Dynamic Behavior of a PWR Core Barrel Using Axisymmetric Thin Shell Finite Elements

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ABSTRACT

In this study we have modeled the core barrel of the HDR (Test Reactor in Germany). A finite element program was developed using axisymmetric thin shell elements under asymmetric loading where the circunferential dependence was removed by Fourier Series expansion. The HDR core barrel is basically a thin vertical cylindrical shell clamped at the top with a rigid ring attached to the bottom.

Two formulations were used to describe the strain-displacement and the resulting stress-displacement relationships of the shell, namelly Flugge's and Timoshenko's equations. The displacement fields were approximated linearly in the axial (u) and circunferential (v) directions and by a cubic polynomial in the radial direction (w). The stiffness and mass matrices were obtained by integrating the strain energy density and the kinetic energy approximated by the interpolation functions over the shell element. The Fourier Series expansion uncouples the stiffness and mass properties in each circunferential mode transforming a large dynamic eigenvalue problem into several eigenvalue problems of dimension 8.

The rigid ring contributes to the inertia properties of the last nodal line only in the circunferential modes 0 and 1 since higher order modes would require deformation of the ring.

For the HDR geometry both Flugge's and Timoshenko's theories agreed well using 35 elements. For Timishenko's formulation we have refined the mesh using 70 elements and little improvement was achieved.

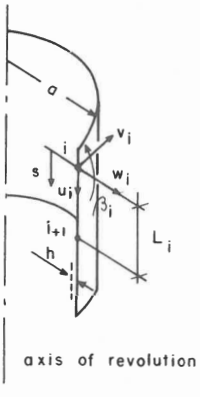

Figure 1 - Shell Geometry

1. Introduction

In the case of a postulated blow down accident in a PWR its core barrel will be submitted to high dynamic loadings. In addition service conditions may cause vibrations leading to fatigue cracks. Therefore the dynamic behavior of the core barrel must be determined.

In the postulated blow down accident the structural behavior of the core barrel is coupled with the fluid inside the reactor vessel to yield the actual dynamic response. Several authors (references /1/, /2/, /3/) tried to model the HDR core barrel by standard finite elements codes and the agreement between them is not very good. Ludwig /4/ presented a quasi-exact solution for the natural modes of vibration of the core barrel representing the core internals by a rigid ring attached to the bottom of a thin cylindrical shell of uniform thickness clamped at the top. The reactor vessel was assumed to be rigid. His results showed that the previously mentioned results yielded some incorrect eigenfrequencies in addition to the omission of some mode shapes.

2. Finite Element Formulation

2.1 Coordinate System and Interpolation Functions

Referring to Figure 1 we describe the shell behavior by four nodal coordinates, namely u_i , v_i , w_i , β_i. The circunferential dependence is removed by Fourier Series expansion

$$\begin{aligned}
u_i(\theta) &= q_1^{(i)o} + \sum_{j=1}^{m} q_1^{(i)j} \cos j\theta + \sum_{j=1}^{m} \bar{q}_1^{(i)j} \sin j\theta \\
v_i(\theta) &= q_2^{(i)o} + \sum_{j=1}^{m} q_2^{(i)j} \sin j\theta + \sum_{j=1}^{m} \bar{q}_2^{(i)j} \cos j\theta \\
w_i(\theta) &= q_3^{(i)o} + \sum_{j=1}^{m} q_3^{(i)j} \cos j\theta + \sum_{j=1}^{m} \bar{q}_3^{(i)j} \sin j\theta \\
\beta_i(\theta) &= q_4^{(i)o} + \sum_{j=1}^{m} q_4^{(i)j} \cos j\theta + \sum_{j=1}^{m} \bar{q}_4^{(i)j} \sin j\theta
\end{aligned} \tag{1}$$

where the q's are the generalized displacements at node (i).

Since both cosine and sine modes yield the same results from now on we will consider only cosine modes. Furthermore since the Fourier Series expansion uncouples the shell response in each mode we need only carry out the analysis for a generic mode, say j. We now can approximate the displacement fields within the elements using the natural coordinate $\zeta = s/L_i$. For cosine mode j we have

$$\begin{aligned}
u^j &= \{q_1^{(i)j} (1-\zeta) + q_1^{(i+1)j} \zeta\} \cos j\theta \\
v^j &= \{q_2^{(i)j} (1-\zeta) + q_2^{(i+1)j} \zeta\} \sin j\theta \\
w^j &= \{q_3^{(i)j} \phi_1 + q_4^{(i)j} \phi_2 + q_3^{(i+1)j} \phi_3 + q_4^{(i+1)j} \phi_4\} \cos j\theta
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
\phi_1 &= 1-3\zeta^2 + 2\zeta^3 \\
\phi_2 &= L_i(\zeta-2\zeta^2 + \zeta^3) \\
\phi_3 &= 3\zeta^2 - 2\zeta^3 \\
\phi_4 &= L_i(-\zeta^2 + \zeta^3)
\end{aligned} \tag{3}$$

note that the ith element DOF for mode j are written in vector notation as $\underset{\sim}{\delta}^{ij} = (q_1^{(i)j}, q_2^{(i)j}, q_3^{(i)j}, q_4^{(i)j}, q_1^{(i+1)j}, q_2^{(i+1)j}, q_3^{(i+1)j}, q_4^{(i+1)j})$. Thus we can write

$$\underset{\sim}{u}^T = \underset{\sim}{H}^i \cdot \underset{\sim}{\delta}^{iT} \tag{4}$$

where $\underset{\sim}{u} = (u^j , v^j , w^j)$; $\underset{\sim}{H} = \underset{\sim}{H}(\zeta , L_i)$

2.2 Mass Matrix

The ith shell element mass matrix , $\underset{\sim}{M}^i$, is obtained by substituing the displacement field description , eq. (2), into the shell element kinetic energy expression

$$T^i = \frac{\rho\, h}{2} \int_0^{L_i} \int_0^{2\pi} (\dot{u}^{j^2} + \dot{v}^{j^2} + \dot{w}^{j^2}) \; a \; d\theta \; ds \tag{5}$$

where ρ - mass density

This results in the consistent mass matrix, $\underset{\sim}{M}^i$, such that

$$T^i = \frac{1}{2} \; \underset{\sim}{\delta}^i \cdot \underset{\sim}{M}^i \cdot \underset{\sim}{\delta}^{iT} \tag{6}$$

The integration along the circunferential variable , is carried out analytically and along the shell element it is done by the Gauss-Legendre procedure.

2.3 Stiffness Matrix

The ith element stiffness matrix , $\underset{\sim}{K}^i$, is computed by integrating the strain energy density approximated by the interpolation functions over the shell element. Here the addopted shell theory comes into play. They are discussed in the next item. Let the strain energy density of the ith shell element be

$$U^i = \frac{1}{2} \int_V \underset{\sim}{\sigma} \cdot \underset{\sim}{e}^T \, dV = \frac{1}{2} \; \underset{\sim}{\delta}^i \cdot \underset{\sim}{K}^i \cdot \underset{\sim}{\delta}^{iT} \tag{7}$$

The stress-strain relations are given by

$$\underset{\sim}{\sigma}^T = \underset{\sim}{E} \cdot \underset{\sim}{e}^T \tag{8}$$

The shell theory used will determine the strain-displacement matrix $\underset{\sim}{W}$ defined as

$$\underset{\sim}{e}^T = \underset{\sim}{W} \cdot \underset{\sim}{u}^T = \underset{\sim}{W} \cdot \underset{\sim}{H}^i \cdot \underset{\sim}{\delta}^{iT} = \underset{\sim}{L} \cdot \underset{\sim}{\delta}^{iT} \tag{9}$$

Substituing eq.(9) into eq.(8) and then into eq.(7) we have the element stiffness matrix

$$\underset{\sim}{K}^i = \underset{\sim}{L}^T \cdot \underset{\sim}{E}^T \cdot \underset{\sim}{L} \tag{10}$$

Next we will discuss the different shell theories.

3. Shell Equations

3.1 Flugge's Equations

The thickness of the shell is taken to be small as compared to the radii of curvature of the middle surface and it is assumed that (1) normals to the middle surface before deformation remain straight and normal to the deformed middle surface; and (2) the length of elements normal to the middle surface remain unchanged. The first derivatives of the displacements are taken to be neglegible compared to unit. The displacements u, v and w of a general point P of the shell can be written, according Figure 2, as

$$u_P = u - z\, w_{,x} \; ; \; v_P = \frac{a+z}{a} v - \frac{z}{a} w_{,\theta} \; ; \; w_P = w \tag{11}$$

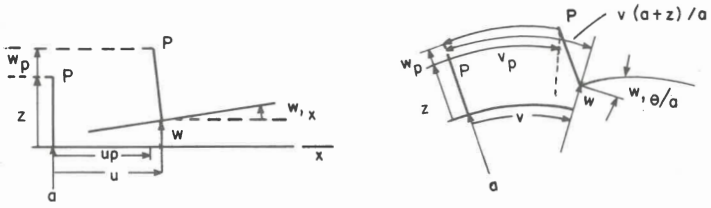

Figure 2 - Displacement of a general point P

The strain-displacement relations are

$$e_{xx} = u_{p,x} = u_{,x} - z\, w_{,xx} = e^o_{xx} - z\, \chi_x$$

$$e_{\theta\theta} = \frac{1}{a+z} v_{p,\theta} + \frac{w_p}{a+z} = \frac{1}{a} v_{,\theta} - \frac{z}{a(a+z)} w_{,\theta\theta} + \frac{w}{a+z}$$

$$= \frac{(v_{,\theta}+w)}{a} - z\left(\frac{w}{a^2} + \frac{1}{a^2} w_{,\theta\theta}\right) = e^o_{\theta\theta} - z\, \chi_\theta \qquad (12)$$

$$\gamma_{x\theta} = 2\, e_{x\theta} = v_{p,x} + \frac{1}{a+z} u_{p,\theta} = \frac{a+z}{a} v_{,x} - \frac{z}{a} w_{,\theta x} + \frac{1}{a+z} u_{,\theta} - \frac{z}{a+z} w_{,\theta x}$$

$$= \left(\frac{u_{,\theta}}{a} + v_{,x}\right) - 2z\left(\frac{u_{,\theta}}{2a} - \frac{v_{,x}}{2a} + \frac{w_{,x\theta}}{a}\right) = \gamma^o_{x\theta} - 2z\, \chi_{x\theta}$$

where the superscript o denotes the middle surface strain.

Eqs. (12) can be written as $\underset{\sim}{e}^T = \underset{\sim}{W} \cdot \underset{\sim}{u}^T$, where

$\underset{\sim}{e} = (e^o_{xx}\ ,\ e^o_{\theta\theta}\ ,\ \gamma^o_{x\theta}\ ,\ \chi_x\ ,\ \chi_\theta\ ,\ \chi_{x\theta})$; $\underset{\sim}{u} = (u\ ,\ v\ ,\ w)$ and

$$\underset{\sim}{W} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{1}{a}\frac{\partial}{\partial \theta} & \frac{1}{a} \\ \frac{1}{a}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & \frac{\partial^2}{\partial x^2} \\ 0 & 0 & \frac{1}{a^2}\left(1+\frac{\partial^2}{\partial \theta^2}\right) \\ \frac{1}{2a^2}\frac{\partial}{\partial \theta} & -\frac{1}{2a}\frac{\partial}{\partial x} & \frac{1}{a}\frac{\partial^2}{\partial x \partial \theta} \end{bmatrix} \qquad (13)$$

By using Hooke's law we can determine the stresses σ_{xx} , $\sigma_{\theta\theta}$, $\sigma_{x\theta}$ from eqs.(12). Integrating the stresses over the shell thickness and assuming that $z/a \ll 1$ we obtain the stress resultant-displacement relationships. Let

N_i - normal stress resultant along i-direction

N_{ij} - shear stress resultant normal to i-axis along j-direction

M_i - bending moment per unit length-vector orthogonal to i-axis

M_{ij} - twisting moment per unit length-vector along i-axis

$$\underset{\sim}{\sigma} = (N_x , N_\theta , N_{x\theta} , M_x , M_\theta , M_{x\theta})$$

Thus we can write the stress-strain relations for the simplified Flugge's equations ($z/a \ll 1$).

$$\underset{\sim}{\sigma}^T = \underset{\sim}{E} \cdot \underset{\sim}{e}^T \tag{14}$$

where

$$\underset{\sim}{E} = \begin{bmatrix} D & \nu D & 0 & 0 & 0 & 0 \\ \nu D & D & 0 & 0 & 0 & 0 \\ 0 & 0 & D(1-\nu)/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & K & \nu K & 0 \\ 0 & 0 & 0 & \nu K & K & 0 \\ 0 & 0 & 0 & 0 & 0 & K(1-\nu) \end{bmatrix} \tag{15}$$

$D = Eh/(1-\nu^2)$; $K = Eh^3/12(1-\nu^2)$; ν - Poisson's ratio ; E - Young's modulus

Eqs.(13) and (15) give the needed matrices to compute the element stiffness matrix using eq. (10). One additional step must be done to take care of the change of variable $\zeta = x/L_i$.

3.2 Timoshenko's Equations

Timoshenko assumed a coordinate system such that the z-axis is positive outwards the shell. Therefore the middle surface strain-displacement relationship becomes

$$\bar{e}^o_{xx} = u_{,x} \; ; \; \bar{e}^o_{\theta\theta} = (v_{,\theta} - w)/a \; ; \; \bar{\gamma}^o_{x\theta} = (u_{,\theta}/a + v_{,x}) \tag{16}$$

The changes in curvature are obtained through geometrical reasonings as

$$\bar{\chi}_x = w_{,xx} \; ; \; \bar{\chi}_\theta = (w + w_{,\theta\theta})/a^2 \; ; \; \bar{\chi}_{x\theta} = (v_{,x} + w_{,x\theta})/a \tag{17}$$

Next Timoshenko assumes that for computing the changes of curvature the middle surface undergoes an inextensional deformation, i.e., $\bar{e}^o_{xx} = \bar{e}^o_{\theta\theta} = \bar{\gamma}^o_{x\theta} = 0$. Therefore the effect of the middle surface strain on the change of curvature is neglected and the effects of strains and change of curvature are simply superposed. If we assume the displacement w as positive if oriented outwards we obtain

$$\begin{aligned} &e^o_{xx} = u_{,x} \; ; \; e^o_\theta = (w + v_{,\theta})/a \; ; \; \gamma^o_\theta = (u_{,\theta}/a + v_{,x}) \\ &\chi_x = -w_{,xx} \; ; \; \chi_\theta = (-w_{,\theta\theta} + v_{,\theta})/a^2 \; ; \; \chi_{x\theta} = -w_{,x\theta}/a + v_{,x}/a \end{aligned} \tag{18}$$

The stress.strain relationships are the same as in the simplified Flugge's formulation given by eq.(15).

4. Ring Mass Matrix

The lower end of the core barrel is connected to a ring which is supposed to be infinitely stiff and to have given inertial properties such as mass , m, and axial and transversal mass moments of inertia I_a and I_t , respectively. The ring is substituted by a circunferential element distant ρ from the last nodal line and rigidly connected to it as shown in Figure 3.

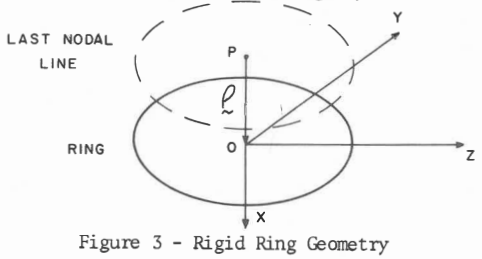

Figure 3 - Rigid Ring Geometry

Letting

$\dot{\underset{\sim}{p}}$- velocity of point P

$\underset{\sim}{\rho}$- vector from point P to the ring mass center O

$\underset{\sim}{\Omega} = (\omega_x , \omega_y , \omega_z)$ - ring angular velocity vector

The kinetic energy takes the form

$$T = \frac{1}{2} m \{\dot{\underset{\sim}{p}}.\dot{\underset{\sim}{p}} + 2\dot{\underset{\sim}{p}}.(\underset{\sim}{\Omega}x\underset{\sim}{\rho}) + (\underset{\sim}{\Omega}x\underset{\sim}{\rho}).(\underset{\sim}{\Omega}x\underset{\sim}{\rho})\} + \frac{1}{2}\{ I_a \omega_x^2 + I_t(\omega_y^2 + \omega_z^2)\} \quad (19)$$

but

$$\dot{\underset{\sim}{p}} = (\dot{q}_1^o , 0 , \dot{q}_3^1) \quad \text{and} \quad \underset{\sim}{\Omega} = (-\dot{q}_2^o /a , \dot{q}_1^1 /a , 0) \quad (20)$$

where the q's in the above expressions denote the last nodal line generalized displacements defined in eq.(1). Therefore the ring mass matrix $\underset{\sim}{M}_R$ is not the zero matrix only for the circunferential modes 0 and 1. Substituing eq.(20) into eq.(19) we can express the ring kinetic energy as

$$T = \frac{1}{2} \dot{\underset{\sim}{q}}^o . \underset{\sim}{M}_R^o . \dot{\underset{\sim}{q}}^{oT} + \frac{1}{2} \dot{\underset{\sim}{q}}^1 . \underset{\sim}{M}_R^1 . \dot{\underset{\sim}{q}}^{1T} \quad (21)$$

where

$$\dot{\underset{\sim}{q}}^o = (\dot{q}_1^o , \dot{q}_2^o , \dot{q}_3^o , \dot{q}_4^o) \; ; \; \dot{\underset{\sim}{q}}^1 = (\dot{q}_1^1 , \dot{q}_2^1 , \dot{q}_3^1 , \dot{q}_4^1)$$

$$\underset{\sim}{M}_R^o = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_a/a^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad ; \quad \underset{\sim}{M}_R^1 = \begin{bmatrix} (I_t+m^2) & 0 & -m/a & 0 \\ 0 & 0 & 0 & 0 \\ -m/a & 0 & m & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

The matrices $\underset{\sim}{M}_R^o$ and $\underset{\sim}{M}_R^1$ must be properly superposed to the global matrices for the circunferential modes 0 and 1.

5. Results and Conclusions

5.1 Results

Considering the following core barrel data

shell radius	1.3185 m
shell thickness	0.023 m
ring mass	12054.0 kg
axial moment of inertia	12190.0 kg m^2
transversal moment of inertia	6096.0kg m^2
mass density	7800 kg/m^3
Young's modulus	0.17x10^{12} N/m^2

We obtained some natural frequencies and mode shapes. These results are presented in Table 1 where

column A - Ludwig's quasi-exact solution (reference /4/)

column B - Timoshenko's formulation 35 elements

column C - Timoshenko's formulation 70 elements

column D - Flugge's formulation 35 elements

column E - Timoshenko's formulation with $w \neq v_{,\theta}$ 35 elements

5.2 Conclusions

Comparing the present results with Ludwig's quasi-exact calculations we found very good agreement at all 19 modes compared except one,i.e., circunferential mode 1 axial mode 3 where

a 4% discrepancy was found. We suspect this singular point was due to the determination of the polynomials roots in Ludwig's procedure by numerical methods. This however is much better than the previously performed finite element calculations.

In the present work we did not consider the change in shell thickness which actually exists at the HDR core barrel. This was simply because Ludwig's method cannot handle this problem although it was of great value in the comparison. For the present method varying the shell thickness presents no obstacle.

Table 1 - Lowest Natural Frequencies for the several Formulation
For n circunferential order, m axial order

n	m	A	B	C	D	E
0	1	82.4	82.36899	82.36820	82.36899	82.36899
	2	328.6	328.8397	328.6820	328.8397	328.8389
1	1	16.2	16.23039	16.22230	16.25883	16.23037
	2	104.2	103.1949	103.0845	103.2186	103.1949
	3	220.7	211.7081	211.4906	211.7172	211.7078
2	1	62.0	62.04764	61.89010	62.47650	62.04817
	2	135.8	134.9030	134.3444	135.1396	134.8999
	3	218.7	215.8889	214.6048	216.0734	215.8776
3	1	41.4	41.38286	41.23970	43.04966	41.39071
	2	85.2	85.37504	84.82013	86.27345	85.37906
	3	141.3	141.4551	140.1612	142.0673	141.4468
4	1	49.0	48.78991	48.71372	51.43016	48.80190
	2	70.5	70.27022	69.82987	72.23892	70.28764
	3	106.3	106.4418	105.2687	107.8564	106.4476
5	1	72.4	72.08546	72.05166	74.96940	72.09685
	2	82.2	81.43211	81.17383	84.10161	81.45626
	3	102.4	101.5118	100.6458	103.7886	101.5345
6	1	104.1	103.8525	103.8366	106.7914	103.8630
	2	109.7	108.6433	108.5079	111.5422	108.6682

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