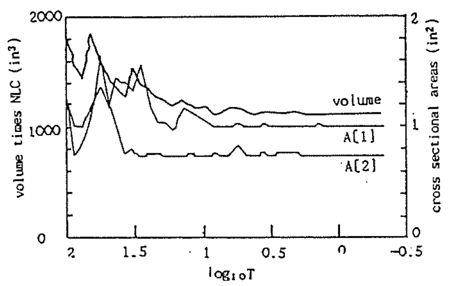


Fig. 4. Convergence to a minimum volume during the annealing process for the ten bar truss with initial areas given from previous solution set. (σ_{yc} = -25 Ksi and σ_{yt} = 25 Ksi).

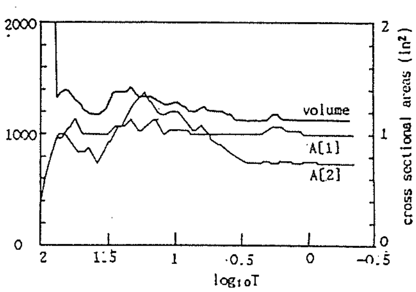


(a) load case I (b) load case II

Fig. 5. Ten-Bar Truss Geometries and Loading Conditions



Convergence trends to a global minimum volume with initial areas equal to 1.3in^2.



Convergence trends to a global minimum volume with initial areas equal to 0.3in^2.

Fig. 6. Ten bar truss: constraints on yield strength, buckling strength, members in group and multiple loading conditions

Table 2. Results from SSA for ten bar truss with considerations on yield strength, buckling strength, multiple loading conditions and members in group.,

initial area	A^o_i = 1.3 in^2							A^o_i = 0.4in^2
group no.	member no.	area (in^2)	min. (σ_B, σ_c)	load case 1		load case 2		area (in^2)
				force	stress	force	stress	
1	1	0.982	-16.1	-5.79	-5.9	-15.8	-16.1	0.982
	5	0.982	-16.1	6.21	6.32	1.17	1.19	0.982
2	2	0.737	-12.11	-1.57	-2.12	-8.92	-12.1	0.737
	4	0.737	-12.11	4.43	6.02	-5.9	-8.03	0.737
3	3	0.679	-11.15	-7.57	-11.15	4.08	6.0	0.679
	8	0.679	-11.15	-1.36	-2.0	9.25	13.6	0.679
4	6	0.741	-6.08	-0.293	-0.396	4.0	5.4	0.741
	7	0.741	-6.08	-0.293	-0.396	-4.49	-6.1	0.741
5	9	0.873	-7.18	2.218	2.54	5.55	6.35	0.873
	10	0.873	-7.18	-6.27	-7.18	-5.77	-6.6	0.873
total volume		561.6 in^2						561.6 in^2