ON SLOW CRACK GROWTH IN FUEL CLADDING BY FINITE ELEMENT ANALYSIS

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ABSTRACT

The present paper deals with the use of a thermoelasto-plastic finite element analysis for the prediction of the instability of a single crack or cracks and their subsequent growth in thin Zircaloy fuel claddings. Special elements called "Breakable elements" were developed for this purpose. This new computational algorithm allows not only considerably larger elements near the crack tip but also the gradual advance of crack front without having to re-number the finite element model. The equivalent rupture strain used as the criterion of initial crack growth is obtained from experimentally determined R-curve for the cladding material. It has been shown that slow growth of a minute crack in a fuel cladding resulting from the application of fission gas pressure can be determined analytically thus avoids costly experimental measurements.

1. Introduction

Nuclear fuel elements normally consist of uranium dioxide pellets encased in leak-tight zirconium alloy cladding. The pellets and cladding form the first two lines of containment for radiative fission products. Abrupt rupture of cladding may cause the dispersion of poisonous fission products into the coolant and henceforth other parts in the reactor system. It is important for the fuel designer to be able to predict quantitatively the rate of growth of minute cracks nucleated in the vicinity of welds and defects in the fuel cladding under various loading conditions. Although the cladding material is not considered to be brittle so that a catastrophic crack propagation associated with brittle fracture does not exist, ductile failure can also be serious if cracks grow to a size sufficient to become self-sustaining or unstable resulting in rapid propagation.

Numerous publications have been appearing on the mathematical treatment of brittle fracture of solids after Griffith's treatment in 1921 [1]*. Among these publications, pertinent references which deal with through cracks in cylindrical shells are due to Folias [2,3]. One major drawback of his work is that the analyses are based entirely on the theory of linear ealsticity and no attempt was made to predict the growth of the cracks.

The present investigation intends to predict the gradual growth of a minute crack or cracks on a fuel cladding. A thermoelasto-plastic analysis based on the finite element variational technique is used to calculate the stress and displacement fields in the vicinity of the growing crack. A special type of element called "Breakable element" is developed to model the gradual propagation of the crack with the crack extension strain established by the experimentally determined "crack growth resistance curve" or R-curve for the cladding material.

2. Theoretical Formulations of Thermoelasto-plastic Analysis

Solutions of the more complicated elastoplastic stress analyses of solid structures have now become reality with the rapid development of the finite element variational technique. Earlier work by Marcal and King [4], Yamada [5] and Zienkiewicz et al [6] made it possible to perform quantitative elasto-plastic analysis on complicated geometries.

The present analysis utilizes a generalized hardening rule and flow law to incorporate both temperature and strain rate dependent material properties. The yield surfaces which comply with von Mises' yield criterion may expand or contract as temperature and strain rate vary. As usual, the inertia effect is neglected and thermal and stress fields are assumed to be uncoupled, but both are made dependent upon the current deformed geometry and are solved quasi-simultaneously. The principle of incremental stationarity of potential energy is used with simplex elements giving in each loading step:

$$\{\Delta Q_\ell\} = [K_\ell]\{\Delta u_\ell\} \tag{1}$$

where $\{\Delta Q_\ell\}$ = incremental loads,

$[K_\ell]$ = instantaneous stiffness matrix of structural material,

$\{\Delta u_\ell\}$ = deformation vectors.

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* Numbers in brackets designate References at the end of the paper.

Thus be referring to the detailed derivations described in Ref. [7] the constitutive equations for the structure is expressed as:

$$\{d\sigma\} = [Cep]\{d\varepsilon\} - [Cep](\{\alpha\}\Delta T + \frac{\partial [Ce]^{-1}}{\partial T}\{\sigma\}\Delta T + \frac{\partial [Ce]^{-1}}{\partial \dot{\bar{\varepsilon}}}\{\sigma\}\, \Delta\dot{\bar{\varepsilon}})$$

$$- [Ce]\,\{\frac{\partial F}{\partial \sigma}\}/[S\,(\frac{\partial F}{\partial T}\Delta T + \frac{\partial F}{\partial \dot{\bar{\varepsilon}}}\Delta\dot{\bar{\varepsilon}})]$$

where $\{d\sigma\}$, $\{d\varepsilon\}$ are the respective incremental stresses and strains; $\{\alpha\}$ is the thermal expansion coefficient matrix; ΔT and $\Delta\dot{\bar{\varepsilon}}$ are the respective temperature and effective strain rate.

The plastic potential function F is defined by the von Mises criterion as:

$$\{\frac{\partial F}{\partial \sigma^T}\} = \{\sigma'\}$$

with $\{\sigma'\}$ to be the deviatoric stress components.

The elasto-plastic matrix [Cep] is:

$$[Cep] = [Ce] - [Cp] \qquad (2)$$

where [Ce] = elasticity matrix, or the coefficients of strain components in a generalized Hooke's law of stress and strain; the plasticity matrix, [Cp] can be expressed as:

$$[Cp] = \frac{[Ce]\{\sigma\}\{\sigma\}^T[Ce]}{S}$$

in which S = 3G + H', with G = shear modulus of elasticity and $H' = d\bar{\sigma}/d\bar{\varepsilon}_p$ from the experimentally determined effective stress $\bar{\sigma}$ vs. effective strain $\bar{\varepsilon}$ relation.

The stiffness matrix in Eq. (1) can be derived by the usual variational procedures identical to the elasticity analysis with the elasto-plasticity matrix in Eq. (2), or as illustrated in [7,8].

3. Algorithms for Crack Propagation

For constant stress/strain elements such as used in the present investigation, difficulty in maintaining smooth propagation of the crack front may be anticipated due to the steep strain gradient near the crack tip. Also, the constant strain element does not predict the strain at the actual crack front, which is assumed to start from an element's free edge, but consistently under estimates. Therefore, it is proposed to extrapolate a "smoothed" strain distribution towards the crack front as shown in Figs. 1(b,c). Upon just exceeding the assigned rupture strain, $\bar{\varepsilon}_{rup}$, the crack growth process is begun. Fig. 1(b) illustrates schmatically the start of crack propagation at load $\sigma_{0,i}$. A portion, x, of the crack tip element is estimated to have exceeded the rupture criterion and to have become incapable of carrying any further load. For the next load step $\sigma_{0,i+1}$, that element's stiffness matrix is evaluated with a proportionally reduced volume. In this subsequent load step an additional portion, x', may be found to exceed the rupture criterion. The same procedure is repeated until at some load $\sigma_{0,i+n}$, the whole element gradually is "broken" and its stiffness simultaneously reduced to zero. The element is maintained as a zero stiffness element so that renumbering and complete matrix rebuilding is not required. Boundary conditions on the nodes behind the crack front then can be changed to free boundary nodes. Thus, the crack front has passed smoothly through a breakable element and incrementally redistributed its load to surrounding elements.

The assigned rupture strain for the cladding material is obtained from the experimentally determined "crack growth resistance curve" or "R-curve" such as shown in Fig. 2. The effective strain at the crack tip corresponding to the applied load at the "zero" stable crack extension point in Fig. 2 can be calculated by the elasto-plastic finite element analysis on the CT specimen shown in the same figure. Physically, this strain represents the threshold strain (or rupture strain), above which crack starts to grow as experimental measurements indicated. This strain value at the crack tip is extrapolated by a 4-point least square curve fitting scheme with the average effective strains in the four elements immediately preceding the crack tip. Four data points scheme was adopted as it gave most satisfactory results for such extrapolation. The distribution of the effective strain ahead of the crack tip thus follows a cubic function.

Once the value of $\bar{\varepsilon}_{rup}$ of the cladding material is established, the computation of crack growth in a fuel cladding can be performed. The determination of the portion of the element that exhibits effective strain greater than the $\bar{\varepsilon}_{rup}$ as illustrated in Fig. 1(b) can be accomplished by solving a cubic equation describing the strain distribution ahead of the crack front. Newton's iteration method is used for this purpose.

An automatic load increment scheme is adopted for better computational efficiency and also avoiding the possibility of skipping the breakable elements by excessive load increments. Referring to Fig. 3, the initial applied load causes the crack to propagate by an amount of ΔL_{i-2} in a breakable element of length L. The position of the crack front due to the next load increment $\sigma_{0,i-1}$ can either be within the element at point A with an additional growth ΔL_{i-1}; or beyond the element at location B with $\Delta L'_{i-1}$ growth. The load increment for the next step to "bring" the crack front to the edge of the element can be estimated by:

$$\sigma_{0,i} = \frac{L - (\Delta L_{i-1} + \Delta L_{i-2})}{\Delta L_{i-1}} \sigma_{0,i-1}$$

or

$$\sigma_{0,i} = \frac{\Delta L_{i-2} - L}{\Delta L'_{i-1}} \sigma_{0,i-1} \qquad (3)$$

4. Numerical Illustration

A representative example is presented to demonstrate the general adaptability of the analytical technique described here for assessing the growth of a crack in a fuel cladding made of Zirconium - 2.5% wt Niobium material. The conditions used in the computation are listed as follows:

Dimensions of the cladding: 19.76 mm O.D. x 18.64 mm I.D. x 500 mm long

Axial initial crack length: 6.35 mm

Material properties:

Specimen No.	1	2
Temperature, °C	20	300
Hydrogen concentration, μg/g	10	200
Modulus of elasticity, GPa	95	78.7
Plastic modulus, MPa	201	96.5
Yield strength, MPa	800	531

The rupture strains for the above CT specimens were found to be 8% for specimen no. 1 and 6% for specimen no. 2 by TEPSA code, with the crack initiation stresses indicated in Fig. 2 [9] and the finite element model described in Ref. [10].

Strictly speaking, the stress analysis of cracks in a tube (cladding) subject to internal pressure loading (e.g. fission gas pressure) should be treated by a 3-dimensional analysis which involves tedious formulation and require a large amount of computer time. However, 2-dimensional models such as plane stress analysis on thin plates can be used as a reasonable approximation provided that the ratio of the diameter of the tube to its wall thickness is large and the size of the crack small [2]. The problem of a thin cladding containing a longitudinal crack subject to internal pressure loading is thus simplified as illustrated in Fig. 4. The same finite element model described in Ref. [11] was used for the present analysis on material number 1.

5. Results and Discussion

The growth of a minute longitudinal crack in a fuel cladding due to the increasing internal fission gas pressure is depicted in Fig. 5. Pertinent information such as the growth rate of this crack can thus be determined with given fission gas release rate. Since relatively high rupture strain of 8% was used as criterion for the crack to propagate, considerable computing time would be required to simulate the graduate progression of the crack front. Measures which would significantly improve the computational efficiency had to be taken and one of such measures was the automatic load application scheme described in Eq. (3). This scheme allowed no more than three load increments for the crack front to pass through one breakable element. These load increments were found to be too coarse, resulting significant under estimation of effective strain in the breakable elements at the vicinity of the current crack front. Results shown in Fig. 5 therefore tend to be on conservative side. Proper modification of this scheme to allow for smoother plastic deformation of the breakable elements according to the given material's stress/strain curve would be desirable.

Another possible error in the present analysis may be introduced by the approximation of a plane stress model for a thin shell type of structure such as fuel claddings. This approximation may prove to be satisfactory for small initial cracks. A simple formula for checking the validity of such approximation is available in Ref. [2].

With the versatility provided by the finite element method described in this paper, this technique can be readily extended to predict the propagation of cracks in fuel claddings with arbitrary orientation, as well as cases involving branch of cracks. The complex situation of interaction between propagating cracks and the leaking coolant could also be included in a similar manner described in [12].

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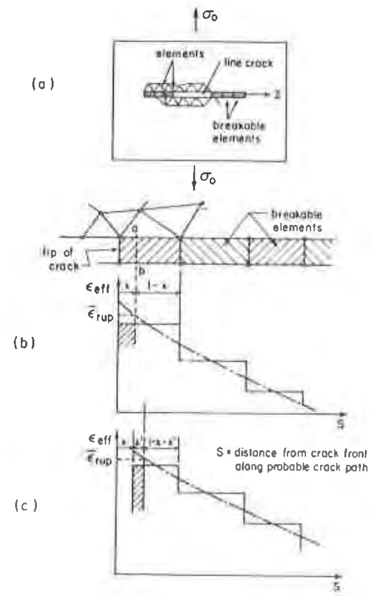

Figure 1 Numerical Scheme for Crack Propagation.

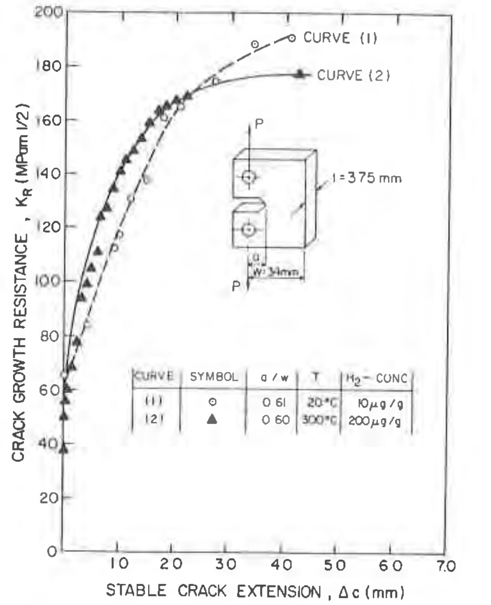

Figure 2 R-curves for Zr-2.5% wt Nb Specimens Determined from COD Measurements.

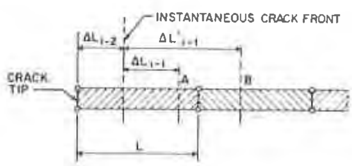

Figure 3 Positions of Crack Fronts for Automatic Load Application.

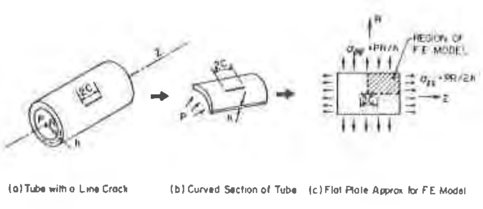

Figure 4 Loading and Geometry of a Crack in a Fuel Cladding.

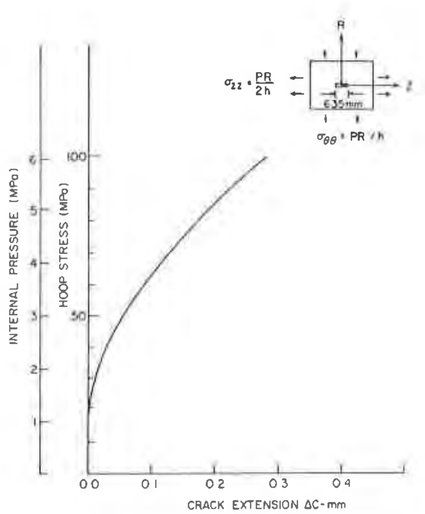

Figure 5 Crack Extension in a Fuel Cladding by Finite Element Analysis.