# Effect of Elastic Follow-up on the Creep Modified Shakedown Limit

M.K.A. Owais, N.R. Otter
*Mechanical Engineering Laboratory, GEC Power Engineering Ltd., Cambridge Road, Whetstone, Leicester LE8 3LH, United Kingdom*

SUMMARY

When structural parts of different flexibility are in series and loaded under displacement control such that only a small part of the structure undergoes inelastic strains while the rest of the structure behaves in an elastic manner, the unit inelastic strain in the highly stressed part is magnified due to the elastic follow-up of the less stressed parts. The developed loads are more constant, stress relaxation is slower and the inelastic strain accumulation is larger. Hence, a purely elastic analysis for components with a large degree of elastic follow-up will underestimate the accumulated strains and creep-fatigue damage.

When structural systems are subjected to cyclic thermal loading at elevated temperature, the stress relaxation during the on load part of the load cycle normally promotes short term plasticity on unloading and reduces the creep modified shakedown limit. Since elastic follow-up reduces the amount of relaxation, it might actually increase the creep modified shakedown limit.

The actual effect of elastic follow-up on the creep modified shakedown limit, for stainless steel type 316 at 600°C, is investigated by the inelastic analysis of a rectangular beam which is fixed from both ends and has a locally reduced cross section and is subjected to a cyclic thermal load. Two beams are chosen, one with a small degree of elastic follow-up and the other with a considerably large degree of elastic follow-up. The accumulated strains and creep-fatigue damage sums are evaluated by a full time stepping inelastic analysis and the results are compared with those predicted by two simplified methods; the first a shakedown analysis and the second the elastic route of ASME III Code Case N-47.

It has been found that the effect of elastic follow-up on the creep-fatigue damage sums and accumulated creep strains is very small. However, with a large degree of elastic follow-up the short term plastic strain component which occurs upon unloading is largely increased due to the effect of creep strains in the elastic parts of the structure. The effect of reduced relaxation on increasing the creep modified shakedown limit is negligible and the resultant effect of elastic follow-up is a decrease in the creep modified shakedown limit.

## 1. INTRODUCTION

When structural parts of different flexibility are in series and loaded under displacement control such that only a small part of the structure undergoes inelastic strains while the rest of the structure behaves in an elastic manner, the unit inelastic strain in the highly stressed part is magnified due to the elastic follow-up of the less stressed parts. This strain magnification may be caused by local lower mechanical properties or local reduced cross sectional area. However, it is not necessary for the area of the critical section to be less than the rest of the system. Systems stressed in bending are subject to this effect even when of uniform properties and size due to the non-uniform stress distribution which prevails.

The effect of elastic follow-up on stress relaxation and creep and plastic strain concentrations has been studied by many authors [1-6]. They showed that when structural systems with a large degree of elastic follow-up are subjected to secondary stresses, the developed loads are more constant, stress relaxation is slower and the inelastic strain accumulation is larger. Hence, a purely elastic analysis for components with a large degree of elastic follow-up will underestimate the accumulated strains and creep - fatigue damage. Therefore, when accumulated strains are evaluated by elastic analysis according to the ASME III B & PV Code Case N-47 (1592) [7] any secondary stress with elastic follow-up should be included as a primary stress.

When structural systems are subjected to cyclic thermal loading at elevated temperature, the stress relaxation during the on load part of the load cycle normally promotes short term plasticity on unloading and reduces the creep modified shakedown limit. Since elastic follow-up reduces the amount of relaxation, it might actually increase the creep modified shakedown limit.

The purpose of this paper is to investigate the actual effect of elastic follow-up on the creep modified shakedown limit. A simple uniaxial model, Fig. (1), is used. It consists of a rectangular beam of a locally reduced cross section fixed from both ends and subject to a cyclic thermal load. Two beams are chosen, one with a small degree of elastic follow-up and the other with a considerably large degree of elastic follow-up. The accumulated strains and creep-fatigue damage sums are evaluated by a full time stepping inelastic analysis and the results are compared with those predicted by two simplified methods; the first is the reference stress from a shakedown analysis which was proposed by Ainsworth and Goodall [8, 9] and the second is the elastic route of ASME III Code Case N-47 [7].

## 2. ELASTIC FOLLOW-UP FACTOR

Consider the uniaxial beam of locally reduced section in Fig. (1-a). When the loading is purely thermal and consists of cycling the temperature at all points through a range $\Delta T$, Fig. (1-b), it can be shown that the elastically calculated stress $\sigma$ and strain $\varepsilon$ in the weaker part (1) and the stronger part (2) are given by :

$$\sigma_1 = -E \alpha \Delta T (1 + k) / (j + k). \quad (1)$$

$$\varepsilon_1 = - \alpha \Delta T(1 - j) / (j + k). \quad (2)$$

$$\sigma_2 = j \sigma_1 \quad (3)$$

$$\varepsilon_2 = -k \varepsilon_1 \quad (4)$$

where E is the modulus of elasticity, $\alpha$ is the coefficient of thermal expansion, $\Delta T$ is the temperature rise, j is the area ratio $A_1 / A_2$ and k is the length ratio $L_1 / L_2$.

When the temperature rise $\Delta T$ is high enough to cause yielding the plastic strain in the weaker part (1) on first loading is given by :

$$\varepsilon_{p1} = \frac{Sy}{E} (1 + \frac{j}{k}) - \alpha \Delta T (1 + \frac{1}{k}) \quad (5)$$

where Sy is the yield strength of the elastic-perfectly plastic material. At the shakedown limit $\sigma_1 = -2$ Sy and it follows from eq. (1) that the shakedown temperature range $\Delta T_{S.D.}$ is given by :

$$\Delta T_{S.D.} = \frac{2\,Sy}{E\,\alpha}\left(\frac{j+k}{1+k}\right) \qquad (6)$$

and from eqs. (5) and (6), the plastic strain in the weak part on first loading is given by :

$$\varepsilon_{po} = -\frac{Sy}{E}\left(1+\frac{j}{k}\right) \qquad (7)$$

Since k can be made indefinitely small, the plastic strain can be infinite. The stress strain response for the weak part is shown in Fig. (1-c) and as suggested by Rose [10] the elastic follow-up could be quantified by the elastic follow-up factor $\psi$ which may be defined as :

$$\psi = \frac{\text{total strain on load at the shakedown limit}}{\text{strain on load at the shakedown limit assuming elasticity}} = \frac{\varepsilon}{e} \qquad (8)$$

For purely thermal load the elastic follow-up factor $\psi$ is given by :

$$\psi = \frac{1}{2}\left(1+\frac{\varepsilon_{po}}{\varepsilon_y}\right) \qquad (9)$$

and it follows that :

$$\varepsilon_{po} / \varepsilon_y = 2\psi - 1 \qquad (10)$$

where $\varepsilon_y = S_y / E$ = elastic strain at the yield stress.

## 3. BEAM GEOMETRY AND LOADING

Two beams of locally reduced cross-section, Fig. (1-a), are chosen to give an elastic follow-up factor $\psi$ of 10 and 2 respectively. It can be seen from eq. (10) that for $\psi = 10$, $\varepsilon_{po} / \varepsilon_y = 19$ and would cause some concern whereas for $\psi = 2$, $\varepsilon_{po} / \varepsilon_y = 3$ which could be acceptable under normal service conditions. The beam is assumed to have a rectangular cross section with a uniform unit width. The length and depth of the weak and strong parts are chosen to give the required elastic follow-up factor $\psi$ at the required shakedown temperature. The beam dimensions are listed in Table (I).

The load cycle, Fig. (1-b), consists of cyclic heating and cooling with a dwell time t = 1000 hours at the hot temperature $T_h = 600^{\circ}C$. The load cycle is repeated for 300 cycles representing a total service life of $3 \times 10^5$ hours. Each beam is subjected to two temperature ranges $\Delta T$, the first is equal to the temperature range at the non-creep shakedown limit and the second is equal to 0.9 n/(n + 1) times this range, i.e. the Creep Modified Shakedown Limit (C.M.S.L.), where n is the secondary creep index.

## 4. MATERIAL PROPERTIES

The beam material is assumed representative of type 316 stainless steel and is modelled as an elastic perfectly plastic material with a yield strength equal to the yield strength values of ASME III N-47 [7] with full temperature dependence, modulus of elasticity $E = 1.5 \times 10^5$ N/mm$^2$, coefficient of thermal expansion $\alpha = 2 \times 10^{-5}\ ^{\circ}C^{-1}$ and creep strain rates as assumed to be represented by the secondary part of the creep law given by Wood [11] as follows :

$$\dot{\varepsilon}_c = 5.21 \times 10^{-6}\ \sigma^4\ \exp\left(\frac{-19700}{T+273}\right)\ hr^{-1} \qquad (11)$$

where $\dot{\varepsilon}_c$ is the creep strain rate, $\sigma$ is the stress in N/mm$^2$ and T is the temperature in degrees C.

## 5. TIME STEPPING LIFE HISTORY INELASTIC ANALYSES

The four cases listed in Table (I) have been analysed using an inelastic finite element beam program [12, 13]. The history of the accumulated strains and creep-fatigue damage sums for the weak section at increments of

20 load cycles is shown in Figs. (2 and 3). The creep and faigue damage sums are evaluated according to the inelastic route (Section T-1420) of ASME N-47 [7]. The design fatigue curves of ASME N-47 Fig. T-1420.1 are used for evaluating the fatigue damage sums whereas the creep damage is evaluated using minimum stress-to-rupture values equal to 80% of the ISO average values.

## 6. RESULTS AND DISCUSSION

The history of accumulated creep, plastic and total strains is shown in Figs. (2a) and (2b) at the shakedown limit and Figs. (2c) and (2d) at the C.M.S.L. for $\psi = 10$ and $\psi = 2$ respectively. As can be seen the effect of elastic follow-up on the accumulated creep strains is very small at both the shakedown limit and the C.M.S.L. However, the accumulated strains at the C.M.S.L. are much less than those at the shakedown limit for both values of $\psi$. The accumulated plastic strain at the shakedown limit for $\psi = 10$, Fig. (2a) is much higher than that for $\psi = 2$, Fig. (2b). At the C.M.S.L., Figs. (2c) and (2d), the effect of $\psi$ on the accumulated plastic strains at the end of the service life of 300 load cycles is not very large, although the rate of accumulation of the plastic strain which occurs upon unloading for $\psi = 10$ is about 4.3 times that for $\psi = 2$ (which is similar to that at the shakedown limit). This is apparently because both the plastic strain on first loading and the plastic strain which occurs upon unloading, after the steady state cyclic condition is attained, are of opposite signs and both are directly proportional to the elastic follow-up factor.

The history of the accumulated creep, fatigue and total damage is shown in Figs. (3a) and (3b) at the shakedown limit and Figs. (3c) and (3d) at the C.M.S.L. for $\psi = 10$ and $\psi = 2$ respectively. The ASME N-47 [7] total damage envelope is also shown in Figs. (3a-3d). As can be seen the effect of $\psi$ on the accumulated damage sums is very small. In all cases the total damage is mainly due to creep damage because of the small number of cycles which causes very small fatigue damage. The accumulated damages at the C.M.S.L. are much less than those at the shakedown limit.

The accumulated inelastic strains and creep damage sums from the time stepping inelastic analyses are compared with those predicted by the shakedown analysis [8, 9] and the elastic route of ASME N-47 [7] in Figs. (4) and (5). It should be noticed that the results from each analysis method at $\psi = 2$ and $\psi = 10$ are connected by a straight line for identification purposes only and it does not mean a linear variation.

At the shakedown limit, the accumulated creep strains for both values of $\psi = 2$ and $\psi = 10$ are conservatively predicted by both shakedown analysis and the elastic route of N-47 as shown in Fig. (4a). The results from N-47 are slightly more conservative than those from the shakedown analysis. The accumulated total strain is also conservatively predicted by both simplified methods at $\psi = 2$. However at $\psi = 10$ the accumulated total strain from the inelastic analysis is larger than that predicted by either of the simplified methods. This is due to the very large accumulated plastic strain as can be seen from Fig. (2a). However, it should be noted that the shakedown analysis is not applicable above the C.M.S.L. At the C.M.S.L., Fig. (4b), the accumulated creep strains and total strains are conservatively predicted by both simplified methods at both values of $\psi = 2$ and $\psi = 10$. The results from the elastic route of N-47 are much more conservative than those from the shakedown analysis. The degree of conservatism of both methods decreases as $\psi$ increases.

The accumulated creep damage at the shakedown limit, Fig. (5a), for both values of $\psi = 2$ and $\psi = 10$ is conservatively predicted by both simplified methods. The results from the shakedown analysis is more conservative than those from the elastic route of N-47. However, it should be noted again that the shakedown analysis is not applicable above the C.M.S.L. At the C.M.S.L., Fig. (5b), the accumulated creep damage at both values of $\psi = 2$ and $\psi = 10$ is conservatively predicted by both simplified methods. The results from the elastic route of N-47 are much more conservative than those from the shakedown analysis. The degree of conservatism of both methods decreases as the elastic follow-up factor $\psi$ increases.

## 7. CONCLUSIONS

i) The effect of elastic follow-up on the accumulated creep strains and damage sums during load cycles with short hold times (e.g. 1000 hours) at high temperature (e.g. 600°C) is very small.

ii) Although the elastic follow-up reduces the amount of relaxation, which normally promotes short term plasticity on unloading and reduces the creep modified shakedown limit, the elastic follow-up does not actually increase the creep modified shakedown limit as might be expected. This is because the actual plastic strain which occurs upon unloading is largely increased due to the effect of the creep strains in the elastic parts of the structure. This effect is much greater than the effect of reduced relaxation and results in a large net increase in the accumulated plastic strains and plastic ratchetting rate.

iii) At the creep modified shakedown limit, the accumulated strains and creep-fatigue damage sums are greatly reduced.

iv) The accumulated inelastic strains in structural systems with a large degree of elastic follow-up subject to cyclic loading at elevated temperature can be underestimated by the elastic route of ASME III N-47 even when considering secondary stresses as primary.

## ACKNOWLEDGEMENT

This paper is published by permission of GEC Power Engineering Limited. The work in this paper is part of a project suggested by Dr. R.T. Rose, National Nuclear Corporation.

## REFERENCES

[1] Robinson, E.L., "Steam-Piping Design to Minimize Creep Concentrations", Trans. ASME, V77, 1955, pp 1147-1162.

[2] Gorczynski, W., "A Method of Assessing Structural Efficiency of Pipework Designed For High Temperature Services", Proc . Instn Mech Engrs, V178, Pt 3L, 1963-64, pp 334-342.

[3] Spence, J., "An Analysis For Pipework Systems Under Creep Conditions", SMIRT-1, Berlin, 1971, F3/5.

[4] Krieg, R. and Ahmed, H., "Some Comments On The Treatment of Thermal Stress in The ASME Nuclear Vessel Code", SMIRT-3, London, 1975, F3/2, pp 1-10.

[5] Johnstone, T.L., "Creep Strain Accumulation in A Typical LMFBR Piperun", SMIRT-3, London, 1975, F3/7.

[6] Watanabe, O. and Ohtsubo, H., "Inelastic Flexibility and Strain Concentration of Pipe Bends in Creep Range with Plastic Effects" J. Pressure Vessel Technology, Trans ASME, V102, 1980, pp 271-277.

[7] ASME Boiler and Pressure Vessel Code, Code Case N-47-17 (1592-17), Section III, Division I, August 1979.

[8] Ainsworth, R.A. and Goodall, I.W., "Proposals for Primary Design Above Creep Threshold Temperature", CEGB Report, BNL-RD/B/N4394, October, 1978.

[9] Ainsworth, R.A. and Goodall, I.W. "Simplified Methods of Inelastic Analysis for Components Operating Within the Creep Range", SMIRT-5, Berlin, 1979, L11/8.

[10] Rose, R.T., Private Communication.

[11] Wood, D.S. et al., Specialist Meeting on Properties of Primary Circuit Materials Including Environmental Effects, International Working Group on Fast Reactors INGFR/22 p 74, Bergish Gladbach, Germany, October, 1977.

[12] Owais, M.K.A. and Otter, N.R., "Nonlinear Analysis of Beams Subjected to Mechanical and Thermal Cyclic Loading", Proc. Int. Conf. Engineering Aspects of Creep, Sheffield, England, September 1980, V2, pp 169-176.

[13] Otter, N.R. and Jones, R.T., "Life Prediction of Simple Structures Subjected to Cyclic Primary and Secondary Loading Resulting in Creep and Plasticity", SMIRT-5, Berlin, 1979, L11/5.

TABLE I ANALYSED CASES

| Case No. | $\psi$ | $T_h$ °C | $\Delta T$ °C | $L_1$ mm | $d_1$ mm | $L_2$ mm | $d_2$ mm | $\varepsilon_{Po}$ % |
|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 600 | 50 | 1 | 1 | 28.17 | 1.57 | -1.47 |
| 2 | 2 | 600 | 50 | 1 | 1 | 3.61 | 1.80 | -0.23 |
| 3 | 10 | 600 | 36 | 1 | 1 | 28.17 | 1.57 | -0.66 |
| 4 | 2 | 600 | 36 | 1 | 1 | 3.61 | 1.80 | -0.10 |

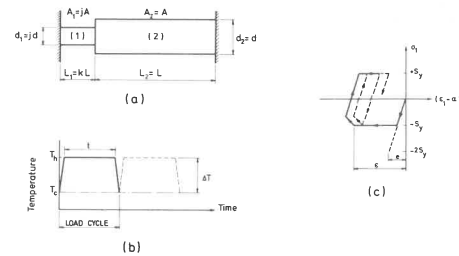
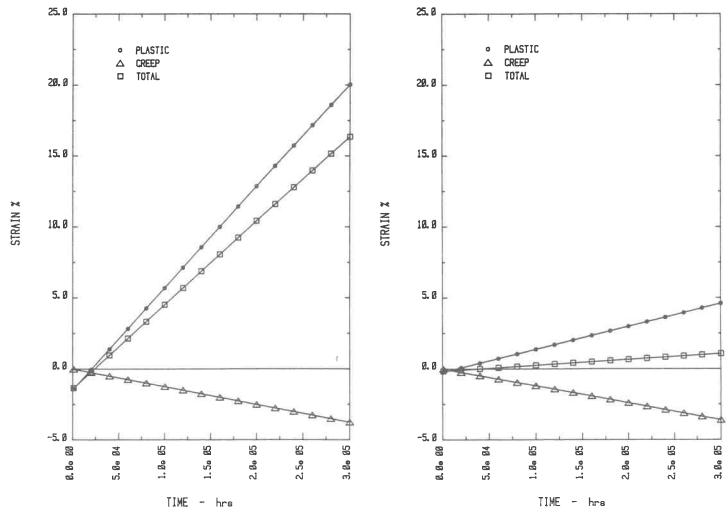
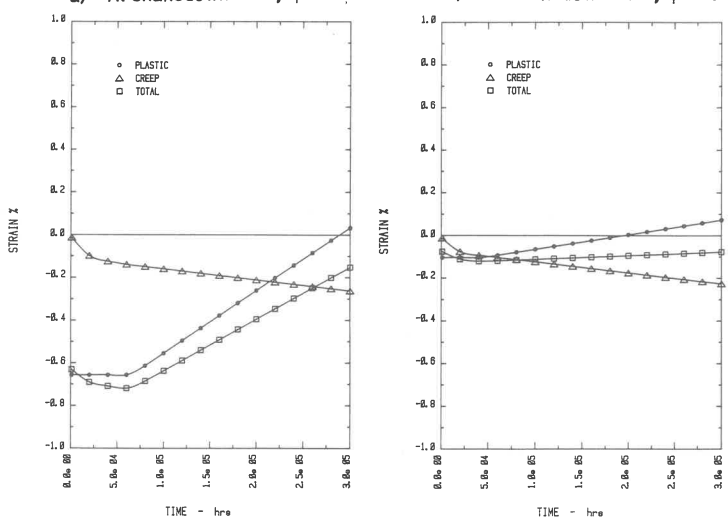



Fig. 1 Beam Geometry and Loading.




a) At Shakedown Limit, $\psi = 10$.

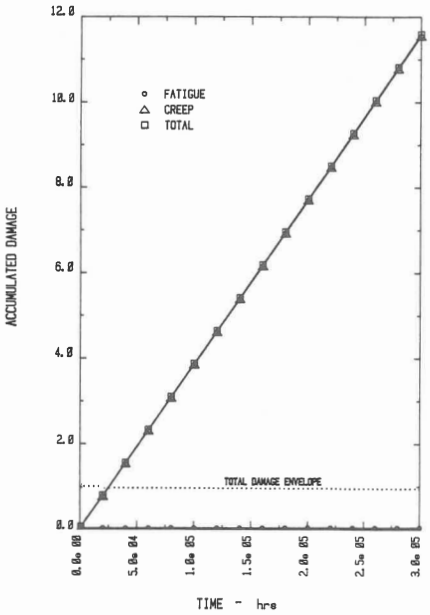
b) At Shakedown Limit, $\psi = 2$.



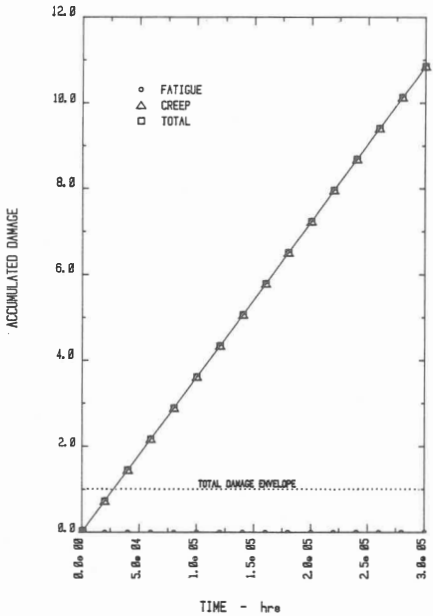

c) At C.M.S.L., $\psi = 10$.

d) At C.M.S.L., $\psi = 2$.

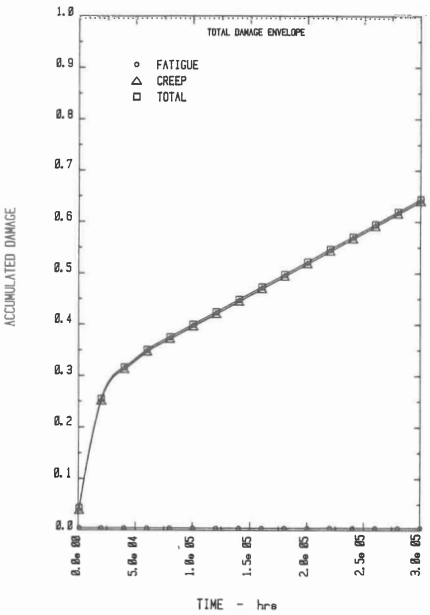
Fig. 2 Accumulated Strains.



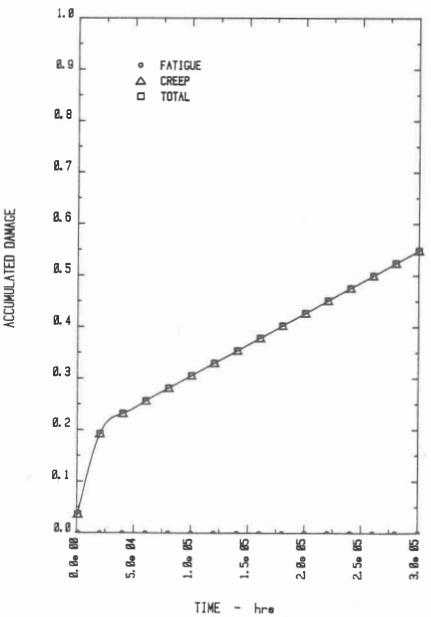

a) At Shakedown Limit, $\psi = 10.$




b) At Shakedown Limit, $\psi = 2.$

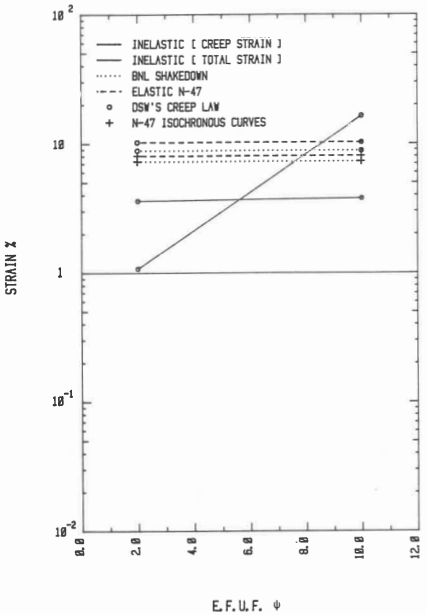



c) At C.M.S.L., $\psi = 10.$

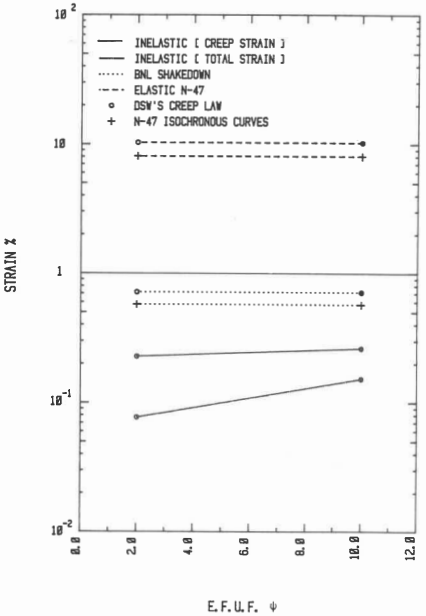



d) At C.M.S.L., $\psi = 2.$

Fig. 3 Accumulated Damage.

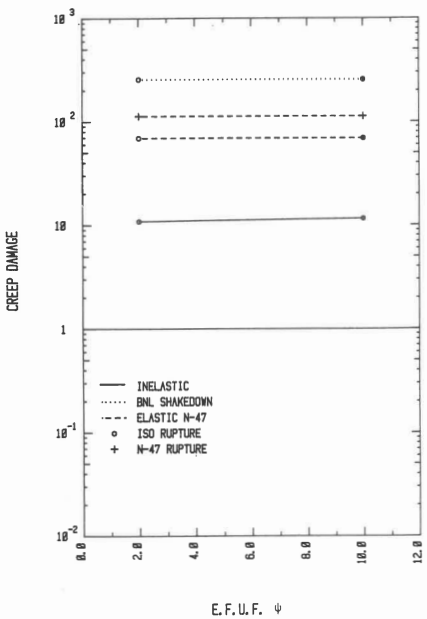



a) At Shakedown Limit.

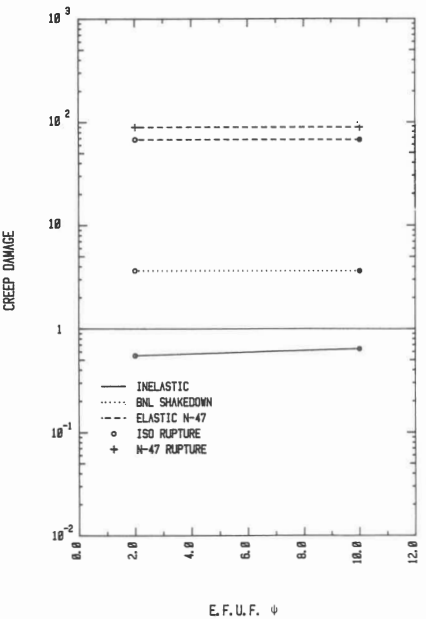



b) At C.M.S.L.

Fig. 4 Accumulated Strains.




a) At Shakedown Limit.




b) At C.M.S.L.

Fig. 5 Accumulated Creep Damage.