



Evaluation of Modal Combination Methods for Seismic Response Spectrum Analysis

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ABSTRACT

Regulatory Guide 1.92 "Combining Modal Responses and Spatial Components in Seismic Response Analysis" was last revised in 1976. The objective of this project was to re-evaluate the current regulatory guidance for combining modal responses in response spectrum analysis, evaluate recent technical developments, and recommend revisions to the regulatory guidance. This paper describes the qualitative evaluation of modal response combination methods.

INTRODUCTION

The United States Nuclear Regulatory Commission (NRC) issues Regulatory Guides (RG) which describe methods acceptable to the NRC staff for satisfying regulations. RG 1.92, "Combining Modal Responses and Spatial Components in Seismic Response Analysis," (Reference 1) was last revised in 1976, prior to a number of significant technical developments for combining modal responses. The 1989 revision to Standard Review Plan (SRP) Section 3.7.2, "Seismic System Analysis," (Reference 2) recognized a number of recent technical developments by reference, and stated that their application to nuclear power plant seismic analysis is subject to review on a case-by-case basis. Also incorporated into SRP Section 3.7.2 as Appendix A was a procedure to address high frequency mode effects, developed by Kennedy (Reference 3).

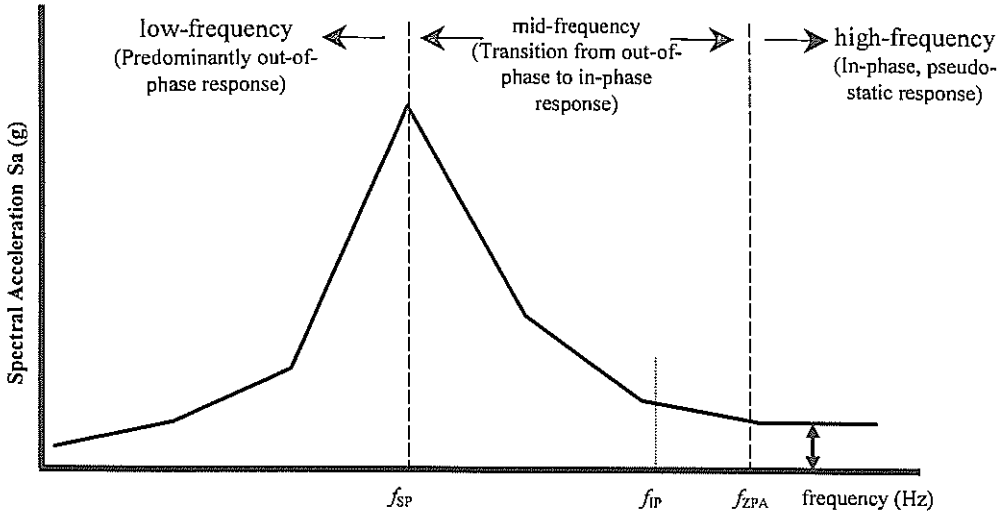
The initial phase of this program focused on review of the technical literature and selection of candidate modal response combination methods for more detailed numerical evaluation. Acceptable methods in RG 1.92 were also included to provide a comparison to more recent technical developments. References 3 and 4 provided an excellent starting point. The methods selected for evaluation were those which have been subjected to the greatest level of prior review and assessment. The methods evaluated for combining out-of-phase modal response components were Square Root of the Sum of the Squares (SRSS), NRC Grouping, NRC Ten Percent, NRC-DSC, Rosenblueth's DSC (Reference 5), and Der Kiureghian's CQC (Reference 6). The methods evaluated for separating in-phase and out-of-phase modal response components were Lindley and Yow (Reference 7), Hadjian (Reference 8), and Gupta (Reference 4). The method evaluated for including high frequency mode effects was Kennedy (Reference 3).

DESCRIPTION OF MODAL RESPONSE COMBINATION METHODS

The major application of seismic response spectrum analysis is for systems and components attached to building structures. A building-filtered in-structure response spectrum depicting spectral acceleration vs. frequency is the typical form of seismic input for

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such analyses. This type of spectrum usually exhibits a sharp peak at the fundamental frequency of the building/soil dynamic system. An idealized in-structure response spectrum is shown below; the spectral regions and key frequencies are indicated.



- f_{SP} ≡ frequency at which the peak spectral acceleration is reached; typically the fundamental frequency of the building/soil system
- f_{ZPA} ≡ frequency at which the spectral acceleration returns to the zero period acceleration
- f_{IP} ≡ frequency above which the SDOF modal responses are in-phase with the time varying acceleration input used to generate the spectrum

The high frequency region of the spectrum ($> f_{ZPA}$) is characterized by no amplification of the peak acceleration of the input time history. A SDOF oscillator having a frequency $> f_{ZPA}$ is accelerated in-phase and with the same acceleration magnitude as the applied acceleration, at each instant in time. A system or component with fundamental frequency $> f_{ZPA}$ is correctly analyzed as a static problem subject to a loading equal to mass times ZPA. This concept can be extended to the high frequency ($> f_{ZPA}$) modal responses of multi-modal systems or components. The mass not participating in the amplified modal responses (i.e., “missing mass”) multiplied by the ZPA is applied in a static analysis, to obtain the response contribution from all modes with frequencies $> f_{ZPA}$.

In the low-frequency region of the spectrum ($< f_{SP}$) the modal responses of SDOF oscillators are not in-phase with the applied acceleration time history, and generally are not in-phase with each other. These are designated “out-of-phase” modal responses. Since a response spectrum provides only peak acceleration vs. frequency, with no phasing information, the out-of-phase peak modal responses for a multi-modal structural system require a rule or methodology for combination. Based on the assumption that the peak modal responses are randomly phased, the square root of the sum of the squares (SRSS) method was developed and adopted. Modifications to SRSS were subsequently developed, in order to account for potential phase correlation when modal frequencies are numerically close (i.e., “closely spaced modes”).

In the mid-frequency region (f_{SP} to f_{ZPA}), it has been postulated that the peak SDOF oscillator modal responses consist of two distinct and separable elements. The first element is the out-of-phase response component and the second element is the in-phase response

component. It is further postulated that there is a continuous transition from out-of-phase response to in-phase response. If $f_{IP} < f_{ZPA}$ can be defined, then the mid-frequency region can be further divided into two sub-regions: $f_{SP} < f < f_{IP}$ and $f_{IP} \leq f \leq f_{ZPA}$.

Past practice in the nuclear power industry has been to assume that individual modal responses in the mid-frequency region ($f_{SP} < f < f_{ZPA}$) are out-of-phase, and combination methods applicable to the low-frequency region are applicable to the mid-frequency region.

Terms used in the following sections are:

- Sa_i = Spectral Acceleration for mode i
- R_i = Response of mode i
- α_i = In-phase response ratio for mode i
- Rr_i = In-phase response component for mode i
- Rp_i = Out-of-phase response component for mode i
- Rr = Total in-phase response component from all modes
- Rp = Total out-of-phase response component from all modes
- Rt = Total combined response from all modes
- C_{jk} = Modal response correlation coefficient between modes j and k.

Combination of Out-of-Phase Modal Responses Components

In generalized form, all of the out-of-phase modal response combination methods can be represented by a single equation:

$$R_p = \left[\sum_{j=1}^n \sum_{k=1}^n C_{jk} R_{p_j} R_{p_k} \right]^{1/2} \tag{Eqn. 1}$$

The coefficients C_{jk} can be uniquely defined for each method.

Square Root of the Sum of the Squares (SRSS)

At the foundation of all methods for combining uncorrelated modal responses is SRSS. All of the methods for combination of the out-of-phase response components are equivalent to SRSS if there are no “closely spaced” modes.

In the case of SRSS,

$$\begin{aligned} C_{jk} &= 1.0 \text{ for } j = k \\ C_{jk} &= 0.0 \text{ for } j \neq k \end{aligned} \tag{Eqn. 2}$$

SRSS Combination reduces to:

$$R_p = \left[\sum_{i=1}^n R_{p_i}^2 \right]^{1/2} \tag{Eqn. 3}$$

NRC Grouping Method

The NRC Grouping Method (Reference 1) is the most commonly applied method of accounting for closely spaced modes in the nuclear power industry. The system modal responses are grouped and summed absolutely before performing SRSS combination of the groups. The modal responses are grouped such that the lowest and highest frequency modes in a group are within 10% and no mode is in more than one group.

$$R_p = \left[\sum_{i=1}^n GR_i^2 \right]^{1/2} \tag{Eqn. 4}$$

The major criticism of the NRC Grouping Method is the use of absolute summation within each group. If modal responses are assumed to be correlated because they have closely spaced frequencies, then summation should be algebraic within each group. In terms of the

coefficients, C_{jk} , the NRC Grouping Method is defined as:

$$\begin{aligned} C_{jk} &= 1.0 && \text{for } j = k \\ C_{jk} &= 0.0 && \text{for } j \neq k, \text{ not in same group} \\ C_{jk} &= 1.0 && \text{for } j \neq k, \text{ in the same group, } R_{pj} * R_{pk} > 0 \\ C_{jk} &= -1.0 && \text{for } j \neq k, \text{ in the same group, } R_{pj} * R_{pk} < 0 \end{aligned} \quad (\text{Eqn. 5})$$

NRC Ten Percent Method

The NRC Ten Percent Method (Reference 1) is a generally more conservative variation of the NRC Grouping Method. Closely spaced modes are defined as modes with frequencies within 10% of each other and absolute summation of the closely spaced modal responses is specified. The difference is that modal responses are not grouped.

In terms of the coefficients, C_{jk} , the NRC Ten Percent Method is defined as:

$$\begin{aligned} C_{jk} &= 1.0 && \text{for } j = k \\ C_{jk} &= 0.0 && \text{for } j \neq k, \text{ and } f_j \text{ and } f_k \text{ separated by } > 10\% \\ C_{jk} &= 1.0 && \text{for } j \neq k, \text{ and } f_j \text{ and } f_k \text{ separated by } \leq 10\%; R_{pj} * R_{pk} > 0 \\ C_{jk} &= -1.0 && \text{for } j \neq k, f_j \text{ and } f_k \text{ separated by } \leq 10\%; R_{pj} * R_{pk} < 0 \end{aligned} \quad (\text{Eqn. 6})$$

The NRC Ten percent Method will always produce results \geq NRC Grouping Method.

NRC Double Sum Combination (NRC-DSC)

The NRC-DSC Method (Reference 1) is an adaptation of Rosenblueth's method. The coefficients C_{jk} are defined by Equation 7. A conservative modification, consistent with the NRC Grouping and Ten Percent methods, is that the product $C_{jk} R_{pj} R_{pk}$ is always taken as positive. In Rosenblueth's method, the product may be either positive or negative, depending on the signs of R_{pj} and R_{pk} . Consequently, NRC-DSC will always produce results \geq Rosenblueth's method.

Rosenblueth's Double Sum Combination (DSC)

Rosenblueth (Reference 5) provided the first significant mathematical approach to evaluation of modal correlation for seismic response spectrum analysis. It is based on the application of random vibration theory, utilizing a finite duration of white noise to represent seismic loading. A formula for calculation of the coefficients C_{jk} as a function of the modal circular frequencies (ω_j, ω_k), modal damping ratios (β_j, β_k), and the time duration of strong earthquake motion (t_D) was derived. Using the form of the equation from Reference 1,

$$C_{jk} = \frac{1}{1 + \left\{ \frac{\omega'_j - \omega'_k}{\beta'_j \omega_j + \beta'_k \omega_k} \right\}^2} \quad (\text{Eqn. 7})$$

where $\omega'_{()} = \omega_{()} \left[1 - \beta_{()}^2 \right]^{1/2}$
 $\beta'_{()} \equiv \beta_{()} + \frac{2}{t_D \omega_{()}}$

Numerical values of C_{jk} were tabulated for the DSC Method as a function of frequency, frequency ratio, and strong motion duration time for constant modal damping of 1%, 2%, 5% and 10%. The most significant result is that C_{jk} is highly dependent on the damping ratio. For 2%, 5% and 10% damping, $C_{jk} \approx 0.2, 0.5$ and 0.8 respectively, at a frequency ratio of 0.9 (modal frequencies within 10%). The definition of closely-spaced modes used in the NRC Grouping and Ten Percent Methods is not damping-dependent.

Der Kiureghian's Complete Quadratic Combination (CQC)

Der Kiureghian (Reference 6) presents a methodology similar to Rosenblueth's Double Sum Combination for evaluation of modal correlation for seismic response spectrum analysis.

It is also based on application of random vibration theory, but utilizes an infinite duration of white noise to represent seismic loading. A formula for calculation of the coefficients C_{jk} as a function of modal circular frequencies and modal damping ratios was derived:

$$C_{jk} = \frac{8 (\beta_j \beta_k \omega_j \omega_k)^{1/2} * (\beta_j \omega_j + \beta_k \omega_k) * \omega_j \omega_k}{(\omega_j^2 - \omega_k^2)^2 + 4 \beta_j \beta_k \omega_j \omega_k (\omega_j^2 + \omega_k^2) + 4(\beta_j^2 + \beta_k^2) \omega_j^2 \omega_k^2} \quad (\text{Eqn. 8})$$

While the form of Equation 8 differs significantly from Equation 7, the two equations produce equivalent results if t_D is assumed very large in Equation 7.

Separation of Modal Responses into Out-of-Phase Components and In-Phase Components

Three methods have received considerable prior review and evaluation: Lindley-Yow (Reference 7), Hadjian (Reference 8), and Gupta (Reference 4). The mathematical statement of each method is not restricted to the mid-frequency range ($f_{SP} < f < f_{ZPA}$) of the response spectrum. However, as discussed previously, it is in the mid-frequency range that the separation of individual peak modal responses into out-of-phase and in-phase modal response components is applicable. The similarities and differences, as well as the limitations, of the three methods are described in the following sections.

Lindley-Yow Method

Mathematically, the Lindley-Yow method (Reference 7) is defined by the following equations:

$$\alpha_i = ZPA / Sa_i \quad 0 \leq \alpha_i \leq 1.0 \quad (\text{Eqn. 9})$$

$$Rr_i = R_i * \alpha_i \quad (\text{Eqn. 10})$$

$$Rp_i = R_i * \sqrt{1 - \alpha_i^2} \quad (\text{Eqn. 11})$$

$$Rr = \sum_{i=1}^n Rr_i \quad (\text{Eqn. 12})$$

$$Rp = \left[\sum_{j=1}^n \sum_{k=1}^n C_{jk} Rp_j Rp_k \right]^{1/2} \quad (\text{Eqn. 13})$$

$$Rt = \sqrt{Rr^2 + Rp^2} \quad (\text{Eqn. 14})$$

The following characteristics of the Lindley-Yow method are observed:

- $\alpha_i \rightarrow 1.0$ as $f_i \rightarrow f_{ZPA}$ ($Sa_i = ZPA$). Therefore, $f_{IP} = f_{ZPA}$.
- The in-phase component of modal response for every mode has an associated acceleration equal to the ZPA.
- The out-of-phase component of an individual peak modal response has an associated modified spectral acceleration given by

$$\bar{S}a_i = \left(Sa_i^2 - ZPA^2 \right)^{1/2} \quad (\text{Eqn. 15})$$

- $R_i = (Rp_i^2 + Rr_i^2)^{1/2}$, which infers that the in-phase and out-of-phase response components of an individual peak modal response are uncorrelated.
- α_i attains its minimum value at $f_i = f_{SP}$, but increases for $f_i < f_{SP}$ until it attains a value of 1.0 when $Sa_i = ZPA$ in the low frequency region of the spectrum. Values of $\alpha_i > 1.0$ have no meaning because $(1 - \alpha_i^2)^{1/2}$ becomes imaginary.

An obvious limitation of the Lindley-Yow method is in the low frequency range ($f < f_{SP}$) of the response spectrum. Therefore, the Lindley-Yow method is applicable to structural

systems which do not have significant modal responses with $f_i < f_{SP}$. Circumventing this limitation in the Lindley-Yow method is straightforward: apply it only to those modes with $f_i \geq f_{SP}$ and set $\alpha_i = 0$ for $f_i < f_{SP}$.

While in theory the Lindley-Yow method includes the in-phase contribution from modes above f_{ZPA} , its practical application is for modal responses below f_{ZPA} , coupled with the missing mass method for modal contributions above f_{ZPA} .

Hadjian Method

The Hadjian Method (Reference 8) is similar in formulation to the Lindley-Yow method, with two notable differences:

- Equation 11 is replaced by
$$R_{p_i} = R_i * (1 - \alpha_i) \quad \text{(Eqn. 16)}$$

- Equation 14 is replaced by
$$R_t = |R_p| + |R_r| \quad \text{(Eqn. 17)}$$

The modified spectral acceleration is given by

$$\bar{S}a_i = Sa_i - ZPA \quad \text{(Eqn. 18)}$$

The Hadjian method has the same limitation as the Lindley-Yow method in the low frequency range, because the definition of α_i is identical. However, the Hadjian Method possesses internal contradictions with respect to the assumed phase relationships between in-phase and out-of-phase response components. Combining Equations 10 and 16 yields

$$R_i = R_{p_i} + R_{r_i} \quad \text{(Eqn. 19)}$$

This implies that the in-phase and out-of-phase response components for each mode are in-phase with each other. However, all R_{r_i} 's are in-phase and summed algebraically, per Eqn. 12, to obtain R_r . Therefore, it would follow that all R_{p_i} 's are also in-phase and should be summed algebraically to obtain R_p . This contradicts Equation 13, in which the R_{p_i} 's are assumed to be predominantly out-of-phase. Kennedy (Reference 3) previously identified this contradiction. On this basis, the Hadjian method is not recommended and was not included in subsequent numerical studies.

Gupta Method

The Gupta Method (Reference 4) is identical in form to the Lindley-Yow method. The one very significant difference is the definition of α_i . Equations 10 through 14 remain the same. In the Gupta method, α_i is an explicit function of frequency. The original basis for definition of α_i is semi-empirical, derived from numerical studies using actual ground motion records. A best fit equation, which defines α_i as a continuous function of frequency, was developed from the results of the numerical studies.

Two spectrum-dependent frequencies (f_1, f_2) are first defined as follows:

$$f_1 = \frac{Sa_{\max}}{2\pi Sv_{\max}} \quad \text{(Eqn. 20)}$$

where Sa_{\max} and Sv_{\max} are the maximum spectral acceleration and velocity, respectively.

$$f_2 = (f_1 + 2 f_{ZPA})/3 \quad \text{(Eqn. 21)}$$

Gupta's definition of α_i is given by:

$$\alpha_i = \begin{cases} 0 & \text{for } f_i \leq f_1 \\ \frac{\ln(f_i/f_1)}{\ln(f_2/f_1)} & \text{for } f_1 \leq f_i \leq f_2 \\ 1.0 & \text{for } f_i \geq f_2 \end{cases} \quad \text{(Eqn. 22)}$$

For a sharply peaked, in-structure response spectrum, $f_1 = f_{SP}$ because $Sv_{max} = \text{Max}(Sa_i/\omega_i) = Sa_{max} / \omega_{SP}$; and the Gupta method has the following characteristics:

- For $f_1 \leq f_{SP}$, $\alpha_i = 0$; treated as out-of-phase.
- For $f_2 \leq f_i \leq f_{ZPA}$, $\alpha_i = 1.0$; this infers that $f_{IP} = f_2$ in the Gupta method.
- Only modal responses with $f_{SP} < f_i < f_2$ are separated into out-of-phase and in-phase response components.

The potential limitations of the Gupta method lie in the semi-empirical basis for definition of α_i as a function of f_i . In Reference 4, Gupta indicates that α_i can be numerically evaluated if the time history used to generate the response spectrum is known. It is implied that numerical evaluation of α_i is more accurate than the semi-empirical definition of α_i given by Equation 22. The overall structure of the Gupta method is superior to the Lindley-Yow method because there is no limitation for modal responses with $f_i < f_{SP}$. In addition, any value of $f_{IP} \leq f_{ZPA}$ can be accommodated by setting $f_2 = f_{IP}$, in lieu of Equation 21.

Contribution of High Frequency Modes

Missing Mass Method

The “Missing Mass” Method is a convenient, computationally efficient and accurate method to (1) account for the contribution of all modes with frequencies above the frequency (f_{ZPA}) at which the response spectrum returns to the Zero Period Acceleration (ZPA) and (2) account for the contribution to support reactions of mass which is apportioned to system support points. It constitutes the total effect of all system mass which does not participate in (i.e., “missing” from) the modes with frequencies below f_{ZPA} . The system response to the missing mass is calculated by performing a static analysis for applied loads equal to the missing mass multiplied by the spectrum ZPA. This method is mathematically rigorous and is considered the only acceptable method to account for high frequency modal contributions ($f > f_{ZPA}$) and mass apportioned to system support points.

Kennedy (Reference 3) documented this method and recommended that it be included in Regulatory Guidance. The 1989 revision to the SRP Section 3.7.2, “Seismic System Analysis,” (Reference 2) incorporated Kennedy’s recommendation as Appendix A. The mathematical details are presented in both References 2 and 3.

Complete Solution for Response Spectrum Analysis

Two methods are defined for constructing the complete response spectrum analysis solution. The coefficients C_{jk} are defined by one of the out-of-phase combination methods.

Method 1

Method 1 represents the common method applied to response spectrum analysis since the 1980’s. Amplified modal responses ($f < f_{ZPA}$) are combined by SRSS with a correction for closely spaced modes. The contribution of unamplified modal responses ($f > f_{ZPA}$) is calculated by the missing mass method. These two components are then combined by SRSS to produce the total solution. Mathematically, this is represented by

$$R_p = \left[\sum_{j=1}^n \sum_{k=1}^n C_{jk} R_j R_k \right]^{1/2}$$

$n = \text{no. of modes below } f_{ZPA}$

$$R_r = R_{\text{missing mass}}$$

$$R_t = \sqrt{R_p^2 + R_r^2}$$

Method 2

Method 2 introduces the concept of in-phase and out-of-phase modal response components for the amplified modes ($f < f_{ZPA}$). Mathematically, the complete solution is represented by

$$\begin{aligned} R_{p_i} &= R_i * (1 - \alpha_i^2)^{1/2} \\ R_{r_i} &= R_i * \alpha_i \\ R_p &= \left[\sum_{j=1}^n \sum_{k=1}^n C_{jk} R_{p_j} R_{p_k} \right]^{1/2} \end{aligned} \quad (\text{Eqn. 24})$$

$n = \text{no. of modes below } f_{ZPA}$

$$R_r = \sum_{i=1}^n R_{r_i} + R_{\text{missing mass}}$$

$$R_t = \sqrt{R_p^2 + R_r^2}$$

Method 2 is equally applicable to both the Lindley-Yow and the Gupta methods. Only the definition of α_i changes.

SUMMARY OF RESULTS

The qualitative evaluation of modal response combination methods provided the foundation for subsequent numerical studies, which quantitatively evaluated the combination methods by comparison to time history analysis results. Together, this provided the basis for technical conclusions and recommendations for revision of regulatory guidance.

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