Post-buckling behavior of steel cylindrical shells during earthquakes

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1 INTRODUCTION

The energy input exerted by earthquakes can be a substantial loading effect of earthquakes, whether the structure remains elastic or not. The energy absorption capacity of a structure can be assumed to be a primary source of resistance of structures against earthquakes. The energy input to structures has been found to be a very stable amount which is mainly dependent on the total mass and the fundamental natural period of structure. On the other hand, even after the occurrence of buckling in elements, structures usually develop a considerable amount of energy absorption. Thus, the occurrence of buckling can not be a decisive criterion for the safety in case of seismic loading, compared to the case of gravity loading under which the structure collapses immediately, when the structure loses its static equilibrium.

This paper aims to clarify the ultimate seismic resistance of steel cylindrical structures the strength of which is much affected by buckling. The non-linear response analysis was made on free-standing cylindrical vessels subjected to horizontal ground motions. The model was assumed to be a single degree of freedom system with restoring-force characteristics inherent to cylindrical structures influenced by buckling. The damage index was selected to be the maximum deflection of structure. The loading index was selected to be the total energy input to a structure due to an earthquake.

2 ENERGY INPUT

In Figure 1, a cylindrical structure is shown. Fundamentally, it can be reduced to a single-mass system shown in the figure, and the equation of motion under a horizontal ground motion can be written as follows.

(1) $M\ddot{x} + C\dot{x} + Q(x) = -M\ddot{z},$

where M is mass of the structure, $C\dot{x}$ is damping force which is equal to $2Mh\omega\dot{x}$, C is damping factor, h is damping constant, ω is natural circular frequency, Q(x) is restoring force, x is lateral deformation, and z is ground motion.

Multiplied by $dx=\dot{x}dt$ on both sides of equation 1, and integrated over the entire duration of an earthquake, equation 1 is reduced to

(2) $W_e + W_p + W_h = E,$

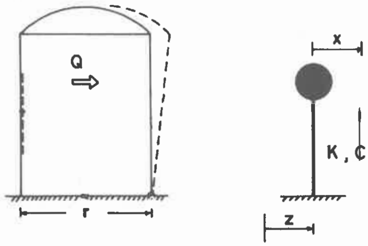

Figure 1. Analytical model.

where W_e is the elastic vibrational energy of structure, W_p is the cumulative inelastic strain energy, W_h is energy absorption due to damping, and E is the total energy input to a structure.

The total energy input to a structure has been found to be a very stable amount which mainly depends on the total mass and the fundamental natural period of structure. More precisely, the total energy input to structures other than the undamped elastic system can be obtained by averaging the energy input to the undamped elastic system, ${}_0E$ with respect to the fundamental natural period, T. Denoting the band of averaging by ΔT, the energy inputs for the damped elastic system and the elastic-plastic system are expressed as follows.

For damped elastic systems,

$$(3) \quad E = \int_{T_0-\Delta T/2}^{T_0+\Delta T/2} {}_0E(T)\, dT \,/\, \Delta T.$$

For elastic-plastic systems,

$$(4) \quad E = \int_{T_0}^{T_0+\Delta T} {}_0E(T)\, dT \,/\, \Delta T,$$

where T_0 is the natural period of a system in the elastic range.

The band of averaging, ΔT expands as the non-linearity increases. The difference between equations 3 and 4 exists only in the difference of the lower and upper bounds of integration. In equation 3, the period of the structure drops at the mid-point of the averaging band,whereas in equation 4, the period of the structure becomes the lower bound of integration. The energy input of the undamped elastic system is highly sensitive to the period of the system and has large fluctuation. On the other hand, the energy input of damped elastic systems and elastic-plastic systems becomes smooth owing to the averaging effect. A clear difference between the damped system and the elastic-plastic system is that the energy input to the damped system is governed mainly by the natural period, while the energy input to the elastic-plastic system is governed by a period longer than the natural period. The total energy input can be converted to the equivalent velocity according to the following equation.

$$(5) \quad V_E = \sqrt{2E / M}\,.$$

The V_E - T relationship can be termed as an energy spectrum. The practical structure can develop a considerable amount of energy absorption due to inelastic deformations until it reaches a structually critical state. Accordingly, the estimate of energy input for practical structures should be made on the basis of an energy spectrum smoothed by applying an appropriate damping constant, and an effective period longer than the natural period. For the purpose of obtaining a smoothed energy spectrum, the author proposed the value of h to be 0.1. When the structure behaves inelastically, the time required for the structure to produce one cycle

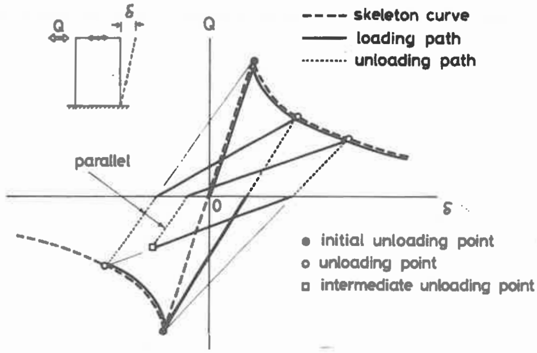

Figure 2. Hysteresis rule

of vibration, which is termed as the temporary period, becomes longer than the natural period, according to the degree of plastification. As an effective period, T_e which should be referred to in applying the energy spectrum, the author proposed the following value.

(6) $T_e = \sqrt{(T_o^2 + T_o T_m + T_m^2)/3}$,

where T_o is the natural period, and T_m is the longest temporary period.

3 HYSTERESIS RULE IN RESTORING FORCE CHARACTERISTICS

To predict the hysteresis rule, some definitions are introduced. The loading path and the unloading path are defined as follows.

(7) $Q\, d\delta > 0$ (loading path), and (8) $Q\, d\delta < 0$ (unloading path).

The skeleton curve is identified by the Q-δ curve under the monotonic loading. The unloading point is defined by the point which rests on the skeleton curve and terminates the loading path. The initial unloading point is defined by the point on which buckling occurs. The intermediate unloading point is defined by the point which terminates the loading path but does not reach the skeleton curve.

Under the assumption that the initial unloading points under positive and negative loading domains have been already experienced, the hysteresis rule is described as follows.

The loading path points to the preceding unloading point in the same loading domain. After the preceding loading point is reached, the loading path follows the skeleton curve. The unloading path from the unloading point points to the initial unloading point in the reverse loading domain. The unloading path from the intermediate unloading point has the same slope as that of the preceding unloading path in the same loading domain. Figure 2 illuatrates a typical pattern of restoring-force characteristics, according to the above-mentiond hysteresis rule.

4 PRINCIPAL RESPONSES

Using various parameters, non-linear response analyses were made. The used ground motion is of Hachinohe record at the Tokachioki Earthquake (1968), the maximum acceleration of which is 183 gal.

As shown in Figure 3, the skeleton curve in the restoring force characteristics can be simplified by a tri-linear relation. Q_{cr} is the buckling load. The second line segment indicates a descending path

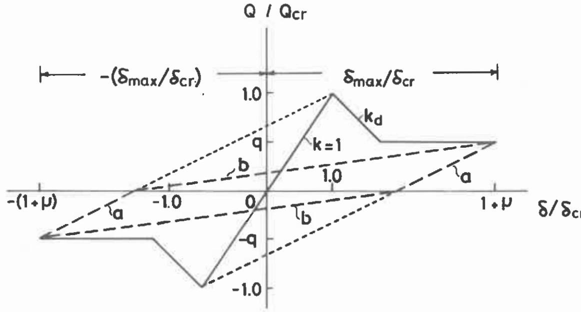

Figure 3. Skeleton curve and last one cycle.

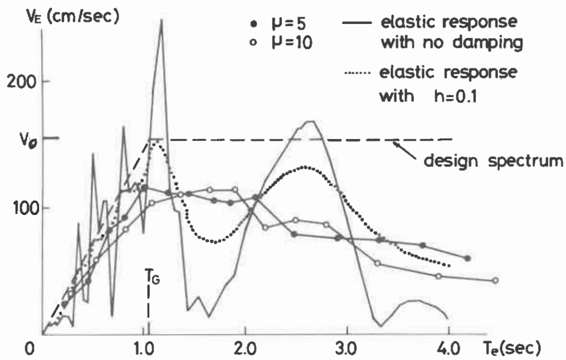

Figure 4. V_E - T_e relationship.

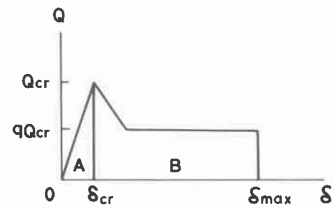

Figure 5. Fundamental scale of strain energy.

from the buckling point, and the third line segment is an asymptote in the range of large deformation. The level of the asymptote is denoted by qQ_{cr}. The slope of the descending path is denoted by $k_d K$, where K is the slope of the first elastic path. In order to estimate the longest temporary period, an assumption is made that the maximum deformation, δ_{max} takes place both in positive and negative directions with a same amount as shown in Figure 3. The temporary longest period is assumed to be the period which corresponds to the hysteresis loop indicated by the broken lines in Figure 3. Then, T_m can be described as

(9) $$T_m = T_o(1/\sqrt{a} + 1/\sqrt{b})/2,$$

where a is the nondimensional slope of the unloading path, and b is the nondimensional slope of the loading path.

A nondimensional lateral strength, α and an index of plastification, μ are introduced as follows.

(10) $\alpha = Q_{cr}/Mg$, and (11) $\mu = (\delta_{max} - \delta_{cr})/\delta_{cr}$,

where g is acceleration of gravity, and δ_{cr} is the elastic deformation at the buckling point in Q-δ diagram.

In Figure 4, V_E - T_e relation is shown, together with the response of elastic systems in which $T_e = T_o$. In each analytical model, α was so selected to produce the same response of μ, irrespective of the difference of T_o by applying iterative calculations. The averaging effect due to plastification is clearly observed. The energy input of the inelastic system almost coincides with that of the damped elastic system. This implies the appropriateness of adopting the effective period defined by equations 6 and 9. In Figure 4, the envelope of the V_E - T_e relation of

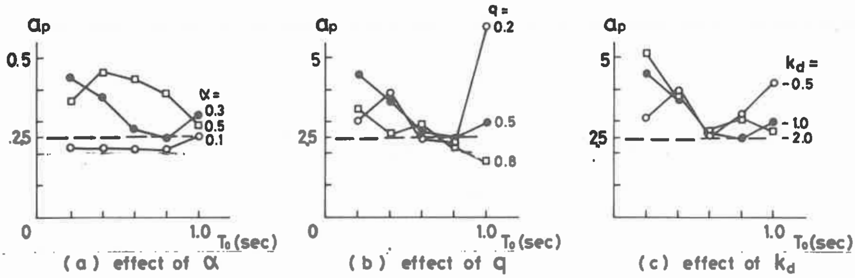

Figure 6. Values of a_p.

the elastic system with h=0.1 is shown by a broken line. This bi-linear relation can cover the energy input to inelastic systems, and can be an energy spectrum for the practical design purpose. Therefore, the energy spectrum for design use can be simply described as follows.

(12) $V_E = V_0 T_0 / T_g$ for the range of shorter period, and

(13) $V_E = V_0$ for the range of longer period,

where T_g is the period which divides the range of period, and V_0 is a constant value in the range of longer period.

The energy absorption capacity of structure, E_a can be formally written as follows.

(14) $E_a = E_e + 2\, a_p E_p$,

where E_e is the elastic strain energy shown by the area-A in Figure 5, E is the apparent plastic strain energy shown by the area-B in the same figure, and a_p is a constant. E_p can be easily related to μ. For undamped systems, it holds that $E = E_a$. Therefore, a_p can be calculated as follows.

(15) $a_p = (E - E_e)/2E_p(\mu)$.

Figure 6 shows the values of a_p. Irrespective of various parameters such as α, q and k_d, the lower bound values of a_p can be found to be approximattely 2.5.

5 EARTHQUAKE RESISTANT DESIGN FORMULATION FOR CYLINDRICAL STRUCTURES

As a conservative evaluation for the energy absorption capacity, it is assumed that h=0 and $k_d = -1.0$. Accordingly, a and b in equation 9 and E_p/E_e are determined as follows.

When $\mu < 1 - q$,

(16) $a = (2 - \mu)/(\mu + 2)$, $b = (1 - \mu)(2 - \mu)/(2 + 3\mu - \mu^2)$, and

(17) $E_p/E_e = (2 - \mu)\mu$.

When $\mu > 1 - q$,

(18) $a = (1 + q)/(\mu + 2)$, $b = q(1 + q)/(2 + 2\mu + \mu q)$, and

(19) $E_p/E_e = 2q(\mu + q - 1) + (1 - q^2)$.

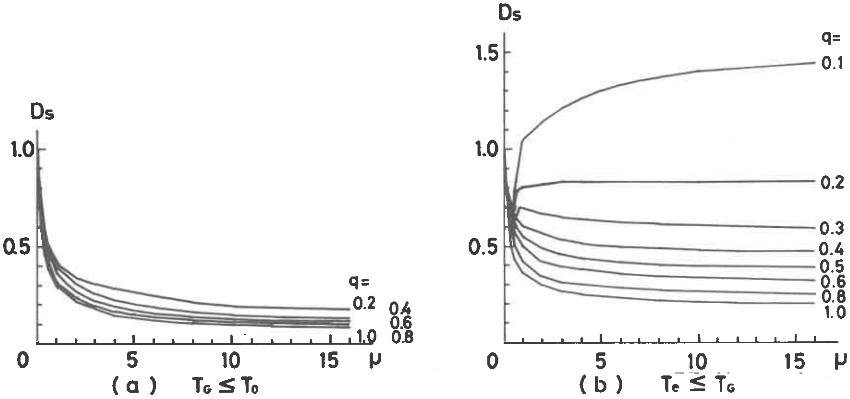

Figure 7. D_s -values.

Referring to equation 14, the balance between the energy input and the energy absorption capacity can be described as follows.

(20) $\quad E_e + 2\,a_p E_p = M\,V_E^2(T_e)/2.$

The elastic vibrational energy, E_e can be written as

(21) $\quad E_e = Q_{cr}\,\delta_{cr}/2 = (M\,g^2\,T^2/4\pi^2)(\alpha^2/2).$

Substituting equation 21 into equation 20, the next relation is obtained.

(22) $\quad \alpha = (1/\sqrt{1 + 2\,a_p\,E_p/E_e})(2\pi\,V_E(T_e)/T_0 g)$

The strength level at which the buckling just occurs without accompanying any inelastic deformations, α_{cr} can be obtained by by setting $E_p=0$ and $T_e=T_0$ in equation 22. α/α_{cr} implies the contribution of inelastic energy absorption to the reduction of the strength required for a structure to resist earthquake, and is termed as D_s-value. The reciprocal of D_s-value means a reserve of strength in the sense of earthquake resistant design for a structure after the buckling has occurred. D_s-value is obtained as

(23) $\quad D_s = \alpha/\alpha_{cr} = (1/\sqrt{1 + 2\,a_p E_p/E_e})(V_E(T_e)/V_E(T_0)).$

D_s-values for $a_p=2.5$ are shown in Figure 7. Figure 7a is the case for structures with longer natural period in which $V_E(T_e)/V_E(T_0)=1.0$, and Figure 7b is the case for structures with shorter natural period, in which $V_E(T_e)/V_E(T_0)=T_e/T_0$. Since most cylindrical structures can be clasified into shorter period structure, Figure 7b should apply to the practical design. It is clearly observed in Figure 7b that a great contribution of inelastic energy absorption shown in Figure 7a is offset by the increase of energy input caused by the expansion of vibrational period. Nevertheless, it can be concluded that the cylindrical structure can reserve a considerable strength even after the buckling has occurred, provided that the level of q is maintained to be greater than 0.2.

REFERENCE

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