SMiRT-12 / K. Kussmaul (Editor)
1993 Elsevier Science Publishers B.V.

BIAXIALLY LOADED COMPOSITE CROSS-SECTIONS

J. Radnić and A. Harapin

Faculty of Civil Engineering, University of Split, 58000 Split, Croatia

SUMMARY : The paper presents a general procedure for the stress-strain analysis of composite cross-sections under short-term eccentric longitudinal force. The sections can have an arbitrary form, can be made of various materials, and formed during several stages. The calculation includes the stress-strain analysis, the ultimate bearing capacity determination, and the determination of the required area of the bar reinforcement. The solution of one practical example illustrates the possibility of the model application to dimensioning of reinforced concrete cross-sections.

1. GENERAL REMARKS

In engineering practice it is frequently necessary to dimension the composite cross-sections under eccentric longitudinal force. Dimensioning includes the stress-strain analysis, the ultimate bearing capacity determination, and the calculation of the required area of the bar reinforcement. The cross-sections may have various forms, may be formed of various materials, and may have different force position (Fig. 1). Therein, some parts of the section may be prestressed, and some of them may be formed in several phases. This paper presents a general procedure for the dimensioning of the previously mentioned sections. One practical problem have been solved in order to illustrate the presented procedure.

2. PROBLEM FORMULATION

2.1. Assumptions

(i) Sections remain plane after the deformations.
(ii) No sliding at the contact surfaces between various materials after their coupling.
(iii) The stress-strain relation is known for all materials.
(iv) Loads, stresses and strains are assumed to have positive values in compression, and negative in tension.

2.2. The strain plane of the section

The graphical representation of the possible strain plane, with respect to the previous equilibrium state, is presented in Fig. 2. The additional strain $\Delta\varepsilon$ of the section point is defined by the plane equation

$$\Delta\varepsilon = \boldsymbol{\rho}^T \mathbf{r} \qquad (1)$$

where:

$$\boldsymbol{\rho}^T = \left[\Delta\varepsilon_0, \rho_z, \rho_y\right] \tag{2}$$

$$\mathbf{r}^T = [1, -y, z] \tag{3}$$

In the above expressions $\boldsymbol{\rho}$ represents the vector of unknown parameters of the additional strain plane, and y, z the coordinates of the point in the Y-Z plane. The strain plane is described by its intersection $\Delta\varepsilon_0$ with the X coordinate axis, and the components of relative rotations ρ_z, ρ_y about the Z and Y axis, respectively.

If the considered section point has previous strain ε_p, its total strain ε is

$$\varepsilon = \varepsilon_p + \Delta\varepsilon \tag{4}$$

Strain ε_p is known and determined by the previous equilibrium position $\boldsymbol{\rho}_p$, i. e. analogous to (1) by

$$\varepsilon_p = \boldsymbol{\rho}_p^T \mathbf{r} \quad ; \quad \boldsymbol{\rho}_p^T = \left[\varepsilon_{op}, \rho_{zp}, \rho_{yp}\right] \tag{5}$$

If (1) and (5) are introduced into (4), it follows

$$\varepsilon = \boldsymbol{\rho}_p^T \mathbf{r} + \boldsymbol{\rho}^T \mathbf{r} \tag{6}$$

i. e.

$$\varepsilon = \boldsymbol{\rho}_u^T \mathbf{r} \quad ; \quad \boldsymbol{\rho}_u = \boldsymbol{\rho}_p + \boldsymbol{\rho} \tag{7}$$

where ρ_u denote the parameters of total deformation plane.

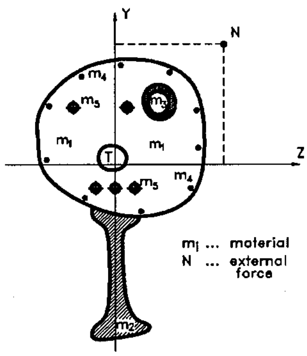


Fig. 1. - Some composite cross-sections

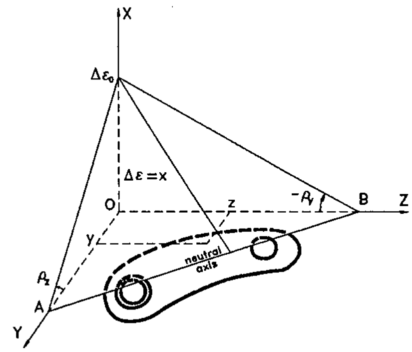


Fig. 2.- Graphical representation of possible deformation plane

2.3. Stress-strain relation

The starting point is the known relation between uniaxial stress σ and strain ε for each material. For real materials this relation is essentially curvilinear, and it is determined by a uniaxial test or by the respective national regulations. Considering the numerical analysis standpoint, it is appropriate to define this relation as linear per each sector (Fig. 3). Thus, the introduced controlled error is negligible with regard to other assumptions. The σ–ε relation between any of the two nodes i, j in the constitutive diagram is defined by

$$\sigma = \sigma_i + E(\varepsilon - \varepsilon_i) \tag{8}$$

If (6) is introduced into (8), and if substitution is introduced

$$\sigma' = \sigma_i - E\varepsilon_i + E\boldsymbol{\rho}_p^T \mathbf{r} \tag{9}$$

stress in the considered sector can be described by the following expression

$$\sigma = \sigma' + E\boldsymbol{\rho}^T \mathbf{r} \tag{10}$$

In the above expressions E represents the modules of elasticity (inclination of the line in the considered sector), whereas the graphical interpretation of stress σ' is presented in Fig. 3. For the known initial state, and the assumption of the current strain between nodes i, j, σ' is constant and determined.

2.4. Equilibrium equation

The vector of the internal resistance forces of section $\mathbf{S}_u$ is the function of the resulting strain plane, and the σ–ε relation for a given material. If they are known, $\mathbf{S}_u$ can be calculated by integration of the stresses in the section area. Thus,

$$\mathbf{S}_u = \left[N_u, M_{zu}, M_{yu}\right] = \sum_m \int_\Omega \sigma\, \mathbf{r}\, d\Omega \tag{11}$$

where N_u represents the longitudinal internal force, M_{zu} and M_{yu} respective force moments with regard to the coordinate axes, Ω the area of given material and $\sum_m$ summation across all materials. If (10) is introduced into (11), it follows

$$\mathbf{S}_u = \mathbf{S}'_u + \mathbf{I}\,\boldsymbol{\rho} \tag{12}$$

where

$$\mathbf{S}'_u = \sum_m \int_\Omega \sigma'\, \mathbf{r}\, d\Omega \tag{13}$$

$$\mathbf{I} = \sum_m \int_\Omega E\, \mathbf{r}\, \mathbf{r}^T d\Omega \tag{14}$$

$\mathbf{S}'_u$ represents one part of the vector of internal forces which is obtained by the integration of stresses σ'. The elements of the matrix $\mathbf{I}$ represent the current mechanical characteristics of the section.

In contest of limit analysis, the vector of external forces $\mathbf{S}_v$ is usually obtained by multiplying the forces of the exploitation load $\mathbf{S}_{vo}$ and the safety coefficients $\boldsymbol{\gamma}_u$. It usually depends upon the type of loading and the position of the deformation plane (see e. q. [1]). Thus,

$$\mathbf{S}_v = \mathbf{S}_{vo}\boldsymbol{\gamma}_u \quad ; \qquad \boldsymbol{\gamma}_u = \boldsymbol{\gamma}_u(\mathbf{S}_{vo}, \boldsymbol{\rho}_u) \tag{15}$$

When the stress–strain state is calculated for exploitation load $\mathbf{S}_v$=$\mathbf{S}_{vo}$. Vector $\mathbf{S}_v$ includes the vector of external forces $\mathbf{S}_{vp}$ which determines the initial strain plane $\boldsymbol{\rho}_p$, and the vector of additional forces $\Delta\mathbf{S}_v$ which causes additional strain plane $\boldsymbol{\rho}$, i. e..

$$\mathbf{S}_v = \mathbf{S}_{vp} + \Delta\,\mathbf{S}_v \tag{16}$$

In order to obtain the equilibrium, $\mathbf{S}_v$ should be equal to $\mathbf{S}_u$, i. e.

$$\mathbf{S}_v - \mathbf{S}_u = 0 \tag{17}$$

In the expanded form, (17) represents a system of three nonlinear equations with unknown $\boldsymbol{\rho}^T = \left[\Delta\,\varepsilon_o, \rho_z, \rho_y\right]$.

3. PROBLEM SOLUTION

3.1. Determination of stress-strain state

For the known external forces and for the defined composite cross–section, it is frequently necessary to determine the equilibrium strain plane and the corresponding stress state of the materials. The solution of this problem is directly reduced to the solution of (17). Employing the iterative solution procedure, the problem can be written in the following form

$$\mathbf{I}^k \Delta\,\boldsymbol{\rho}^{k+1} = \Delta\,\mathbf{S}^k\,; \qquad \Delta\,\mathbf{S}^k = \mathbf{S}_v^k - \mathbf{S}_u^k \tag{18}$$

$$\boldsymbol{\rho}^{k+1} = \boldsymbol{\rho}^k + \Delta\,\boldsymbol{\rho}^{k+1} \tag{19}$$

where k represents the iteration counter, $\Delta \mathbf{S}_v^k$ vector of the current non–equilibrium forces, and $\Delta \boldsymbol{\rho}^{k+1}$ the increment of the additional strain plane parameters. The solution of the nonlinear problem is reduced to the solution of the linear problem in each iteration. After the parameters of additional strain plane $\boldsymbol{\rho}$ are determined, the total deformation plain $\boldsymbol{\rho}_u$ is calculated by (7).

3.2. Computation of the ultimate bearing capacity

In order to obtain the ultimate bearing capacity $\mathbf{S}_{ug}$ of the cross–sections, for the given direction $\mathbf{S}_v$ the strain plane should be in the limit position. It is defined by reaching the limit strain ε_g in a given material. The external forces vector, which caused this state, will be in that case

$$\mathbf{S}_{vg} = \alpha^n \, \mathbf{S}_v \tag{20}$$

where α^n is the coefficient. $\mathbf{S}_{vg}$ should be in equilibrium with $\mathbf{S}_{ug}$, i. e.

$$\mathbf{S}_{vg} - \mathbf{S}_{ug} = 0 \tag{21}$$

The solution of (21), for every increase of external forces α^j, is analogous to the solution of (17). Therein, the vector of current non–equilibrium forces $\Delta \mathbf{S}^k$ is

$$\Delta \mathbf{S}^k = \alpha^j \, \mathbf{S}_v^k - \mathbf{S}_u^k \tag{22}$$

where $\alpha^j \mathbf{S}_v$ represents the external forces vector in j[th] increment. A possible relation among $\alpha^j \mathbf{S}_v$ and competent limit strain ε_{max} in some material is presented in Fig. 4. It is similar to $\sigma-\varepsilon$ relation of the given material.

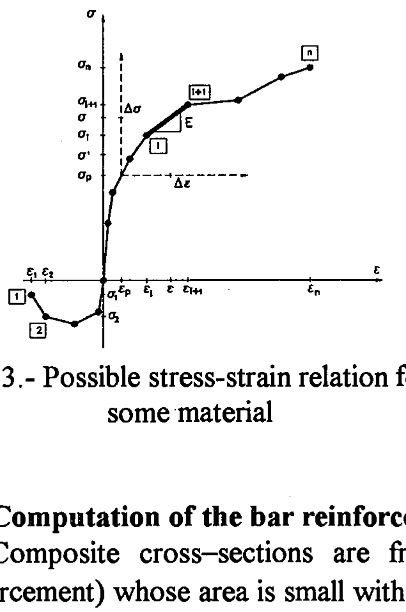


Fig. 3.- Possible stress-strain relation for some material

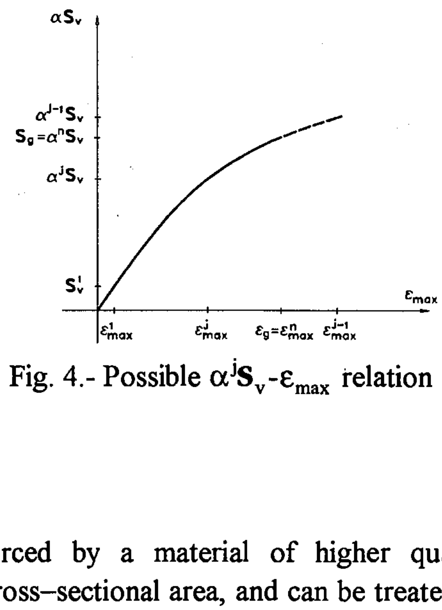


Fig. 4.- Possible $\alpha^j \mathbf{S}_v$-ε_{max} relation

3.3. Computation of the bar reinforcement area

Composite cross–sections are frequently reinforced by a material of higher quality (reinforcement) whose area is small with regard to the cross–sectional area, and can be treated as condensed to the its center of gravity point. It is often necessary to determine the required area of that reinforcement for the known dimensions of the cross–section, the quality and distribution of the materials, and the external forces vector. Therein, there exist many possibilities. Computation of reinforcement area for the known bar spacing were considered here. Two computation procedures were used:

(i) The ultimate bearing capacity of the section $\mathbf{S}_{vg}^1$ is computed according to the given position and at the beginning of the assumed area A_a of a single bar, and according to $\mathbf{S}_{vg}^1$ which should be supported by the section. If $\mathbf{S}_{ug}^1 < \mathbf{S}_{vg}^1$, it is necessary to increase the initial reinforcement and repeat the procedure. Conversely, if $\mathbf{S}^1_{ug} > \mathbf{S}^1_{vg}$, then the reinforcement should be reduced (decreased) and the ultimate bearing capacity should be computed again.

(ii) The stress–strain state of the section is determined according to the known A_a i $\mathbf{S}^1_{vg}$. If the ultimate strain is important (to be taken into account), the reinforcement should be reduced and the ultimate bearing capacity should be computed again. If $\varepsilon^1_{max} < \varepsilon_g$ then the reinforcement should be increased and the computation should be repeated. The procedure for correcting the reinforcement area is continued until $\varepsilon^n_{max} = \varepsilon_g$.

Evidently, the relations between the areas of single bars which were adopted at the beginning, remain constant at the end in both procedures.

4. EXAMPLE

It is necessary to determine the stress-strain state and the ultimate bearing capacity of the prestressed concrete cross-section presented in Fig. 5. The prefabricated prestressed beam was loaded by the bending moments due to dead exploitation load: M_{zg}= -8.0 kNm and M_{yg}= 2.0 kNm. Initial presstresing force in the steel tendons is N_{pp}= 500.0 kN (corresponding initial strain ε_{pp}= -0.0025 ‰). The stress-strain diagram for the concrete is presented in Fig. 6a, and for the prestressed tendons in Fig. 6b.

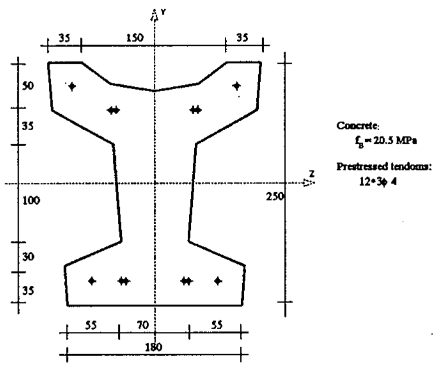


Fig. 5 - Cross-section analyzed in Example

The stress state of the section for the exploitation load is presented in Fig. 7. The ultimate carrying bending moments of section are M_{zu}= -22.5 kNm and M_{yu}= 5.6 kNm. The corresponding stress-strain state of the section is presented in Fig. 8.

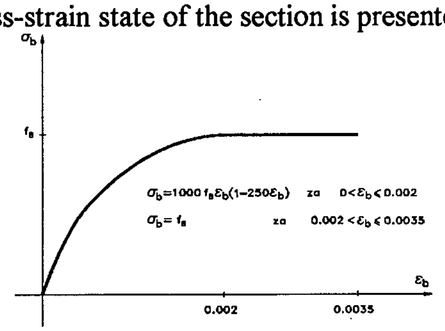


Fig. 6a - Stress-strain diagram for concrete according to [1]

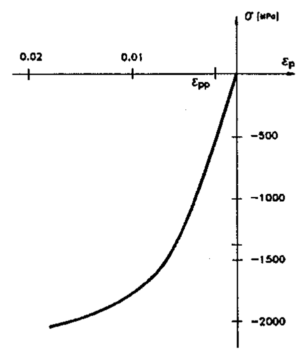
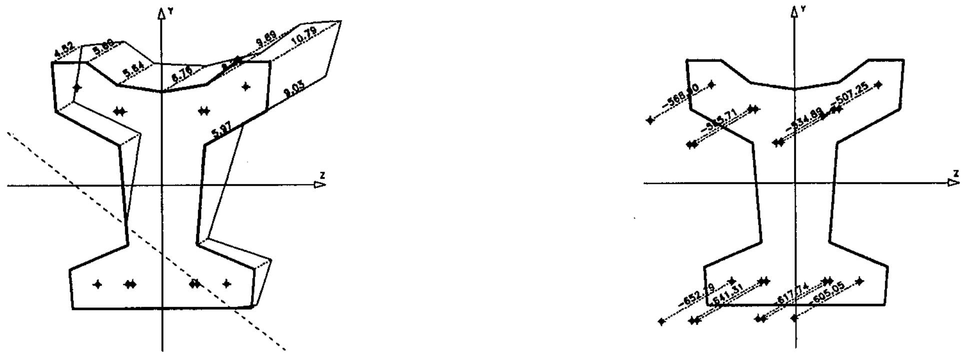
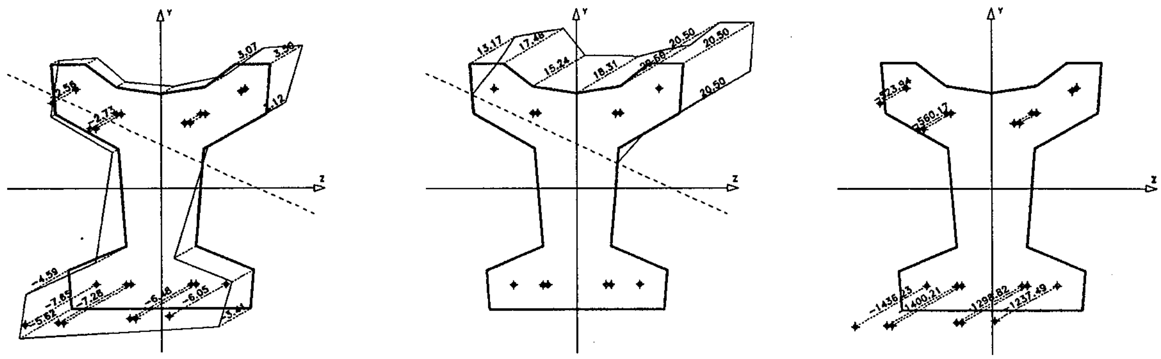


Fig. 6b - Stress-strain diagram for prestressed tendons



a) Concrete b) Steel tendons

Fig. 7 - Stress state of the cross-sections for the exploitation load



a) Strains b) Concrete stresses c) Steel stresses

Fig. 8 - Ultimate stress-strain state of the cross-section

5. REFERENCES

[1] Croatian technical regulations for concrete and reinforced concrete (1987). S.l. no. 11, from 23.02.1987. (On Croatian).

[2] Croatian technical regulations for prestressed concrete (1982). S.l. no. 51, from 18.11.1972. (On Croatian).

[3] Marin, J. (1980) "Computing unidimensional normal stress resultants". Journal of Structural Division, 106, 233–245.

[4] Pinto, M. (1979). "Biaxially loaded Concrete Structures", Journal of Structural Division, 106, 2639–2656.