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On the problem of determining the stochastic variation of the eigenfrequencies of layered soil

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ABSTRACT: Often power spectra of several accelerograms at the same station show significant stable peaks during different earthquakes or aftershocks. This indicates that the eigenfrequencies of the soil are mainly determined by the local soil system. In order to prove these empirical findings Monte Carlo Simulations are carried out with random variables covering the influence of the location of the source and soil layers. The smoothing method as an analytical solution for this problem with the necessary extensions especially for random layering is presented.

1 INTRODUCTION

For the structural and earthquake engineer as well as for the seismologist the problem of determining the eigenfrequencies of the soil is of upmost importance. This is because man made structures whose natural frequencies are near the eigenfrequencies of the soil

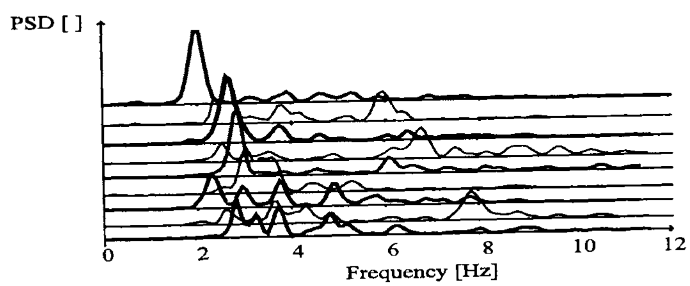


Fig. 1 PSDs (bandwidth 1.0 Hz) of EW-components of 9 accelerograms recorded at Forgaria station at 11.5.76 4:35h, 15.9.76 3:15h, 15.9.76 9:21h, 13.5.76 13:04h, 18.5.76 2:29h, 8.6.76 17:21h, 17.6.76 14:28h and 11.6.76 17:16h (top to bottom)

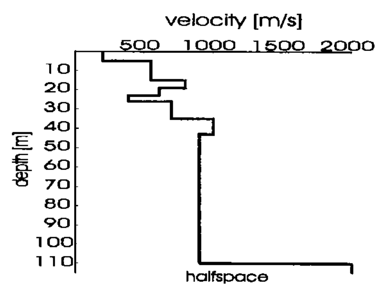
will suffer great damage in the case of an earthquake. Motivated by the 1986 Mexico earthquake, where the basin of the city was driven into resonance, many researchers have dealt with the deterministic calculation of the eigenfrequencies of the soil with a special geometry [Wirgin 1994]. If the geometry of the problem allows a separation of the wave equation, then the problem is solvable because the wave equation is solvable. But often there are no separable geometries (e.g. layered soil). On the other hand the uncertainties of the local soil conditions have to be taken into account. Thus in this paper the influence of the stochastic variation on the eigenfrequencies of layered soil is analysed.

This paper is divided into two main parts. First, Monte Carlo Simulations (MCS) are carried out. Conditions at Forgaria station are simulated based upon the cross-hole measurement carried out by ENEA-ENEL [Fontanive 1985]. The results are compared with empirical results of the 1976 Friuli earthquake. In the second part an analytical solution of the stochastic wave equation is sought. The smoothing method [Karal 1964] is found best suited. The necessary extensions for the problem of stochastic earthquake waves are discussed.

2. EMPIRICAL FINDINGS

Often PSDs of one station show very stable peaks during different earthquakes (INCERC-station during Vrancea earthquakes in Romania [Lungu 1994]) or during different aftershocks of the same earthquake (see Forgaria station during the 1976 Friuli earthquake in Italy in fig.1). This indicates that the eigenfrequencies of the soil are mainly determined by the local soil system and that the influence of the source is of minor importance. As a characteristic case example Forgaria station is chosen, because many aftershocks were recorded at this station. Besides large-scale cross-hole measurements were carried out by ENEA-ENEL in 1981. These results and a profile of the SH-wave velocities are given in tab. 1.

layer	v_p[m/s]	v_S[m/s]	$\xi_p=\xi_S$[%]	ρ[g/cm³]	h[m]
1	1000	200	20	1.8	5
2	2000	550	10	2.1	10
3	1500	800	10	2.3	4
4	2200	600	10	2.1	4
5	1800	400	15	2.0	3
6	2200	700	10	2.2	9
7	2500	1000	4	2.3	8
8	2200	900	5	2.2	ca.67
Halfsp.	3400	2000	0.1	2.5	



Tab. 1 Model of the soil layers at Forgaria station with v_s-profile

3 MONTE CARLO SIMULATIONS

The MCS are used to simulate Forgaria conditions and calculate synthetic PSDs for the model shown in tab.1. Because of the high computational efforts of the MCS a relatively simple method is chosen. [Schuëller and Scherer 1988] developed it for the assessment of the seismic risk. It contains the solution of the one-dimensional wave equation to model the soft soil deposits, which is of interest in this context.

Modelling of the source: The source spectrum is assumed as a white noise spectrum, i.e. the seismic energy is not concentrated in special frequency ranges. This theory can be justified by empirical facts. Stations located directly above the source on rock show PSDs with nearly constant spectral amplitudes over the whole frequency range (up to 30 Hz). The rock does not alter the shape of the spectrum significantly, because the damping is very low and there are no significant layers. What can be seen at the receiver is dominantly stamped by the influence of the source and the source spectrum does not

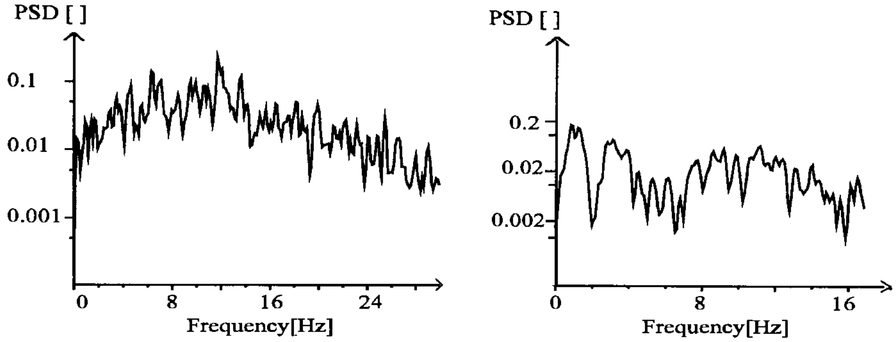


Fig. 2 PSDs (bandwidth 0.25Hz) of accellerograms (EW) of Lucerne Valley station (left) recorded at 2:30 July24 1992 during Landers earthquake and Somplago Turbine station (right) recorded 16:35 Sep11 1976 during Friuli earthquake

have very dominant spectral peaks. Two characteristic examples (fig.2) are the PSDs of accelerograms recorded at Lucerne valley station during the 1992 Landers earthquake and at Somplago Turbine station during the 1976 Friuli earthquake.

Results of the MCS: The results of [Zsohar 1995] are shortly repeated. The thickness, wave velocities and damping of the layers and the angle of inclination of the wavetrace into the bedrock are chosen as normal random variables (RV) with mean values based on the values in fig.1 and a variation coefficient of 30% (1000 simulations). The resulting parameters of interest are the mean and standard deviation of the dominant eigenfrequency f_{eigen} [Hz], which is the abscissa of the maximum peak of the PSD. The eigenfrequencies (mean, mean+standard deviation) are plotted against the layers/angles.

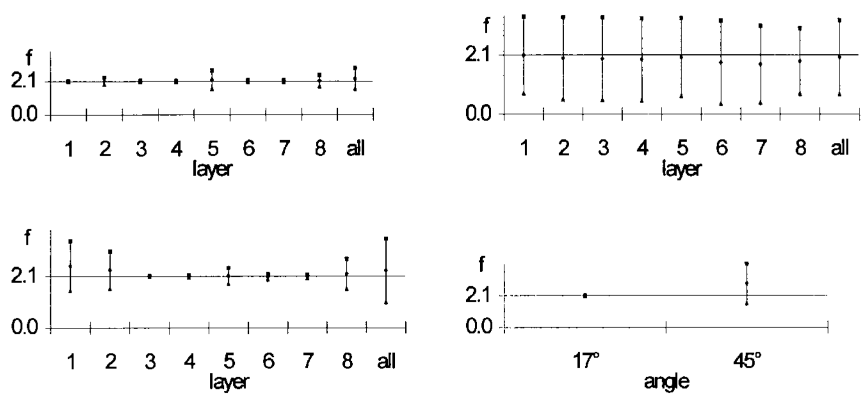


Fig. 3 Variation of the eigenfrequencies (mean, mean+s.d.), if the thickness of each layer and of all layers simultaneously (transversal: top left, longitudinal: top right), if the SH-wave velocity of each layer and all layers simultaneously (transveral: bottom left) and the angle of incidence (transversal: bottom right) are modelled as RVs

The thickness of one layer and for all layers simultaneously correlated in such a way that the thickness of the whole layer package stays approximately constant are modelled as RVs (fig.3). The mean values of the longitudinal case are smaller (< 2.1) and moreover this component shows less stable eigenfrequencies than the transversal one. The reason is that the longitudinal component is composed by two kinds of waves, the P- and SV-waves. For the transversal component the largest deviation (0.575) is found for layer 5. Here the dominant eigenfrequency is very sensitive on variations of the RV because the first and second peak of the PSD have similar heights. The second largest

deviation (0.36) occurs for the thickest layer 8. However, if each thickness varies simultaneously, a higher mean (2.3) and deviation (0.64) result. For the case of the wave velocities the results are nearly the same. For the damping no significant influence on the eigenfrequencies was found. For the angle of incidence of the wave trace into the bedrock only a mean value of 45° in order to cover the whole possible range leads to a large deviation (1.32) and a mean of 2.9, because the range of critical angles is entered.
Discussion of the results: In the deterministic case for the transversal component a f_{eigen} of 2.1 and for the longitudinal component of 2.2 is calculated. The quality of the model used for MCS is acceptable, because at Forgaria (fig.1) the dominant peak of nearly all PSDs lies in between 2.0 and 2.5 (mean value ca. 2.3). In interpreting the results of the MCS one has to be very careful. They are valid only in this limited range where the necessary assumptions can be justified. Under these circumstances the following about the influence on the eigenfrequency of the soil can be said:

- The influence of the angle of incidence of the wave into the bedrock is significant only when large deviations of the angle are assumed, i.e. extreme cases are included.
- The influence of the damping is weak.
- The influences of the wave velocities and thickness of the soil layers are dominant.

4 ANALYTICAL METHODS

In the literature various methods to generate synthetic seismograms in a random medium exist. They can be separated in analytical and numerical methods. Among the first ones are perturbation methods [Zhang 1993], ray theory [Fisk 1991] or the discrete wave-number method [Kijono 1994], among the second ones are Finite Differences [Frankel 1986], Finite Elements [Waubke 1994] or Boundary Elements [Burczynski 1992]. If one is interested in a general applicable model for different locations one has to resort to analytical methods, because of the major drawback of all numerical methods, which is the extensive computational efforts.

A very elegant perturbation method of second order is the smoothing method [Karal 1964]. The key of this method is to calculate the mean wave of an ensemble of waves. It allows to model the density and the Lamé constants as RV, e.g. $\lambda(x,\gamma)=\lambda_0+\varepsilon\lambda_1(x,\gamma)$ with $\langle\lambda_1\rangle=0$. If the equation for the mean wave

$$\left\{L_o+\varepsilon^2\left[\langle L_2\rangle-\langle L_1L_0^{-1}L_1\rangle\right]\right\}\langle u\rangle=g+O(\varepsilon^3). \tag{1}$$

is to be applied, the wave equation

$$(\lambda+\mu)\nabla(\nabla\cdot u)+\mu\nabla^2u+\nabla\lambda(\nabla\cdot u)+\nabla\mu\times(\nabla\times u)+2(\nabla\mu\cdot\nabla)u+\omega^2\rho u=0 \tag{2}$$

has to be written with the three differential operators

$$Lu=(\lambda_0+\mu_0)\nabla(\nabla\cdot u)+\mu_o\nabla^2u+\omega^2\rho_0u \tag{3}$$

$$L_1u=(\lambda_1+\mu_1)\nabla(\nabla\cdot u)+\mu_1\nabla^2u+\nabla\lambda_1(\nabla\cdot u)+\nabla\mu_1\times(\nabla\times u)+2(\nabla\mu_1\cdot\nabla)u+\omega^2\rho_1u \tag{4}$$

and L_2 = 0. Applying a mean-value theorem with the aid of the Green's function $G(x,x')=\dfrac{-e^{ik_o|x-x'|}}{4\pi}|x-x'|$ of L and doing some extensive calculations the equation for a plane wave solution $\langle u(x)\rangle=Ae^{ik\cdot x}$ (amplitude A, propagation vector k, unit vector $\hat{k}$)

$$\left[\left\{\mu_0k^2-\omega^2\rho_0+\varepsilon^2D_1\right\}+\left\{(\lambda_0+\mu_0)k^2+\varepsilon^2D_2\right\}(\hat{k}\cdot A)\hat{k}\right]e^{ik\cdot x}=0 \tag{5}$$

results. D_1 and D_2 are defined by integrals containing eight quantities, which depend upon the wave number, distance, frequency and the correlation functions of the RVs. Equation (5) can be separated and thus the problem is solvable by splitting the amplitude into two components A_p and A_s parallel and perpendicular to k

$$A_p\left[(\lambda_0 + 2\mu_0) - \omega^2\rho_0 + \varepsilon^2(D_1 + D_2)\right]e^{ik\cdot x} = 0 \qquad (6)$$

$$A_s\left[\mu_0 k^2 - \omega^2\rho_0 + \varepsilon^2 D_1\right]e^{ik\cdot x} = 0 \qquad (7)$$

When $A_p \neq 0$ then the average wave is a *longitudinal* motion, when $A_s \neq 0$ then the average wave is a *transversal* motion. The integrals D_1 and D_2 have to be evaluated.

In order to get a general applicable model there are three fundamental extensions necessary, (i) the source mechanism must be implemented, (ii) the correlation functions must be determined and (iii) the random soil layering must be included. The first point will be done as white noise spectrum as shown in this paper. For the second point the mentioned two integrals have to be evaluated which was done up to now only in the case of one RV [Hryniewicz 1989]. The third point is quite difficult. To the author's knowledge nobody has examined the thickness of the layers as a RV for earthquake waves. A possible solution of this problem is now shortly presented following the ideas of [Kotulski 1990].

An inclined (angle θ_1) longitudinal wave $A^1 n^1 e^{ik_l^1 n^1\cdot r}$ generates at the interface four waves, the reflected longitudinal wave $B^1 m^1 e^{ik_l^1 m^1\cdot r}$, the reflected transversal wave $D^1 a\times h^1 e^{ik_t^1 h^1\cdot r}$, the transmitted longitudinal wave $A^2 n^2 e^{ik_l^2 n^2\cdot r}$ and the transmitted transversal wave $C^2 a\times g^2 e^{ik_t^2 g^2\cdot r}$, where A,B,C and D are the amplitudes, n,m and g the unit vectors parallel and a perpendicular to propagation. All directions of propagation can be determined with the aid of Snellius′ law (θ_1 known). The amplitudes can be determined with the conditions of continuity at the interfaces

$$u_1(r) = A_j^i\cos\theta^i e^{ik_l^i n^i\cdot r} + B_j^i\cos\theta^i e^{ik_l^i m^i\cdot r} - C_j^i\sin\chi^i e^{ik_t^i g^i\cdot r} + D_j^i\sin\chi^i e^{ik_t^i h^i\cdot r} \qquad (8)$$

$$u_2(r) = A_j^i\sin\theta^i e^{ik_l^i n^i\cdot r} - B_j^i\sin\theta^i e^{ik_l^i m^i\cdot r} + C_j^i\cos\chi^i e^{ik_t^i g^i\cdot r} + D_j^i\cos\chi^i e^{ik_t^i h^i\cdot r} \qquad (9)$$

$$t_1(r) = (\lambda^i + 2\mu^i)\frac{\partial u_1}{\partial x_1} + \lambda^i\frac{\partial u_2}{\partial x_2},\quad t_2(r) = \frac{1}{2}\mu^i\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \qquad (10)$$

for the displacements u and the normal stress t or in matrix form

$$\begin{pmatrix} u_1(r)\\ u_2(r)\\ t_1(r)\\ t_2(r)\end{pmatrix} = T_i(r)\begin{pmatrix} A_j^i\\ B_j^i\\ C_j^i\\ D_j^i\end{pmatrix},\quad i = 1,2\quad,\ j = 1,2,\ldots N+1 \qquad (11)$$

with $T_i(r) = T_i(0)\cdot Q_i(r)$ being a product of a diagonal matrix $Q_i(r)$ and a matrix independent of r. For N+1 parallel layers with thickness h_i (11) becomes

$$\begin{pmatrix} A^0\\ B^0\\ 0\\ D^0\end{pmatrix} = T_0^{-1}(0)T_1(-\Delta_1)\prod_{j=1}^{N} F_j T_1^{-1}(0)T_0\left(\sum_{k=1}^{N+1} h_k + \sum_{k=1}^{N}\delta_k\right)\begin{pmatrix} A^f\\ 0\\ C^f\\ 0\end{pmatrix} \qquad (12)$$

where the amplitudes of the upper layer can be determined from the other layers Neglecting the inclusions of other materials δ_k the h_i can be chosen as RVs.

5 CONCLUSIONS

In this paper we tried to find out, which parameters of the soil mostly influence the dominant eigenfrequency of layered soil by carrying out huge MCS based upon physical reliable facts. The source modelling as a white noise spectrum was justified by some well selected stations located directly above the source on bedrock. As a result for our specific assumptions it is found that the location of the source as well as the damping of the layers do not influence the eigenfrequencies to that extend than the wave velocities and thickness of the layers. For a general stochastic model the analytical smoothing method will be applied. It was shown that the wave equation can be separated analogously to the deterministic case and thus is solvable. A possible treatment of the case of the thickness of the layers as RVs was presented.

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