CALCULATION OF ADDED WATER MASS EFFECTS FOR REACTOR SYSTEM COMPONENTS

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SUMMARY

The vibrations of many components of reactor systems such as heat exchanger tubes and rectangular channels are greatly affected by the interaction of these components with the surrounding water. The inertial properties of the water can be represented by an added mass matrix, which is an expression of such interaction. A computational approach is presented, based on the finite element method, for deriving the added mass matrix of complex systems of components. The method is two-dimensional, and is valid for long axial bodies of arbitrary cross section which are defined by a nodal grid. The results are expressed in mass per unit axial length. No account is taken of end leakage and similar effects. The method is based on the successive solutions of Laplace's equation for a number of fundamental boundary conditions (in which each body in turn is given a unit acceleration in two orthogonal directions, one at a time). Examples are given concerning the added mass matrices of well known configurations (for instance, concentric cylinders) in order to demonstrate the accuracy of the method. Examples are also given of more complex configurations involving pipes and channels found in reactors.

1. Introduction

When an object which is submerged in a fluid vibrates, its motion causes some of the fluid to move with it. The mass of the moving fluid has an important effect on the dynamics of the vibrating object, particularly on its natural frequencies. The dynamics of structural components in fluids are often studied by introducing the concept of the added mass (otherwise known as virtual mass, or, hydrodynamic mass). This additional mass represents inertial effects that the component incurs due to the motion of the adjacent fluid. It is possible to estimate the amount of the added mass and a method of so doing will be presented in this paper.

When several structural components are submerged in the same fluid, the motion of the fluid during vibration causes an interaction between the components. Their vibrational responses become coupled. The added mass concept can be extended to this case by introducing an added mass matrix, which expresses the interaction influence of the fluid on the vibrating components. We shall limit the discussion to structural systems in which the flow of fluid is two-dimensional, although the basic concepts can be readily extended to three dimensions. Examples of such systems include boilers, heat exchangers, and other piping systems whose lengths are substantially greater than the cross-sectional dimensions.

2. Potential Flow -- A Variational Principle

For many purposes, the motion of the fluid may be adequately represented by the potential flow theory of an incompressible fluid. In the case of small amplitude motions, it can be shown [1,2] that the fluid pressure p is represented by Laplace's equation

$$\nabla^2 p = 0 \tag{1a}$$

subject to the boundary condition that on the surfaces of the submerged structure,

$$\frac{\partial p}{\partial n} = -\rho a_n \tag{1b}$$

Here n is the normal from the boundary into the fluid, and a_n is the acceleration of the boundary in the normal direction. The fluid density is given by ρ.

In the two-dimensional case that we consider here, Laplace's equation is written as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \tag{2a}$$

and the boundary condition can be stated as

$$\ell_x \frac{\partial p}{\partial x} + \ell_y \frac{\partial p}{\partial y} = -\rho a_n \tag{2b}$$

Here, x and y are the coordinates of the two-dimensional plane, while ℓ_x and ℓ_y are the direction cosines of the normal to the boundary directed into the fluid.

We shall first discuss a method for the solution of Laplace's equation in two dimensions, without specific reference to the added mass determination. To solve this equation, we shall use the finite element method. A related finite element approach to added mass calculations has been described by Hartz and Schmid [3]. It is shown by Zienkiewicz [2] that the solution of eq.(2) is equivalent to the variational problem of minimizing the functional

$$\Psi = 1/2 \iint_A \left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2 \right] dxdy + \int_C \rho a_n p ds \qquad (3)$$

where A represents the area of the fluid section, and C denotes the contour of the fluid boundaries. The integral around the boundaries is such that when the right hand faces the fluid, the direction of integration is forward. Thus, integration is carried out in an anti-clockwise direction on interior boundaries and in a clockwise direction on the outer boundary. To demonstrate that the variational problem of minimizing Ψ in eq.(3) is equivalent to eq.(2), we compute the variation $\delta\Psi$. Denoting $\partial p/\partial x$ by p_x and $\partial p/\partial y$ by p_y, we have

$$\delta\Psi = 1/2 \iint_A \left[\frac{\partial}{\partial p_x} (p_x^2 + p_y^2)\delta p_x + \frac{\partial}{\partial p_y} (p_x^2 + p_y^2)\delta p_y \right] dxdy$$

$$+ \int_C \rho a_n \delta pds \qquad (4)$$

which reduces to

$$\delta\Psi = \iint_A (p_x \delta p_x + p_y \delta p_y) dxdy + \int_C \rho a_n \delta pds \qquad (5)$$

Now the commutation rule applies between the operators δ and $\partial/\partial x$, so that

$$\delta p_x = \delta\left(\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial x} \delta p$$

and similarly

$$\delta p_y = \frac{\partial}{\partial y} \delta p$$

Thus

$$\delta\Psi = \iint_A \left[p_x \frac{\partial}{\partial x} \delta p + p_y \frac{\partial}{\partial y} \delta p \right] dxdy + \int_C \rho a_n \delta pds \qquad (6)$$

Now carry out an integration by parts. Observing that

$$\iint_A p_x \frac{\partial}{\partial x} \delta p \, dxdy = \int_C p_x \delta p \ell_x ds - \iint_A \frac{\partial}{\partial x} p_x \delta p \, dxdy \qquad (7)$$

we have

$$\delta\Psi = \iint_A \delta p \left[-\frac{\partial}{\partial x} p_x - \frac{\partial}{\partial y} p_y \right] dxdy + \int_C \delta p \left[\rho a_n + \ell_x p_x + \ell_y p_y \right] ds \qquad (8)$$

In order to minimize the functional Ψ, we require that for all variations δp,

$$\delta\Psi = 0 \tag{9}$$

This condition can only be satisfied if the expressions inside the integrals of eq.(8) vanish. Thus we require

$$-\frac{\partial}{\partial x} p_x - \frac{\partial}{\partial y} p_y = 0 \text{ in A} \tag{10a}$$

$$\rho a_n + \ell_x p_x + \ell_y p_y = 0 \text{ on C} \tag{10b}$$

and these equations can be rewritten as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \text{ in A} \tag{11a}$$

$$\ell_x \frac{\partial p}{\partial x} + \ell_y \frac{\partial p}{\partial y} = -\rho a_n \text{ on C} \tag{11b}$$

These equations (11a) and (11b) are the same as those which we originally used to define the fluid pressures. Thus, we have demonstrated that the minimization of the functional Ψ of eq.(3) is equivalent to the solution of the field eq.(2). The pressure field that minimizes eq.(3) is the same pressure field that satisfied Laplace's equation (2a) and the pressure gradient at the boundary that minimizes eq.(3) is the same pressure gradient at the boundary that satisfies the boundary condition, eq.(2b).

3. The Finite Element Method Applied to Potential Flow

Having now shown that the pressure field that minimizes the functional Ψ of eq.(3) also satisfies the appropriate field equation and boundary conditions, we shall proceed to carry out the minimization by means of the finite element method.

Let the area A of the fluid surface be represented by a number of triangular elements as shown in Figure 1. There, we show an interior body surrounded by fluid, and enclosed by an outer boundary. It is assumed that the accelerations of the boundaries are known. Now, attention is fixed on a single triangle, shown in Figure (2a). The nodes of the triangle are numbered by the letters k, l, n, as shown. We assume that within the triangular area, the pressure varies linearly in the coordinates x and y. Thus, we write

$$p = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{12}$$

where α_1, α_2, and α_3 are constants to be determined. The pressure may also be written in terms of the pressures at the nodes p_k, p_l, p_n. We observe that

$$\begin{Bmatrix} p_k \\ p_l \\ p_n \end{Bmatrix} = \begin{bmatrix} 1 & x_k & y_k \\ 1 & x_l & y_l \\ 1 & x_n & y_n \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} \tag{13}$$

so that, solving for α_i, we have

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_k & y_k \\ 1 & x_l & y_l \\ 1 & x_n & y_n \end{bmatrix}^{-1} \begin{Bmatrix} p_k \\ p_l \\ p_n \end{Bmatrix} = [d_{ij}] \begin{Bmatrix} p_k \\ p_l \\ p_n \end{Bmatrix} \tag{14}$$

In addition, we note from eq.(12) that

$$\frac{\partial p}{\partial x} = \alpha_2 , \quad \frac{\partial p}{\partial y} = \alpha_3$$

As a result,

$$\begin{Bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \end{Bmatrix} = \begin{bmatrix} d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{Bmatrix} p_k \\ p_l \\ p_n \end{Bmatrix} = [D] \{ p_e \} \tag{15}$$

We have defined the pressures at the nodes of the element as $\{p_e\}$

$$\{p_e\} = \begin{Bmatrix} p_k \\ p_l \\ p_n \end{Bmatrix} \tag{16}$$

Since

$$\left[\frac{\partial p}{\partial x} \ \frac{\partial p}{\partial y} \right] \begin{Bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \end{Bmatrix} = \left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 \tag{17}$$

we also have

$$\left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 = \{p_e\}^T [D]^T [D] \{p_e\} \tag{18}$$

The first portion of the functional Ψ in this element is therefore given by

$$\Psi^e = 1/2 \iint_{A_e} \left[\left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 \right] dxdy = 1/2 \{p_e\}^T [D]^T [D] \{p_e\} A_e$$

$$= 1/2 \{p_e\}^T [h_e] \{p_e\} \tag{19}$$

where A_e is the area of the element. The area is related to the coordinates of the element nodes, and it can be shown that

$$2A_e = \det \begin{bmatrix} 1 & x_k & y_k \\ 1 & x_l & y_l \\ 1 & x_n & y_n \end{bmatrix} \tag{20}$$

The remaining integral of the functional Ψ has a contribution only at the fluid boundary. Imagine that the j^{th} segment of the boundary is formed by the line of length L_j between two nodes k and l, as shown by Figure 2b. The normal acceleration of the segment is denoted by a_j. Then the contribution Ψ^b of the segment is

$$\Psi^b = \int_{C_j} \rho a_j p ds = 1/2 \ \rho a_j L_j (p_k + p_\ell) = \{g_b\}^T \{p_b\} \tag{21}$$

where

$$L_j = [(x_k - x_l)^2 + (y_k - y_l)^2]^{1/2}$$

and the elements of $\{g_b\}$ are made up of the coefficients of the segment boundary pressures $\{p_b\}$. The process of assembly follows by the addition of the element contributions. Thus, if we compute the sum of all element contributions we have

$$\Psi^E = \sum_{\text{elements}} \Psi^e \tag{22}$$

and similarly, the sum of all boundary segment contributions is

$$\Psi^B = \sum_{\text{segments}} \Psi^b \tag{23}$$

Now the total functional Ψ is the sum of contributions from the elements and the boundaries,

$$\Psi = \Psi^E + \Psi^B \tag{24}$$

and as a result of these additions, we have

$$\Psi = 1/2 \{p\}^T [H] \{p\} + \{G\}^T \{p\} \tag{25}$$

where $\{p\}$ is the vector of all nodal pressures. The matrices $[H]$ and $\{G\}$ are determined during the assembly process by the appropriate addition of the contributions $[h_e]$ and $\{g_b\}$, respectively.

In order to minimize the functional Ψ with respect to each unknown nodal pressure p, we require that

$$\frac{\partial \Psi}{\partial \{p\}} = 0 \tag{26}$$

By carrying out the indicated differentiation, it will be found that

$$[H]\{p\} + \{G\} = 0 \tag{27}$$

$[H]$ is singular, so a small constant is added to each diagonal term, before solution for $\{p\}$ by Gaussian elimination. This operation has the effect of making the average pressure zero.

4. The Finite Element Method Applied to Added Mass Calculations

The determination of the added mass matrix involves a special case of the general field eq. (2). In this case, we wish to determine the forces induced on all the submerged boundaries by a unit acceleration of each body in turn. The forces are a direct expression of the added mass. Thus, consider two bodies immersed in a fluid, as shown in

Figure 3. Body 1 is given an acceleration a_{1x} in the x-direction. The resulting pressures in the fluid are calculated. The forces experienced by each body in the x and y directions are obtained by integration of the resultant boundary pressures in those directions. Thus, we have because of the acceleration a_{1x}, the forces F:

$$\begin{Bmatrix} F_{1x,\ 1x} \\ F_{1y,\ 1x} \\ F_{2x,\ 1x} \\ F_{2y,\ 1x} \end{Bmatrix} = \begin{Bmatrix} M_{11} \\ M_{21} \\ M_{31} \\ M_{41} \end{Bmatrix} a_{1x} \tag{28}$$

Now, if we apply other accelerations a_{1y}, a_{2x}, and a_{2y}, in turn, we develop the forces

$$[F] = [M]\,[a] \tag{29}$$

where

$$[a] = \begin{bmatrix} a_{1x} & 0 & 0 & 0 \\ 0 & a_{1y} & 0 & 0 \\ 0 & 0 & a_{2x} & 0 \\ 0 & 0 & 0 & a_{2y} \end{bmatrix}$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

$$[F] = \begin{bmatrix} F_{1x,\ 1x} & F_{1x,\ 1y} & F_{1x,\ 2x} & F_{1x,\ 2y} \\ F_{1y,\ 1x} & F_{1y,\ 1y} & F_{1y,\ 2x} & F_{1y,\ 2y} \\ F_{2x,\ 1x} & F_{2x,\ 1y} & F_{2x,\ 2x} & F_{2x,\ 2y} \\ F_{2y,\ 1x} & F_{2y,\ 1y} & F_{2y,\ 2x} & F_{2y,\ 2y} \end{bmatrix}$$

In general, with n bodies, we can generate relationships like eq.(29) excepting that the matrices are (2n x 2n) in size. The elements of the matrix [M] are the hydrodynamic mass coefficients and represent the inertia coefficients that give the forces on each body when the accelerations of each body are specified.

To determine the added mass matrix, we apply individually a unit acceleration to each body in the two coordinate directions. The development of the element matrices and their assembly is done only once for all these variations in acceleration, since the matrix [H] of eq.(27) is the same for all the boundary value problems: only the vector {G} changes. The vector {G} for a single acceleration condition is expanded to form a matrix [G] having twice as many columns as the number of bodies. Successive pairs of columns of [G] correspond to unit acceleration of a body in the x and y directions. Suppose we number consecutive nodes along a boundary as shown in Figure 4. We find that

$$\{g_b\}^T = 1/2\,\rho a_n L_n\,[1,\ 1] \tag{30}$$

The body accelerations for the pair of columns in [G] being considered is 1 for a_x and 0 for a_y for the first of the pair, and 0 for a_x and 1 for a_y for the second. For all other bodies, a_x and a_y are both zero. For the normal acceleration a_n of the segment we have

$$a_n = a_x \cos\theta + a_y \sin\theta \tag{31}$$

and since $\cos\theta = (y_k - y_\ell)/L_n$

$$\sin\theta = (x_\ell - x_k)/L_n$$

then

$$\{g_b\}^T = 1/2\,\rho[a_x(y_k - y_\ell) + a_y(x_\ell - x_n)]\ [1,\ 1] \tag{32}$$

Having the segment matrices for each column in [G] permits the assembly of [G].

A solution of the system

$$[H]\{p\} + [G] = 0 \tag{33}$$

gives columns of nodal pressures for each body acceleration condition. The forces that the body exerts on the fluid in the x and y directions are found by integrating the pressures around the boundaries. We have, for example, for the contribution f from a specific boundary segment between the two nodes k and ℓ, the forces

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = \begin{bmatrix} 1/2\,(y_k - y_\ell) & 1/2\,(y_k - y_\ell) \\ 1/2\,(x_\ell - x_k) & 1/2\,(x_\ell - x_k) \end{bmatrix} \begin{Bmatrix} p_\ell \\ p_k \end{Bmatrix} \tag{34}$$

Addition of all contributions around the boundary of a body yields the total integrated forces on that body. These forces in turn make up a column of the added mass matrix.

It is interesting to note that the same terms that enter into the assembly of [G], eq.(32), are needed in going from pressure to virtual mass except for the factor ρ. The added mass matrix is obtained from the computed pressure matrix [p] and boundary matrix [G] as

$$[M] = 1/\rho\,[G]^T\,[p] \tag{35}$$

Thus, the added mass matrix is found directly once the pressures are determined, without the specific need to actually perform the force integrations discussed previously.

5. Example -- Concentric Cylindrical Vessels

To illustrate the type of results that can be expected from the finite element approach described, we consider a two-inch diameter cylinder situated concentrically in a four-inch diameter cylinder, which is filled with a fluid of density 1.00 (for purposes of illustration). It has been shown [4] that the added mass matrix for concentric cylinders of radius a and b is given by

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \end{Bmatrix} = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ 0 & m_{22} & 0 & m_{24} \\ m_{13} & 0 & m_{33} & 0 \\ 0 & m_{24} & 0 & m_{44} \end{bmatrix} \begin{Bmatrix} a_{1x} \\ a_{1y} \\ a_{2x} \\ a_{2y} \end{Bmatrix} \tag{36}$$

where

$$\begin{aligned} m_{11} &= m_{22} = \rho\pi a^2 \left(\frac{b^2 + a^2}{b^2 - a^2}\right) \\ m_{13} &= m_{24} = -2\rho\pi\ a^2 b^2 / (b^2 - a^2) \\ m_{33} &= m_{44} = \rho\pi\ b^2 \left(\frac{b^2 + a^2}{b^2 - a^2}\right) \end{aligned} \tag{37}$$

For the case considered here, we get

$$\begin{aligned} m_{11} &= m_{22} = 5.23 \\ m_{13} &= m_{24} = -8.38 \\ m_{33} &= m_{44} = 20.95 \end{aligned}$$

Upon applying the finite element method with the mesh shown in Figure 5, it will be found that

$$\begin{aligned} m_{11} &= m_{22} = 5.169 \\ m_{13} &= m_{24} = -8.293 \\ m_{33} &= m_{44} = 20.792 \end{aligned}$$

The difference in the exact and finite element solutions is about 1%.

If the cylinders are placed eccentrically by moving the inner cylinder so its center lies 0.5 in. along the x-axis from the center of the outer cylinder, the finite element results become:

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \end{Bmatrix} = \begin{bmatrix} 5.605 & 0 & -8.729 & 0 \\ 0 & 5.598 & 0 & -8.722 \\ -8.720 & 0 & 21.228 & 0 \\ 0 & -8.722 & 0 & 21.221 \end{bmatrix} \begin{Bmatrix} a_{1x} \\ a_{1y} \\ a_{2x} \\ a_{2y} \end{Bmatrix}$$

It appears that there is therefore a moderate increase in all the coefficients.

Many other configurations of a simple geometric nature have been studied, and some exact solutions are available for some of them. Some have been compiled by Patton [5] and by Fritz [6]. Experimental analyses that show the effects of end leakage and viscosity are discussed by Liu [4], Fritz [6], and Mavis [7].

6. Example -- Array of Square Beams in a Cylindrical Container

As an example of a more complex situation of irregular geometry, consider an array of four square beams, shown in Figure 6. In this case, we shall obtain the added mass of the four beams when they are contained in a cylindrical vessel of water of radius 15 inches.

The finite element mesh used for the calculation of the added masses is shown in Figure 6. The resulting pressure fields corresponding to unit accelerations of one of the beams are shown in Figure 7. Of interest are the relatively high pressures in the narrow passages between the beams. Such pressures are due to the rapid flow of water as it is squeezed out of the passageway due to the movement of the beams. The added mass of such passageways has been studied by Sharp and Wenzel[8].

The added mass matrix is shown in Table I in the form that it is printed by the computer. This matrix is symmetric and the diagonal terms tend to dominate.

7. Summary

This paper has been devoted to some aspects of the vibrations of structural components submerged in fluid. We developed the concept of the added mass due to the motion of the fluid, and showed how the added mass could be calculated by using potential flow theory for some general cases. The finite element method was used to make such calculations and the results of the finite element calculations were compared to analytical results for the case of concentric cylinders, where good agreement was obtained. The method was also used to generate the added mass matrix for a complex configuration of beams in a container of water.

References

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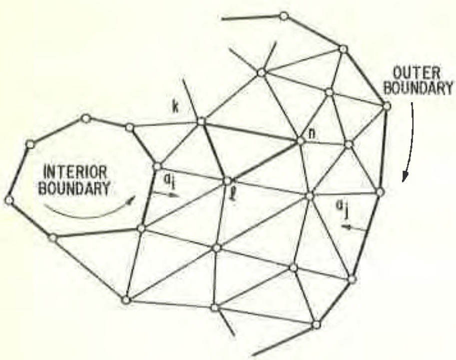
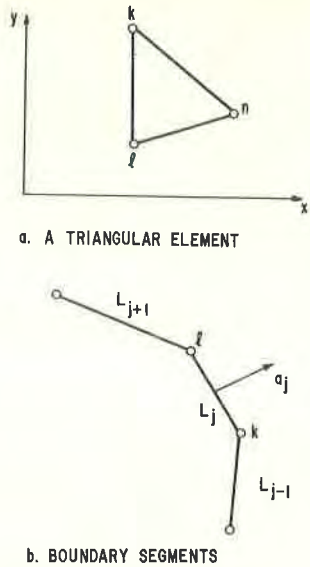
Table I: Hydrodynamic Water Mass Matrix for Four Beams and Cylindrical Container

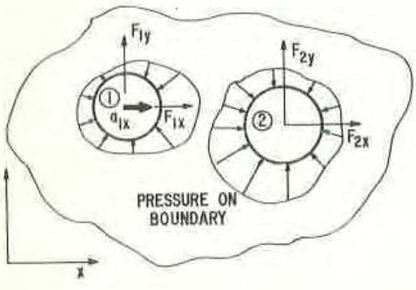
1X BODY	1Y BODY	2X BODY	2Y BODY	3X BODY	3Y BODY	4X BODY	4Y BODY	5X BODY	5Y BODY
0.00654	0.00000	-0.00037	-0.00372	-0.00250	-0.00000	-0.00037	0.00372	-0.00607	0.00000
0.00000	0.00524	-0.00182	-0.00037	-0.00000	0.00065	0.00182	-0.00037	0.00000	-0.00792
-0.00037	-0.00182	0.00524	0.00000	-0.00037	0.00182	0.00065	-0.00000	-0.00792	0.00000
-0.00372	-0.00037	0.00000	0.00654	0.00372	-0.00037	-0.00000	-0.00250	0.00000	-0.00607
-0.00250	-0.00000	-0.00037	0.00372	0.00654	0.00000	-0.00037	-0.00372	-0.00607	0.00000
-0.00000	0.00065	0.00182	-0.00037	0.00000	0.00524	-0.00182	-0.00037	0.00000	-0.00792
-0.00037	0.00182	0.00065	[illegible]	-0.00037	-0.00182	0.00524	0.00000	-0.00792	0.00000
0.00372	-0.00037	-0.00000	-0.00250	-0.00372	-0.00037	0.00000	0.00654	0.00000	-0.00607
-0.00607	0.00000	-0.00792	0.00000	-0.00607	0.00000	-0.00792	0.00000	0.09391	-0.00000
0.00000	-0.00792	0.00000	-0.00607	0.00000	-0.00792	0.00000	-0.00607	-0.00000	0.09391



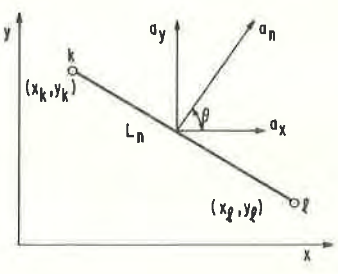
Representing the Fluid Field by Finite Triangular Elements



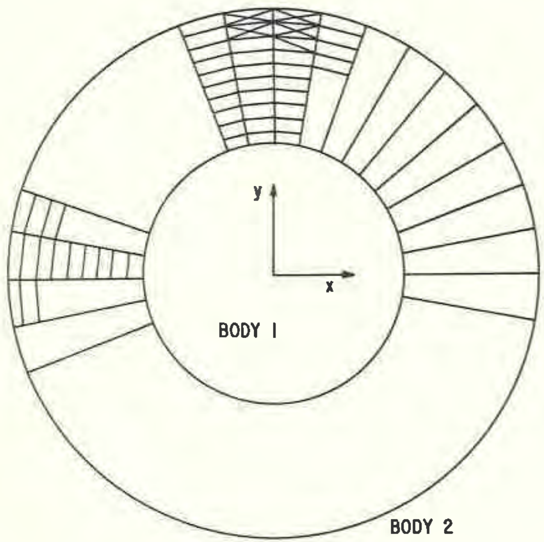
Element and Boundary Segment Geometries



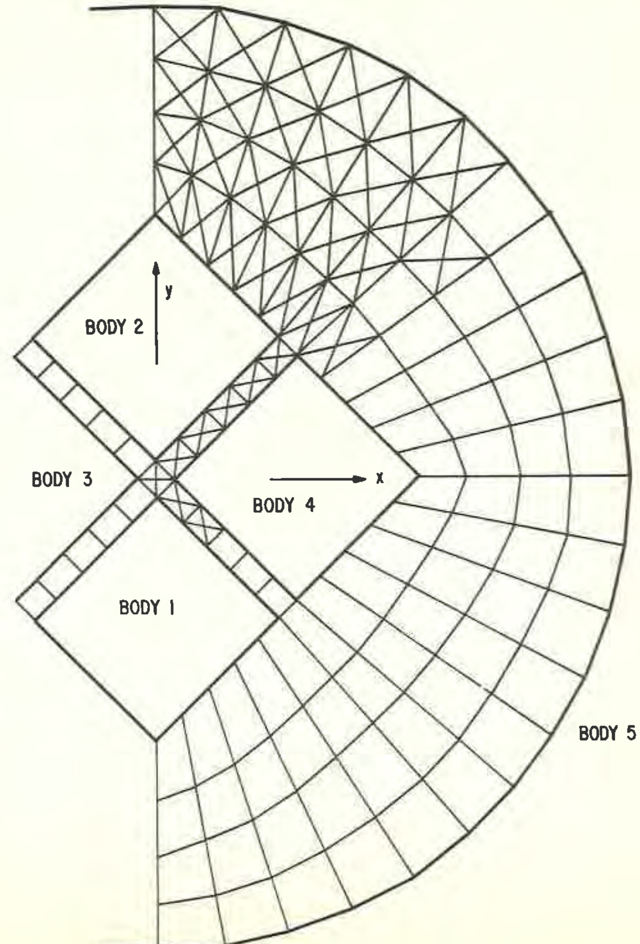
An acceleration a_{1x} of Body 1 in the x-direction causes a pressure field in the fluid. The forces the bodies exert on the fluid in the x and the y directions can be computed by integrating the boundary pressures in those directions.



Geometrical Relationship between Acceleration Components on a Triangle Boundary

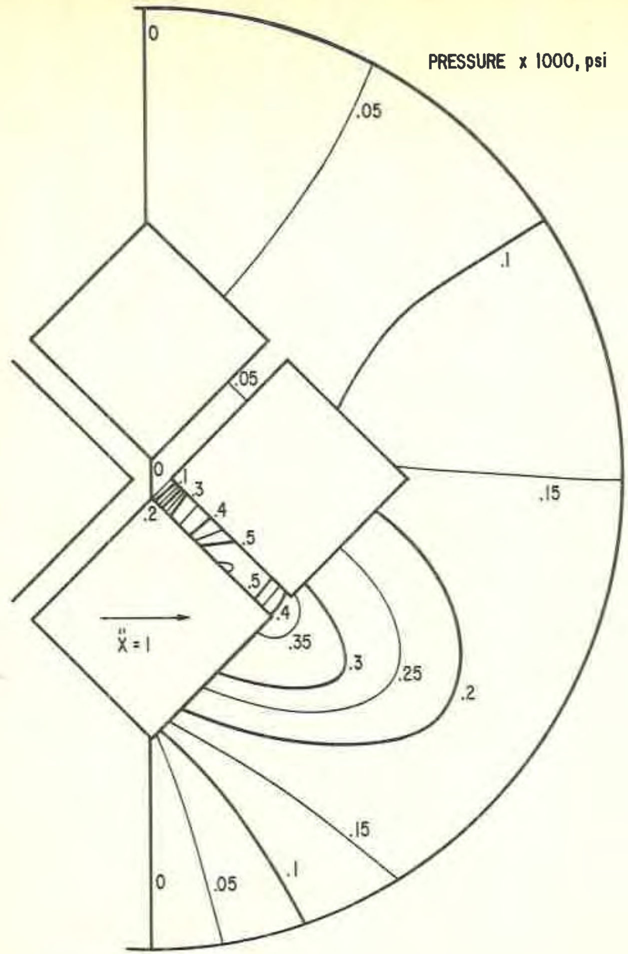


Finite Element Mesh Used for Two Concentric Cylinders

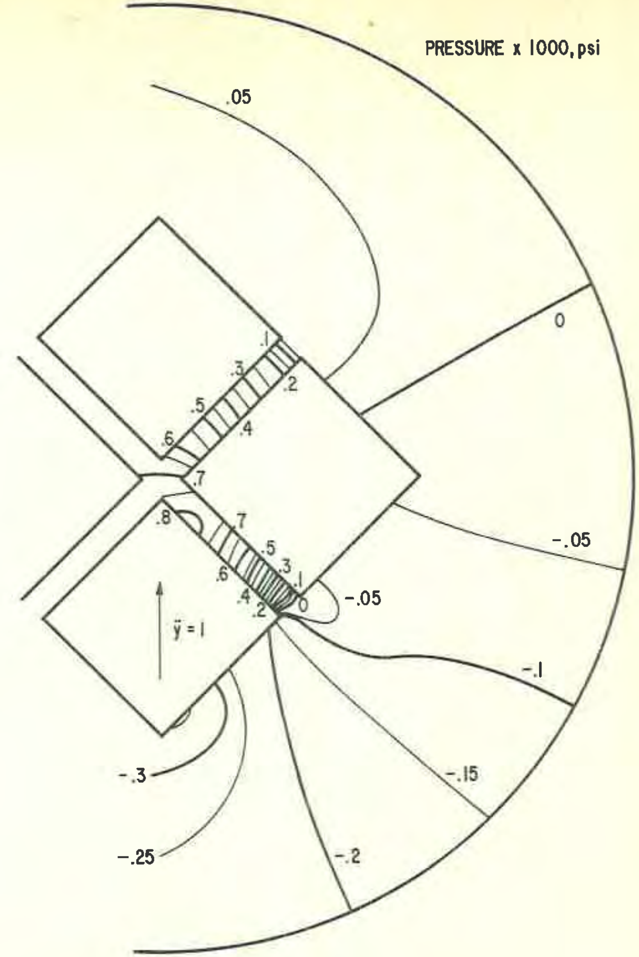


Section of Beams and Outer Cylindrical Container, Showing Finite Element Mesh

F 2/5



Pressure Distribution for Unit Acceleration of Body 1 in the x-Direction



Pressure Distribution for Unit Acceleration of Body 1 in the y-Direction