

NONLINEAR FINITE ELEMENT ANALYSIS OF RC STRUCTURE SUBJECTED TO THERMAL LOAD

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A nonlinear finite element analysis of a reinforced concrete containment structure subjected to thermal and other loads is presented. The design of a reinforced concrete containment structure is usually based on a linear elastic analysis in which the concrete is assumed to behave like a homogeneous, isotropic and linearly elastic material. However, the concrete behaves differently in actuality. The stress-strain relationship of concrete has been proven to be nonlinear. To consider the nonlinearity of the concrete, a nonlinear analysis is obviously needed.

Perhaps the most important contribution to the nonlinear behavior of a reinforced concrete structure is the cracking of the concrete under low tensile stress. These cracks are initiated by various kinds of loads; thermal load is probably the most important one to initiate these cracks in nuclear containment structures. The intent of this paper is to emphasize the effects of the thermal loads on the nonlinear analysis of containment structures. The nonlinearity of the concrete is accounted for in tension as well as in compression. The accuracy of the method used in this analysis is validated by comparing the analytical results with other published data. The practical application of the method is also demonstrated by analyzing a simplified nuclear containment structure.

1. Introduction

The design of a reinforced concrete containment structure is usually based on a linear elastic analysis in which concrete is assumed to behave like a homogeneous, isotropic and linearly elastic material. In reality, concrete behaves differently. The stress-strain relationship of concrete has been proven to be nonlinear. To consider the nonlinearity of concrete, a nonlinear analysis is needed. Perhaps the most important contribution to the nonlinear behavior of a reinforced concrete structure is the cracking of the concrete under low tensile stress. These cracks are initiated by various kinds of loads; thermal load is probably the most important one to initiate these cracks in nuclear containment structures.

The intent of this paper is to investigate the effects of thermal loads on the nonlinear analysis of containment structures. The nonlinearity of concrete is accounted for in tension as well as in compression. The versatility of the finite element method in solving this class problem has already been demonstrated elsewhere (1). A list of references on the application of the finite element method to the nonlinear analysis of various types of reinforced concrete structures has been given by Scordelis (2) in a state-of-the-art paper. Wanchoo and May (3) and Hand and Pecknold (4) have solved plate problems using rectangular plate elements, while Lin and Scordelis (5) studied the plates and shell problems using quadrilateral shell elements.

A nonlinear finite element analysis of reinforced concrete structures due to axisymmetric, thermal and pressure loads is studied in the present paper. A simple two-noded conical finite element (6) is used in this analysis. Due to the nonlinear stress-strain relationship of the concrete, the properties of concrete change across the thickness as loading progresses. To account for these differences, the conical element is assumed to be composed of concentric rings of concrete and steel layers. Each layer is assumed to have its own material properties and to be in plane stress condition. This approach was also used successfully by several other authors (3, 4, 5).

The constant stiffness method (1) is used to obtain load deflection curves for the structures. The accuracy of the method is validated by comparing the analytical results with other published results (5). The practical applicability of the method is demonstrated by analyzing a concrete containment structure subjected to thermal and pressure loads.

2. Stress Strain Relationships

To account for the variable material properties across the thickness, the conical shell element is divided into concrete and steel layers as shown in Fig. 1. Each layer may have different material properties. The advantages of this kind of discretization are that (1) it allows discretized variation of elastic properties across the thickness as loading progresses, and (2) only a biaxial failure criterion for concrete and steel need to be known because each layer is assumed to be in plane stress condition. A detailed description of the layer's stress-strain relationship is given in Reference 7.

2.1 Yield Criteria for Steel Layers

Each layer of reinforcement bars is modeled as an equivalent smeared, uniformly distributed steel layer with orthotropic properties (Fig. 1). The stiffness in the equivalent steel layer is assumed to be in the direction of reinforcement only. The thickness of the equivalent steel layer is determined assuming that the cross-sectional area of the steel in the element remains unchanged. The steel is assumed to be elastic-perfectly plastic in both

compression and tension as shown in Fig. 2. The ultimate failure of steel is determined by uniaxial ultimate strain.

2.2 Yield Criteria for Concrete Layers

The concrete layers are assumed to behave like an elastic-perfectly plastic material in biaxial compression with a limited tensile strength f_t (Fig. 3). The stress-strain relationship of the cracked concrete is hypothetically assumed as follows:

$$= E_c \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\beta}{2(1+\nu_c)} \end{bmatrix} \quad (1)$$

in which is a cracked shear constant accounting for aggregate interlock and any dowel action. In the present study, β is assumed to be 0.5.

The Von Mises yield criteria are used in this analysis to define the failure surface in biaxial compression. A detailed description of this is given in Reference 7.

3. Finite Element Description

In the present finite element analysis, the axisymmetric structure is divided into a number of conical frustums. A detailed description of the finite element matrices for these conical frustums is given in Reference 6.

The thermal load due temperature can be expressed as

$$\{F\} = \int_V [B]^T [D] \{\epsilon_t\} dv \quad (2)$$

in which $\{\epsilon_t\}$ is thermal strains. The thermal strain for a typical j^{th} layer can be expressed as

$$\{\epsilon_t\}_j = [\sigma_s A, \sigma_\theta A, \sigma_s C]^T \quad (3)$$

in which

$$A = \left[\frac{T_o + T_I}{2} \quad + \quad \frac{T_o - T_I}{2} \right] (\zeta_{j+1} - \zeta_j) \\ + \frac{T_o + T_I}{4} (\zeta_{j+1} - \zeta_j) \\ C = \frac{T_o - T_I}{2} (\zeta_{j+1} - \zeta_j) \times \frac{1}{h_j}$$

where σ_s, σ_θ are the coefficients of thermal expansion in the s and θ directions, respectively, T_o and T_I are the temperatures on the outside and inside surfaces of the shell element, respectively, and B is the strain-displacement matrix.

4. Numerical Results

To verify the method of analysis, a right circular concrete cylinder with a steel liner (Fig. 4) is analyzed for pressure and temperature loadings. For simplicity, the boundary conditions at the end of the cylinder are assumed as shown in the figure and the Poisson's

ratio is assumed to be zero. First, an internal pressure of 1.44 ksf is gradually applied to the cylinder. As loading progresses, the concrete in the cylinder starts cracking in the meridional direction; at one stage the entire concrete cross section fails, thereby transferring the entire load to the steel liner. At this stage, the problem is reduced to a steel cylinder subjected to an internal pressure of 1.44 ksf. The circumferential stress and strain in the steel obtained from a simple hand calculation for the given load are 4570 ksf and 0.1058×10^{-2} , respectively. These results compare favorably with the stress and strain obtained from the crack analysis: 4532 ksf and 0.1049×10^{-2} , respectively.

As a result of an assumed temperature drop of 100°F , the concrete cylinder will contract in all directions. However, because of the slenderness of the cylinder, the contraction in the longitudinal direction will be more significant than in the circumferential direction. Due to the composite action, the concrete will crack in the circumferential direction and the entire concrete section will eventually crack as loading progresses, thereby transferring the entire load to the steel cylinder. In this situation, the prescribed temperature drop for the steel stress and strain can be computed by simple hand calculations. The computed steel stress is 2808 ksf. The crack analysis also shows a stress of 2808 ksf.

In the second example, the plate shown in Fig. 5 is analyzed for a temperature variation of 100°F at the top face and -100°F at the bottom face. The plate is in pure bending situation since the temperature at the centerline of the plate is zero which is equal to the assumed ambient temperature. This problem was checked at different stages of the loading by solving the cracked plate elastically (ignoring the cracked layers) and by comparing the results with those obtained from the crack analysis.

In the third example, a simplified model of a containment of a nuclear power plant (Fig. 6) is investigated under the following load cases: internal pressure, accidental temperatures, and internal pressure combined with accidental temperatures. The model consists of 48 elements with each element divided into six layers of concrete and four layers of reinforcement. An internal pressure of 45 psi is gradually applied. Fig. 7 shows the variation of the meridional moment with the height of the cylinder for both the elastic and nonlinear analyses. To analyze the model for temperature changes, it is assumed that the inside of the containment is subjected to a temperature of 135°F while the outside is kept under 80°F . The ambient temperature is 60°F . Fig. 7 shows the variation of the meridional moment with the height from the base for the elastic and nonlinear analysis. In general, the cracked section moments are much lower than the elastic, except near the junction of the base slab where the difference is small.

Fig. 7 also compares the elastic and nonlinear meridional moments induced by the pressure load combined with the specified temperature variation. The moment obtained from the nonlinear analysis is much smaller than the elastic moment along the height except at the fixed end, where the moment for the nonlinear analysis is higher.

Table 1 summarizes the differences between the maximum moments obtained by the elastic and nonlinear methods for the three specified load cases. If the pressure load is considered alone, it is clear that the nonlinear analysis results in much higher maximum moments than the elastic analysis, especially at the junction to the base mat. For the temperature loading, the negative moment along the height drops considerably except at the junction, where it stays almost the same. For the combined loading, the negative moment along the height is much smaller than that obtained elastically, while the positive moment at the base junction is

about the same as the elastic moment.

The results of this analysis show that an elastic analysis is not always conservative. Therefore, it is recommended that the redistribution of the moments due to cracking of the concrete be considered in the design.

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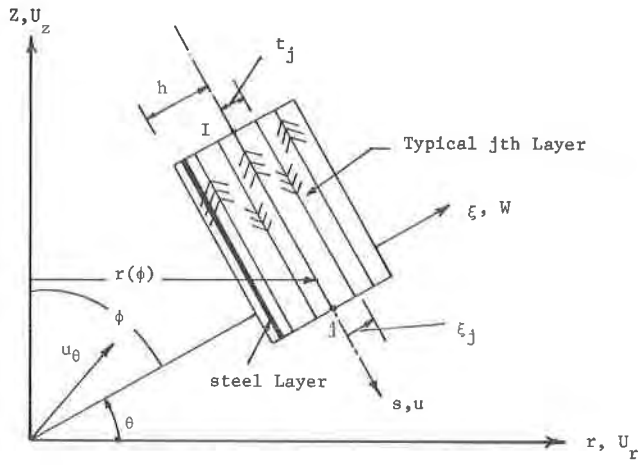


Figure 1 Geometry, Displacements and Layer Systems For Shell Element

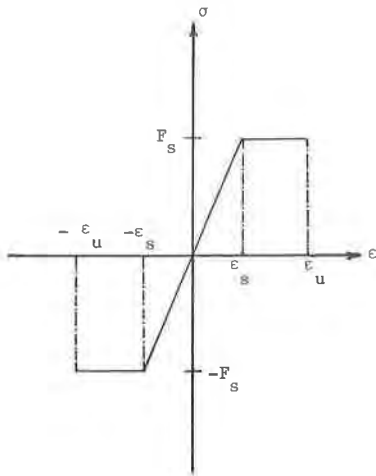


Figure 2 Stress-Strain Relationship of Steel

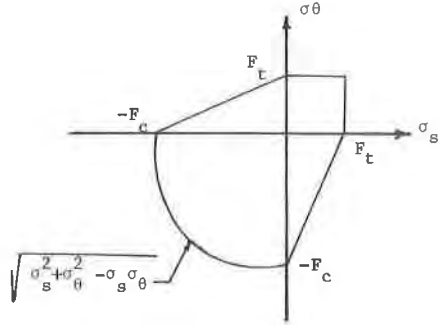


Figure 3 Failure Surface For Concrete

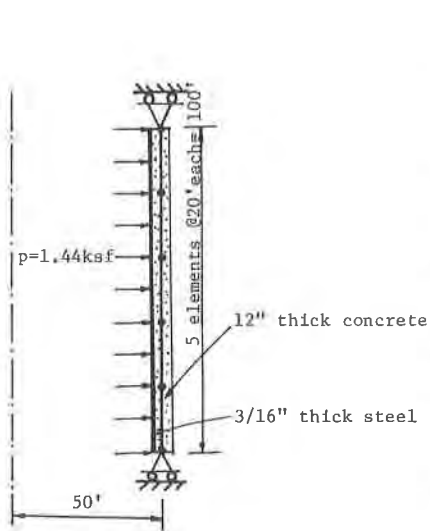


Figure 4 Steel Cylinder With Concrete Protection
(1 in. = 2.54 cm)

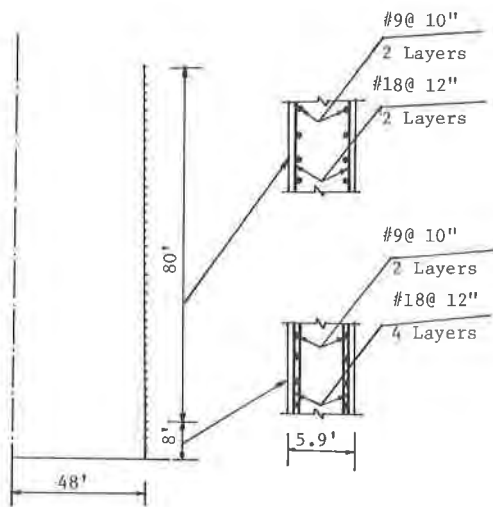


Figure 6 Simplified Model of a Containment Shell
(1 in. = 2.54 cm)

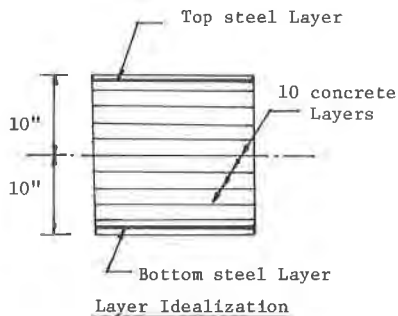


Figure 5 Circular Plate Under Temperature Loading
(1 in. = 2.54 cm)

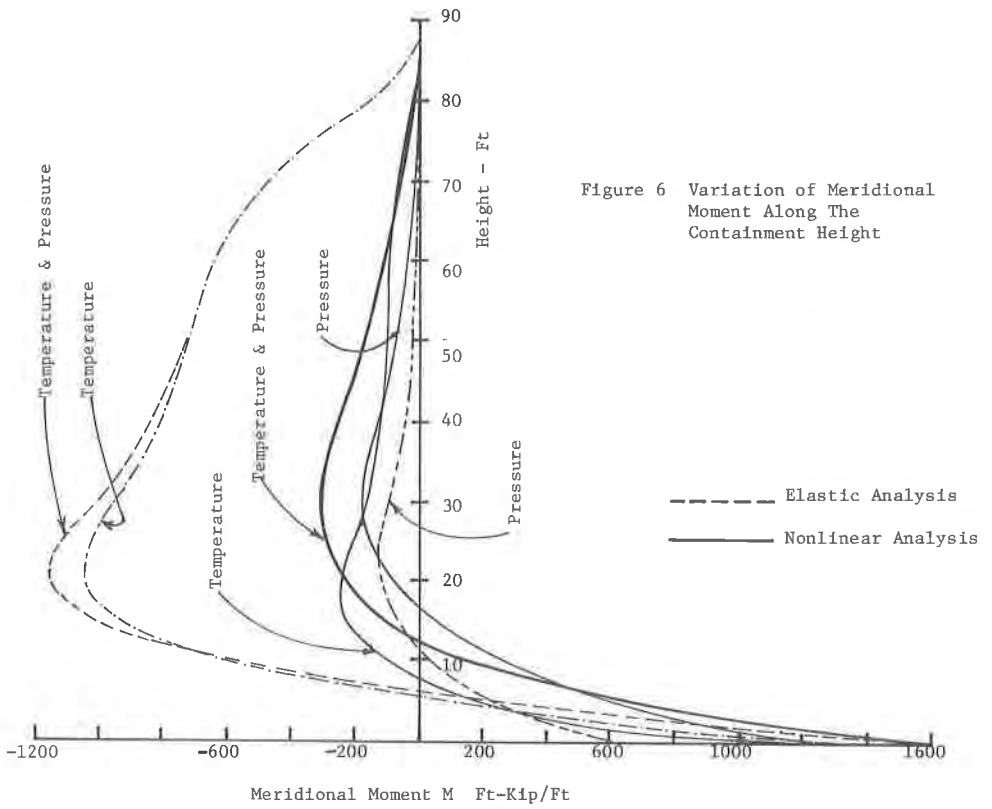


Figure 6 Variation of Meridional Moment Along The Containment Height

Table - 1
Comparison of Moments

Loading Case	Elastic Analysis		Nonlinear Analysis	
	Maximum Negative M_ϕ	Max. Positive M_ϕ at Base	Max. Negative M_ϕ	Max. Positive M_ϕ at Base
Pressure	113	537	164	1201
Temperature	930	926	231	926
Pressure & Temperature	1043	1463	311	1643