SMiRT-12 / K. Kussmaul (Editor)

MG09/2

APPLICATION OF PROBABILISTIC FRACTURE MECHANICS TO THE RISK ASSESSMENT OF PRESSURE VESSELS

C.-H. Tsai[1] and W.-F. Wu[2]

[1]Dept. of Nuclear Engineering, National Tsing Hua University, Hsinchu, Taiwan, R.O.C.
[2]Dept. of Mechanical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

ABSTRACT

The major causes for the reactor pressure vessel failure include fatigue, corrosion fatigue and brittle fracture. They are, in turn, greatly affected by the stress loading conditions, material properties (aged by neutron damage) and defects embedded in the structure. All these factors are actually random variables rather than deterministic variables. Therefore, in the present study, a probabilistic fracture mechanics model is proposed to analyze the failure probability of a typical high power density reactor pressure vessel. Based on the model, both analytical analysis and Monte Carlo simulation are employed in the study. The NDE reliability is also considered. Through the study, it is found that failure probability of the pressure vessel lies in the order of 10^{-6} per vessel year, which agrees the value of 10^{-4} to 10^{-6} as reported in the literature based on real-world statistics. If inspection is considered, the failure probability then decreases significantly.

1. INTRODUCTION

Conventionally, the assessment of pressure vessel reliability has been performed based on the statistical analysis of previously accumulated vessel failure data [1]. While the method is quite reasonable for non-nuclear pressure vessels, it is not appropriate for nuclear reactor pressure vessels due to the rare occurrences of their failures. According to a report by Marshall, it was estimated that by the end of 1980, the accumulated total light water reactor years of operation was some 3300 reactor years in the world. However, no disruptive failure of any reactor pressure vessel has been reported [2]. Obviously statistics on nuclear pressure vessels will not provide us with direct information on the quantitative reliability measurement for a very long time to come. In order to overcome this difficulty, the probabilistic fracture mechanics (PFM) concept evolved in the aircraft and satellite industry has been adopted for the reliability analysis of nuclear reactor pressure vessels.

One of the difficulties in applying the PFM to the risk assessment of structures arises in the complication of the analytical analysis if one wants to abide by all rules developed in fracture mechanics and reliability engineering. Monte Carlo simulation is therefore frequently used in the reliability estimation. For a nuclear reactor pressure vessel, since the failure probability is so small, the number of Monte Carlo trials needed would be very large before a reasonable prediction can be obtained. To avoid the time-consuming Monte Carlo simulation, several approximation methods developed for the reliability analysis of complex systems are considered in the risk assessment of nuclear reactor pressure vessels. As expected, the accuracy of the prediction obtained from the approximation methods depends on the degree of approximation, the numerical techniques employed in the analysis as well as the complication of the system itself. It may occur that the approximation result deviates very much from the real situation.

In the present analysis, a hybrid method is proposed to find the failure probability of a nuclear reactor pressure vessel. The advantage of the proposed method is that tedious direct Monte Carlo simulation can be avoided and approximations used in the reliability estimation can be reduced to the smallest amounts. The effect of non-destructive inspection (NDI) on the failure probability can also be included in the present analysis through the introduction of probability of detection (POD). The result of a sample numerical study is presented in this paper.

2. FORMULATION

For a pressure vessel, flaws are very likely to initiate at the circumferential welds in the beltline region and then grow to the so-called initial crack sizes. If we focus on the most critical circumferential defect in the weld and neglect the effect of number of defects, then final fracture will usually result from the growth of this critical defect beginning from its initial crack size. Because of its uncertainty, the initial crack size could be better described by a probability density function. According to Pennell [3], the following initial crack depth distribution is assumed in the present analysis,

$$f(a_0) = \lambda \exp\left[-\lambda\left(a_0 - a_d\right)\right] \tag{1}$$

where a_0 is the initial crack size and a_d and λ are constants.

From linear elastic fracture mechanics theory, an initial fatigue crack size will propagate due to applied cyclic stresses. It is assumed that the crack propagation follows the famous Paris law

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{2}$$

where da/dN is the crack growth rate, ΔK is the range of the stress intensity factor, and C and m are material constants. In the application of PFM, the last two parameters can be considered random representing the statistical nature of material properties. The other input parameter in Eq. (2) that should be considered random is the stress intensity factor range ΔK which is the result of stress transients a reactor pressure vessel may encounter during its years of operation. Referring to the stress analysis report of a boiling water nuclear power plant, a finite difference computer code is written to calculate the transient temperature distribution for the beltline region of the pressure vessel. After the temperature distribution is calculated, MARC finite element code is employed to find the extreme stresses for each of the various transient conditions. The calculated transient stresses as well as the possible times of occurrence for each transient as predicted in the stress analysis report are classified into several categories and summarized in Table 1.

Based on the described conditions, a Monte Carlo algorithm is employed to simulate the fatigue crack growth under transient stresses. The simulation begins with the generation of an initial crack depth as described by Eq. (1). For each randomly generated initial crack depth, random transient stresses cause it to propagate following a growth law shown by Eq. (2). During the crack growth simulation, it should be noted that the three pre-operational hydrostatic tests listed in the first raw of Table 1 have to be performed at the beginning of the operation. In other words, their effect on the crack growth is considered to be deterministic in both their occurrences and stress magnitudes. For other three normal transients listed in Table 1, since they are expected to occur almost regularly in each fuel cycle, their occurrences are assumed to follow uniform distributions in forty years. The occurrence rates of these transients as well as their stress magnitudes can either be deduced from or be read directly from Table 1. For the other two abnormal transients in Table 1, Poisson counting processes are used to model their occurrences. Therefore, an exponentially distributed inter-arrival time is assumed for each of the two abnormal transients. The occurrence rates and the possible stress magnitudes are again obtainable from Table 1. In performing each set of the Monte Carlo simulation, the fatigue crack size can be monitored and recorded at any time. The stress intensity factor around the crack can also be calculated at any time by the following formula,

$$K_I = \beta(g) S \sqrt{\pi a} \tag{3}$$

In the above equation, a is the crack size and S should be explained as the nearest occurred maximum transient stress, i.e. S_{max} in Table 1. $\beta(g)$ is a correction factor which depends on the geometry of the crack and its surrounding structural configuration.

TABLE 1
Summary of various transient stresses

Operating Condition	**Category**	S_{max} (MPa)	S_{min} (MPa)	ΔS (MPa)	**Number of Cycles in 40 yrs.**
Pre-operational hydrostatic test	normal	296.48	68.95	227.53	3 prior to initial start-up
Start-up	normal	208.49	45.48	163.02	120
Shut-down	normal	240.52	68.95	171.57	111
Hydrostatic test to design pressure	normal	236.90	68.95	167.95	130
1. Turbine generator trip feedwater on, isolation valves stay open 2. Pre-op. blow-down, 3. All other scrams	abnormal	277.12	145.80	132.03	190
Loss of AC power natural circulation restart	abnormal	268.14	117.54	150.60	5

The simulation process can be repeated for many times, and the results for K_I can be plotted as a histogram. Statistic methods can then be applied to find the probability density function of K_I, which is directly related to the failure probability or reliability of the pressure vessel. From fracture mechanics viewpoint, the vessel failure will occur under the condition that $K_I \geq K_{IC}$ in which K_{IC} is the fracture toughness of the material. For a nuclear reactor pressure vessel, it should be noted that neutron radiation is potentially the most important external influence on the degradation of the material fracture toughness. This is especially true for vessels having welds in the central beltline region nearest to the active core. Many studies, however, have shown that the principal effect of neutron radiation is to increase the temperature of the ductile-to-brittle transition without appreciably altering the upper-shelf toughness. Therefore, it would be sufficient to allow the mean toughness to shift in accordance with the change in transition temperature but without change to the other aspects of the toughness distribution. According to Becher and Pedersen, the fracture toughness above the transition temperature may be calculated on the basis of the upper shelf Charpy-V (C_V) fracture toughness and the yield strength, S_Y, at the temperature in question as [4]

$$K_{IC} = \sqrt{5\left(C_V S_Y - 0.05 S_Y^2\right)} \tag{4}$$

The probability density function of K_{IC} is thus readily determined from the probability densities of C_V and S_Y. In the present study, another simple Monte Carlo algorithm is used to generate C_V and S_Y based on their assumed probability density functions. Values of K_{IC} are then calculated based on Eq. (4) and a histogram is constructed for the result. Statistic tests are then employed to find the appropriate probability density function of K_{IC} with a certain confidence level. After the probability densities of K_I and K_{IC} are obtained, the failure probability can be calculated using the method of inteference. The failure rate can also be calculated accordingly. The latter has the dimension of failure probability per unit time (usually per year for the case of pressure vessels) and is frequently used in the statistical analysis of pressure vessels and piping [1].

To study the effect of NDI on the failure probability of a pressure vessel, the following probability of detection (POD) curve is assumed for a specific ultra-sonic testing (UT) device,

$$\mathrm{POD}(a) = 1 - \exp(-\mu a) \tag{5}$$

The POD curve acts as a filter which filters out the detected crack sizes in a probabilistic manner. After the crack size is detected, complete repair can be employed to insure the integrity of the structure. The probability density function of the crack size can therefore be re-distributed after the inspection-and-repair process. This practice can, of course, be incorporated into the Monte Carlo simulation algorithm.

3. NUMERICAL EXAMPLES

In the present study, according to available literatures and data, $\lambda = 0.10/\,\mathrm{mm}$ (2.54/in) and $a_d = 2.54\,\mathrm{mm}$ (0.1 in) are used in Eq. (1) for the initial crack depth of the pressure vessel. The material constants are found from ASTM A533B steel crack growth data to be $m = 2.47$ and $C = 5.22 \times 10^{-8}$ mm/cycle (2.60×10^{-9} in/cycle). The latter has a coefficient of variation 8.67%. Based on the initial crack distribution and the material's crack growth constants, Monte Carlo simulations are performed. A thousand samples for the crack sizes and stress intensity factors of the cracked vessel at the 10th, 20th, 30th and 40th year of operation are thus obtained. The result for the stress intensity factors are plotted in normal, log-normal and Weibull probability paper respectively. Chi-square tests are then employed to test the goodness of fits. It is found that log-normal distribution is the best description for the stress intensity factor K_I. The fitted curve is acceptable at the 0.5% significance level.

According to the published data from the HSST program on vessel steel plate A533B, the probability distribution functions of the Charpy-V toughness and yield stress can all be fitted to three-parameter Weibull forms [4]. Following those parameter values used by Becher and Pedersen [4], another simple Monte Carlo algorithm is written to generate samples for the Charpy-V toughness and yield stress. Equation (4) is then applied to calculate the fracture toughness for each pair of the generated C_V and S_Y. A thousand K_{IC} values are thus obtained and the results are plotted in three different probability papers. Chi-square tests on the best fitted curves are then performed. It is found that normal probability is the most appropriate form to represent the fracture toughness of the material.

After the probability density functions of K_I and K_{IC} are obtained, the vessel failure probability can then be calculated by integrating the "interference" area of two probability density curves. The result for the 0th, 10th, 20th, 30th and 40th year of operation are summarized in Table 2. The average failure rates (per vessel) per year are also shown in the same table. It is found that the vessel failure probability is in the order of 10^{-4} and the failure rate is in the order of 10^{-6} per vessel year.

In considering the non-destructive inspection, $\mu = 0.113\,/\mathrm{mm}$ is used in Eq. (5), which results in a POD curve shown in Fig. 1. If we assume that a complete NDI and repair is applied to the vessel every ten years, then it is found through the Monte Carlo simulation that the crack size distribution before the inspection follows a shifted exponential distribution as that described by Eq. (1) and is shown in Fig. 2. According to Bayes' theorem, this prior distribution function together with Eq. (5) determines another shifted exponentially distributed posterior probability density function for the crack size after inspection, which is also shown in Fig. 2. The present Monte Carlo simulation does prove this conclusion. It also gives us the value of 4.90×10^{-12} for the failure probability of the vessel at the 40th year, if it is inspected every ten years and the detected crack is completely repaired following each inspection. If it is inspected and repaired every twenty years, then the failure probability becomes 6.62×10^{-7} at the 40th year of operation.

TABLE 2
Failure probabilities and failure rates

Year	Failure Probability	Failure Rate
0	2.32×10^{-4}	
10	2.75×10^{-4}	4.78×10^{-6}
20	3.21×10^{-4}	4.47×10^{-6}
30	3.60×10^{-4}	4.29×10^{-6}
40	4.10×10^{-4}	4.46×10^{-6}

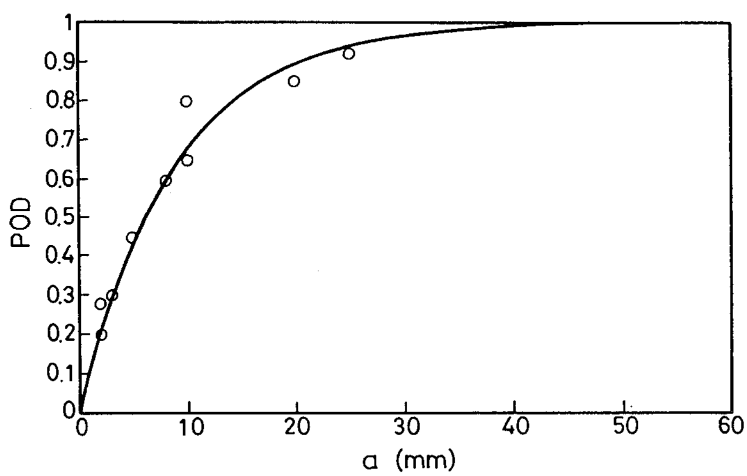

Fig. 1 POD curve for an UT device

4. CONCLUSIONS

A hybrid method consisting of a computer simulation algorithm as well as an analytical interference technique has been proposed and employed to estimate the potential failure probability of a nuclear reactor pressure vessel. In performing the Monte Carlo simulation of the fatigue crack growth, 1,000 samples were obtained which were latter used in the calculation of the failure probability. To check whether the sample size is enough, 1,500 samples has been generated instead and it was found that the stress intensity factor remained log-normally distributed and the fracture toughness remained normally distributed for the best fits. The failure probability at the 40th year of operation was found to be 4.3×10^{-4} as compared to 4.1×10^{-4} obtained from the calculation based on 1,000 samples. Therefore, it is concluded that 1,000 samples are enough in our present analysis.

In the calculation of transient stresses based on the stress design report, the maximum stress during each possible transient situation was considered in the present study. From

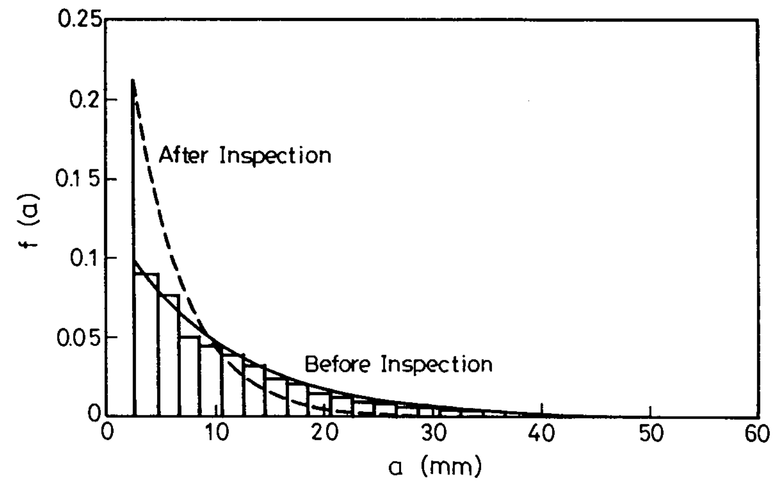

Fig. 2 Crack size distributions before and after an inspection (at the 10th year)

probabilistic point of view, the combination of temperature, pressure and other affecting factors may not always be such worse as to give us the calculated maximum stresses. Therefore, the results we obtained in the present analysis should be explained on the conservative side. Suitable randomization of each transient stress can be introduced and incorporated into the present simulation algorithm. The result should give us smaller value of failure probability. Residual stress was also included in our analysis. The consideration of residual stress gave us even more conservative estimation. If residual stress was ignored in the analysis, the failure probability was found to be 1.7×10^{-5} at the 40th year of operation.

Finally, a preliminary study of the effect of NDI on the pressure vessel failure probability indicates that an effective inspection-and-repair practice does improve the reliability of the pressure vessel significantly. According to the present study, the vessel failure probability at the 40th year decreases from the order of 10^{-4} to 10^{-12} or 10^{-7} if inspection is applied every ten or twenty years respectively.

ACKNOWLEDGEMENTS

This work is supported by the Atomic Energy Council of the Republic of China. The authors would like to express their appreciation to the Materials Research Lab., ITRI for the assistance of performing the stress calculation. The authors especially enjoyed the fruitful discussions with Dr. D.-I. Hsia and Mr. S.-C. Chu of the AEC-ROC.

REFERENCES

1. Bush, S. H. 1988. "Statistics of Pressure Vessel and Piping Failures." *ASME Journal of Pressure Vessel Technology*, Vol. 110: pp. 225–233.
2. Proven, J. W. 1987. *Probabilistic Fracture Mechanics and Reliability*. Dordrecht, Netherlands: Martinus Nijhoff Publishers.
3. Pennell, W. E. 1992. "Heavy-Section Steel Technology Program: Fracture Issues." *ASME Journal of Pressure Vessel Technology*, Vol. 114: pp. 255–264.
4. Becher, P. E. and Pedersen, A. 1974. "Application of Statistical Linear Elastic Fracture Mechanics to Pressure Vessel Reliability Analysis." *Nuclear Engineering and Design*, Vol. 27: pp. 413–425.