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Energy-Balanced Approach to Evaluate Local Effects of Impact of Non Deformable Missiles on Concrete Structures

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Abstract

The energy-balanced approach is used in this study to evaluate the local effects of nondeformable missiles on concrete structures in terms of penetration depth. For nondeformable missiles, the kinetic energy of the missiles are dissipated into local strain energy in terms of penetration, back-face scabbing, or perforation, as well as overall structural response. This study confirms that the energy dissipation due to the overall structural response is very small compared to the local effects in most cases, and can be neglected. The energy-balanced approach is a viable alternative approach to estimate the penetration depth of nondeformable missiles striking concrete structures. The equations proposed in this paper can be used for this purpose. Once the penetration depth is known, the target thickness necessary to prevent back-face scabbing can be easily estimated.

1. Introduction

Over the last decade, a considerable amount of progress has been made in estimating the local effects of nondeformable missiles, in particular turbine missiles, on safety-related concrete structures in a nuclear power plant. The classical paper by Kennedy [1] accelerated the process. Subsequently, research and testings were carried out by several researchers considering various aspects of the problem [2,3,4,5,6]. Haldar and Hamieh [6] summarized most of the damage-predicting equations commonly used to estimate the local effects. In the same paper, they also discussed the deficiencies in these equations. To improve the predictability of the equations, Haldar and his associates [7] proposed a nondimensional impact factor. Using consistent units, the impact factor, I, is defined as

$$I = \frac{12}{32.2} \times \frac{NWv^2}{d^3 f'_c} \tag{1}$$

where W = weight of the missile in lbs, v = velocity of the missiles in fps, d = diameter of the missile in inches, f'_c = compressive strength of concrete in psi, and N = missile shape factor [7].

Using all the available test results for small as well as large missiles including bullets (625 test results for pure penetration and 176 test results where back-face scabbing was observed), Haldar and Hamieh [6] proposed functional relationships between the impact factor and the penetration depth, x, and the impact factor and the scabbing thickness, s. They can be summarized as:

$$\frac{x}{d} = -0.0308 + 0.2251 \ I \ ; \ 0.3 \leq I \leq 4.0 \qquad (2)$$

$$\frac{x}{d} = 0.6740 + 0.0567 \ I \ ; \ 4.0 < I \leq 21.0 \qquad (3)$$

$$\frac{x}{d} = 1.1875 + 0.0299 \ I \ ; \ 21.0 < I \leq 455.0 \qquad (4)$$

and

$$\frac{s}{d} = 3.3437 + 0.0342 \ I \ ; \ 21.0 \leq I \leq 385.0 \qquad (5)$$

The predictability of these equations are found to be superior to the corresponding National Defense Research Council (NDRC) equations [6].

The conservation of energy is one of the earlier approaches for estimating the local effect. Thus, it is prudent to see if the energy-balanced approach using the available test results can be used to evaluate the local effects of impact. This is the purpose of this paper. Only the local effects of the impact of nondeformable missiles on concrete structures are discussed in this paper.

2. Problem Description

When a missile hits a target with a known kinetic energy, the structure undergoes deflection, penetration, perforation, scabbing and/or spalling, depending on many factors such as the weight and velocity of the missile, the impact area of the missile, the time history of impact, the strength of the concrete structures, etc. The exact energy dissipation mechanism due to the impact of a missile on a concrete structure is a difficult problem. For this discussion, the kinetic energy of the missile is considered to be dissipated into the following three forms: (1) local effects in terms of penetration, scabbing and perforation of the target due to the impact, (2) the overall structural response, and (3) the strain energy of the missile. Since nondeformable missiles are under discussion in this paper, the strain energy of the missile will not be discussed here. In the following sections, methods to evaluate the local effects and the overall structural response are developed.

3. Local Effects

A complete mathematical formulation to evaluate the local effects or the loss of energy associated with local missile penetration, scabbing or perforation is extremely complicated. However, this energy could be estimated by the analysis of test data already available.

To evaluate the loss of energy due to impact, several cases need to be considered. They can be summarized as: (1) the missile does not penetrate the target, (2) the missile penetrates a distance x into the target, (3) the missile penetrates a distance x into the target and produces a back-face crater, and (4) the missile perforates the target and exits with a known velocity. In the case of no penetration, the loss of energy due to the impact is zero or very small compared to the other factors, and is not discussed further here. Moreover, to protect safety-related structures in a nuclear power plant against missiles, the penetration and back-face scabbing damage criteria are generally used. For this reason, these two damage criteria are specifically discussed here.

3.1 Loss of Energy During Pure Penetration

To evaluate the loss of energy during penetration, all test results available in the literature for bullets as well as solid missiles were collected [6]. 590 test results for bullets and 33 test results for missiles were found to cause pure penetration. The kinetic energy, KE, of each of these bullets and missiles in kips-ft can be plotted against the ratio, x/d (the observed penetration depth to the bullet or missile diameter), in the form of a scatter diagram. They cannot be shown here due to lack of space. All bullet data can be grouped into four sub-groups in terms of their weights where the KE versus x/d plottings are linear. Similarly, missile test data are grouped into three sub-groups. Denoting LE as the loss of energy due to the local effect of impact, the results can be summarized as follows:

For Bullets

$$LE = 0.587 \frac{x}{d} - 0.312 \quad ; \quad 0.0441 \leq W \leq 0.13 \text{ lb} \tag{6}$$

$$LE = 20.096 \frac{x}{d} - 23.766 \quad ; \quad 1.67 \leq W \leq 1.91 \text{ lb} \tag{7}$$

$$LE = 154.43 \frac{x}{d} - 187.39 \quad ; \quad 12.34 \leq W \leq 15.0 \text{ lb} \tag{8}$$

$$LE = 879.23 \frac{x}{d} - 819.304 \quad ; \quad 85.0 \leq W \leq 108.0 \text{ lb} \tag{9}$$

Equations (6), (7), (8), and (9) are developed by considering 241, 293, 33, and 23 test results, respectively. The corresponding r^2-values of the regression equations are 0.77, 0.97, 0.96, and 0.94. The r^2-value is also known as the coefficient of determination. It assumes values between 0 and 1.0. When r^2 is close to 1, it implies that most of the variability in the dependent variable, in this case x/d, is explained by the regression model. Except for eq. (6), the r^2-values for all the equations are very high. The predictability of these equations is expected to be very good.

For Missiles

$$LE = 4.437 \frac{x}{d} - 2.953 \quad ; \quad 0.24 \leq W \leq 0.26 \text{ lb.} \tag{10}$$

$$LE = 17.48 \frac{x}{d} - 6.37 \quad ; \quad 1.0 \leq W \leq 5.0 \text{ lb} \tag{11}$$

$$LE = 50.547 \frac{x}{d} - 0.06 \quad ; \quad 12.5 \leq W \leq 23.5 \text{ lb} \tag{12}$$

Equations (10), (11), and (12) are developed by considering 9, 19, and 5 test results, respectively. The corresponding r^2-values are 0.98, 0.81, and 0.67, respectively.

3.2 Lack of Energy During Back-Face Scabbing

115 test results can be identified where both the penetration depth and the back-face crater depth are available. All these test results are for bullets. In this case, as proposed by Haldar and Hamieh [6], the dependent variable of the regression equation is considered as $(x/d - s_0/d)$. S_0 is the observed back-face crater depth. Proceeding as in the previous section, the following regression equations are proposed:

$$LE = 0.616 \left(\frac{x}{d} - \frac{s_0}{d}\right) - 0.0735 \; ; \; 0.0441 \leq W \leq 0.13 \text{ lb} \tag{13}$$

$$LE = 22.486 \left(\frac{x}{d} - \frac{s_0}{d}\right) - 23.507 \; ; \; 1.67 \leq W \leq 1.91 \text{ lb} \tag{14}$$

$$LE = 154.541 \left(\frac{x}{d} - \frac{s_0}{d}\right) - 127.655 \; ; \; 12.35 \leq W \leq 15.0 \text{ lb.} \tag{15}$$

$$LE = 1057.494 \left(\frac{x}{d} - \frac{s_0}{d}\right) - 692.361 \; ; \; 85.0 \leq W \leq 108.0 \text{ lb} \tag{16}$$

Equations (13), (14), (15), and (16) are developed by considering 7, 54, 26, and 28 test results. The corresponding r^2-values are 0.67, 0.91, 0.96, and 0.96, respectively. The validity of eqs. (6) through (16) will be discussed further later.

4. Overall Structural Strain Energy

The structure being impacted by the missile will deform, and thus will dissipate some energy. It is important to evaluate this energy loss. The structure considered in this paper is idealized as a plate supported at all four sides. The plate is initially flat, of uniform thickness, and the thickness is small compared to the other dimensions. Most of the tests conducted for large missiles fit this idealization.

The equation of motion for the nondeformable impacting missile at any time t_i can be represented as

$$\frac{W}{g} \frac{d^2 x_i}{dt_i^2} = - p_i A_c \tag{17}$$

where A_c = the contact area of the projectile; x_i = the penetration depth at time t_i; p_i = the impact pressure per unit contact area; and W = the weight of the missile. Kennedy [1] suggested that reasonable results can often be obtained by assuming that the impact force remains constant throughout the entire duration of impact. With this assumption, the duration T and the average constant force P can be estimated as [1]:

$$T = \frac{2x}{d} \tag{18}$$

and

$$P = \frac{Wv}{gT} \tag{19}$$

where v = missile's striking velocity. All the other parameters in eqs. (18) and (19) were defined earlier.

The strain energy, stored in the plate during deformation, is found by integrating (over the entire middle surface) the negative work of internal forces. In general, the strain energy of the plate consists of bending, U_b, which is [8]:

$$U_b = \frac{1}{2} \iint D \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2 (1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy \tag{20}$$

However, if all edges are supported, the second term of eq. (20) drops out and we obtain

$$U_b = \frac{1}{2} \iint D \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 dxdy \qquad (21)$$

and

$$D = \frac{E\,h^3}{12(1-\nu^2)} \qquad (22)$$

where ν = the Poisson Ratio; w = the deflection in the direction of the impact normal to the plate; h = the thickness of the target; E = modulus of elasticity, $E = 57\sqrt{f'_c}$; and f'_c = ultimate concrete strength.

The potential of the external force, V, is defined as the negative work done by the external force, P (eq. 19), at point (x,y) and can be written as

$$V = - P\, w \Big|_{xy} \qquad (23)$$

where w = deflection at the point of application of the external load P. The total potential energy of the plate under consideration, π, is the summation of eqs. (21) and (23).

The Rayleigh-Ritz method can be used to solve the potential energy. The deflected shape of a simply supported plate can be expressed as:

$$w = C \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \qquad (24)$$

where C = a constant to be determined and a and b = the dimensions of the plate.

Substituting eq. (24) into eqs. (21) and (23), U_b and V can be shown to be:

$$U_b = \frac{D}{8} \pi^4 C^2 ab \left(\frac{1}{a^2} + \frac{1}{b^2} \right)^2 \qquad (25)$$

$$V = - PC \sin \frac{\pi\alpha}{a} \sin \frac{\pi\beta}{b} \qquad (26)$$

Differentiating the total potential energy, π, with respect to C and equating it to zero, the constant C can be estimated. The value of C for the problem under consideration is found to be:

$$C = \frac{4\, Pa^3 b^3 \sin \frac{\pi\alpha}{a} \sin \frac{\pi\beta}{b}}{D\pi^4 (a^2 + b^2)^2} \qquad (27)$$

All the parameters in eq. (27) are known. Thus, the overall strain energy of the plate can be obtained by using eq. (25).

5. Evaluation of Proposed Relationships

For a given missile with all properties known, the energy dissipated locally due to the impact can be estimated by using eqs. (6) through (16). The total energy dissipated can be estimated by adding the contribution of eq. (25) to this information. This concept can be evaluated by comparing the predicted loss of energy due to impact to the observed kinetic energy before the impact. Two factors R_1 and R_2 are introduced here such that:

$$R_1 = \frac{LE}{KE} \tag{28}$$

and

$$R_2 = \frac{LE + SE}{KE} \tag{29}$$

where LE = the loss of energy due to the local effect; SE = the loss of energy due to structural response; and KE = the kinetic energy of the incoming missiles. SE is calculated by considering a simply supported slab of dimensions 9 ft x 9 ft being hit at the center of the panel. The Poisson ratio for concrete is assumed to be 0.2. The modulus of elasticity of concrete, E, is considered to be $57000\sqrt{f'_c}$ in psi.

Using all the 738 available test results, the R_1 and R_2 factors are calculated using the appropriate equations. The mean values of R_1 and R_2 are found to be 1.033 and 1.051, respectively.

6. Conclusions

This study confirms that the energy dissipation due to the overall structural response is very small compared to the local effects in most cases and can even be neglected. The energy-balanced approach is a viable alternative approach to estimate the penetration depth of nondeformable missiles on concrete structures. Once the penetration depth is known, the target thickness to prevent back-face scabbing can be easily estimated, as discussed by Haldar and Hamieh [6] elsewhere.

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