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Computational rate dependant localization analysis in concrete

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ABSTRACT: The fundamental difference between localized failure predictions of elasto-plastic and rate dependent elasto-viscoplastic material formulations are analized. At the material level the study focus on the features regarding loss of ellipticity in the form of discontinuous bifurcation of the rate dependent algorithmic tangential operator, obtained from the underlying algebraic problem. It is shown that in the limit cases, spatial discontinuities will emerge because of localized failure in analogy to rate-independent elasto plasticity. This observation completely agree with the results obtained on the FE level for the IBVP.

1. INTRODUCTION

Proper predictions of structural behavior need constitutive formulations for the inherent materials, which cover the entire spectrum response in the pre-peak and, particularly, in the post-peak regime.

Aside from the issue of proper softening measures for describing the degradation of strength and/or stiffness, most smeared crack formulations introduce material branching and localization due to the loss of hiperbolicity of the field equations describing the motion of the body. In other words, the IBVP becomes ill-posed and, as a consequence, the numerical predictions of failure processes suffer from strong mesh dependence. In that sense, the inclusion of rate effects in smeared crack-based material formulations were often advocated as a regularization strategy of softening behavior, regarding both size and orientation of the mesh.

This paper focus on the numerical prediction of dynamic localization processes in concrete, which are obtained by means of an elastic-viscoplastic constitutive formulation. To this end, the inviscid model for concrete by Etse and Willam (1994) is recast into a Duvaut-Lions viscoplastic formulation.The numerical results indicate that with increasing viscosities the spatial discontinuities of the associated rate-independent material are suppressed and the corresponding FE solutions of relaxation processes in concrete do not suffer from mesh dependence. However, when large time steps are considered, the failure predictions of elastic-viscoplastic material descriptions exhibit similar deficiencies as the inviscid material, such as loss of uniqueness and loss of objectivity of the FE predictions.

2. CONSTITUTIVE RELATIONS FOR DUVAUT-LIONS VISCOPLASTICITY

Similar to the elastoplasticity, the total strain rate in the viscoplastic theory is decomposed into an elastic and a viscoplastic part $\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_{vp}$. In the Duvaut-Lions formulation (1972) the viscoplastic strain rate and the rate of hardening/softening variables are defined as

$$\dot{\varepsilon}_{vp} = \boldsymbol{E}^{-1}:(\sigma - \bar{\sigma})/\eta, \qquad \dot{\mathbf{q}} = (\mathbf{q} - \bar{\mathbf{q}})/\eta \tag{1}$$

where η denotes the viscosity and $(\bar{\sigma}, \bar{\mathbf{q}})$ stand for the rate-independent instantaneous stress tensor and tensor of state parameters if we consider the concomitant plasticity problem with zero viscosity.

We combine the Duvaut-Lions formulation with the Extended Leon Model (ELM) for concrete, see Etse & Willam (1994). In this case, the yield surface to be considered, $F = F[\mathrm{p}, \rho, \theta, \mathrm{h}, \mathrm{c}, \mathrm{m(c)}]$, is a function of the High-Westergaard stress coordinates p, ρ, θ and of the state variables h, c and $\mathrm{m(c)}$ representing the hardening, the cohesion and the frictional parameter respectively.

2.1. Algebraic Format for Extended Leon-Duvaut-Lions Viscoplasticity

Integrating eq.(1) with the Euler Backward algorithm we get, after some algebra

$$^{n+1}\sigma = \frac{\eta}{\eta + \Delta t}\left({}^{n}\sigma + \boldsymbol{E}:{}^{n+1}\Delta\varepsilon\right) + \frac{\Delta t}{\eta + \Delta t}\,{}^{n+1}\bar{\sigma} \tag{2}$$

and similarly

$$^{n+1}\mathbf{q} = \frac{\eta}{\eta + \Delta t}\,{}^{n}\mathbf{q} + \frac{\Delta t}{\eta + \Delta t}\,{}^{n+1}\bar{\mathbf{q}} \tag{3}$$

where the subscript $n+1$ signifies the actual time step and $({}^{n}\sigma, {}^{n}\mathbf{q})$ denote the converged stress state and set of state parameters from the last time step ${}^{n}t$.

In order to complete the algebraic format of the model, we consider the inviscid solution of the ELM, given by

$$^{n+1}\bar{\sigma} = {}^{n}\sigma + \boldsymbol{E}:{}^{n+1}\Delta\varepsilon - \Delta\lambda\,\boldsymbol{E}:{}^{n+1}\left(\frac{\partial Q}{\partial \sigma}\right) \tag{4}$$

where the plastic potential $\mathrm{Q} = \mathrm{Q}(\mathrm{p}, \rho, \theta, \mathrm{h}, \mathrm{c}, \mathrm{m_Q})$ is based on a volumetric modification of the yield condition of the ELM.

Substituting eq.(4) in eq.(2) we finally get

$$^{n+1}\sigma = {}^{n+1}\sigma^{t} - \frac{\Delta t}{\Delta t + \eta}\Delta\lambda\,\boldsymbol{E}:{}^{n+1}\left(\frac{\partial Q}{\partial \sigma}\right) \tag{5}$$

where ${}^{n+1}\sigma^{t} = {}^{n}\sigma + \boldsymbol{E}:{}^{n+1}\Delta\varepsilon$ is the elastic trial stress state at $t = {}^{n+1}t$.

From the algebraic problem (e.g. the Euler backward approach), the algorithmic tangent operator consistent with the Newton-type finite element solution can be derived (see Ju

(1990) and Willam et. al. (1993)). The viscoplastic consistent tangent operator E^{alg}_{vp} is based on the inviscid algorithmic tangent moduli E^{alg}_{ep} and is defined, for the general case, by the relation $E^{alg}_{vp} = E\,\eta / (\eta+\Delta t) + E^{alg}_{ep}\,\Delta t / (\eta+\Delta t)$

Note that the limiting conditions for the algorithmic elastic-viscoplastic tangent operator of $\eta \to \infty$ or $\Delta t \to 0$ result in instantaneous elasticity $d\sigma = E{:}d\varepsilon$, while $\eta \to 0$ or $\Delta t \to \infty$ results in instantaneous elastoplasticity $d\sigma = E^{alg}_{ep}{:}d\varepsilon$. The algorithmic tangent operator of the ELM is given by Etse (1992).

3. LOCALIZED FAILURE PREDICTIONS

In this section we analyze the predictions of localized failure modes of the rate-dependent elastic-viscoplastic material, both at the material level as well as at the FE or structural level.

3.1. *Material Level*

In the smeared crack approach localizad failure modes are described by means of discontinuities in the field of velocity gradients, rather than of the velocities. Therefore, a mathematical condition for localization at the material level can be defined, corresponding to the singularity of the acoustic or localization tensor $Det(\mathbf{Q}_{ep}) = Det(\mathbf{N}\cdot E_{ep}\cdot\mathbf{N}) = 0$. Thereby represents the first order tensor **N** the normal direction to the so-called discontinuity surface across which the jump or discontinuity has developed. It must be observed that in the case of viscoplastic materials a continuum tangent operator can not be obtained. However, considering the integration of viscoplastic material processes with time intervals which are truly finite, an algorithmic tangent operator exists. By doing so, also an algorithmic localization tensor can be obtained.

To analize the localization features of the viscoplastic tangent operator we consider the load cases of the uniaxial compression test under plane strain. Fig. 1 illustrates the variation of the localization indicator $\mu = Det(\mathbf{Q}^{alg}_{vp}) / Det(\mathbf{Q}_e)$ as a function of the angle Φ of the normal vector to the discontinuity surface.

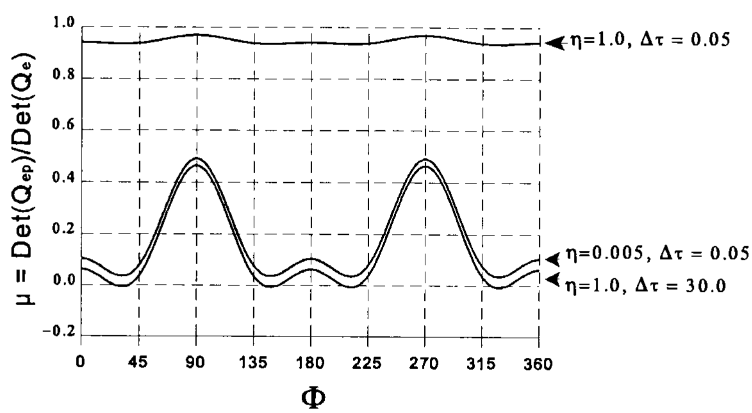


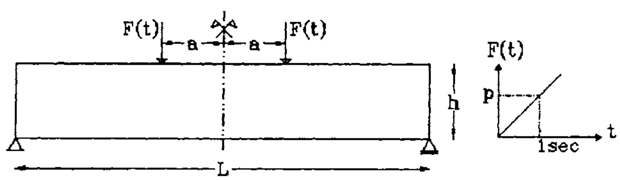
Figure 1. Variation of the localization indicator of ELM under uniaxial compression considering some values.

As can be observed in Fig. 1, when the viscosity is big enough and for small values of the time step, the viscoplastic material not only does suppress the localization effects, but also the localization indicator approaches the horizontal line $\mu=1$ associated with the fully isotropic elastic solution. On the other hand, we see that with increasing time step, while keeping η constant, the localization indicator becomes singular, i.e. the elasto-plastic localization at $\Phi=\pm 32°$ is recovered. The same conclusions are obtained for decreasing η while keeping constant Δt.

The results for the uniaxial tensile and pure shear test leads to the same conclusions as those obtained from Fig. 1.

3.2. *Finite Element Level (IBVP)*

In this section we illustrate firstly the capabilities of the viscoplastic formulation to regularize the predictions of the smeared crack approach when appropiate values (not from the physical point of view) of the viscosity and of the time step are considered. However we will show in the second part that rate-type regularization does fail in the limit, when the time interval is increased.



L = 2.00 m.
h = 0.36 m.
b = 0.18 m.
a = 0.30 m.
E = 3.8E4 MN/m^2
f'_c= 58 MN/m^2
f'_t= 4.5 MN/m^2
p = 1.6
η = 2.5E-5

Figure 2. Plane strain beam problem

We consider the beam problem of Fig. 2 under plane stress conditions. Fig. 3 shows the distribution of lines of equal equivalent plastic strains at $t = 0.015$ sec obtained with the inviscid material and with the viscoplastic material. In the case of the elasto-plastic model, the localization zone width coincide with the element width. However, when the viscoplastic material is considered, the localization zone width exceeds the element size. Moreover, this material formulation leads to objective predictions of the localization zone, as can be concluded from the comparison between the results in Figs. 3 (b) and 4.

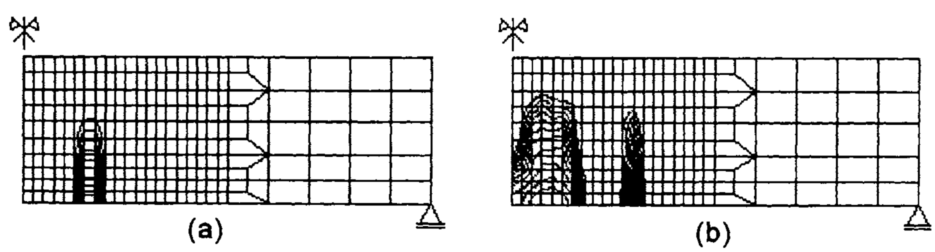


Figure 3: Isolines of equivalent plastic strains for (a) elastoplastic and (b) viscoplastic material.

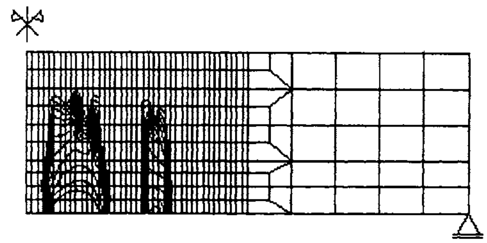


Figure 4: Equivalent plastic strain isolines for viscoplastic material using a 435 elements mesh.

In Fig. 5 we illustrate the results obtained with the viscoplastic material but considering $\Delta t = 5{\cdot}10^{-3}$ and $\Delta p = 0.8$. The other parameters, including the viscosity, were not varied with respect to those indicated in Fig. 2. We observe that in this case the viscoplastic material renders the same localization characteristics of the elastoplastic formulation. That means in the limit, for very large time steps, the analyses of transient creep and relaxation processes are susceptible to the same difficulties as strain softening computations of râte-independent material models.

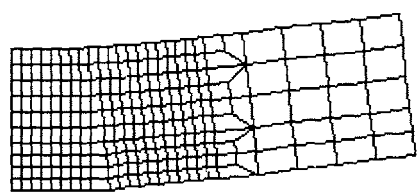


Figure 5. Deformed mesh obtained with viscoplastic material, considering $\Delta t = 5{\cdot}10^{-3}$ and $\Delta p = 0.8$.

4. CONCLUSIONS

Numerical analyses were carried out toward determining the localization characteristics of viscoplastic materials. The results indicate that with increasing viscosities the spatial discontinuities of the associated rate-independent material are supressed and the corresponding FE solutions of relaxation processes in concrete do not suffer from mesh dependence. However, when large time steps are considered, the failure predictions of elastic-viscoplastic formulations exhibit similar deficiencies as the inviscid material, such as loss of uniqueness and loss of objectivity of the FE predictions.

5. ACKNOWLEDGMENT

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6. REFERENCES

Duvaut, G. and Lions, J. (1972). Les Inequations en Mechanique et en Phisique. Dunod, Paris, France.

Etse, G. (1992). Theoretische und numerische Untersuchung zum diffusen und lokalisierten Versagen in Beton. Dr.-Ing. Dissertation, University of Karlsruhe, Karlsruhe, Germany.

Etse, G. and Willam, K. (1994). Fracture Energy Formulation for Inelastic Behavior of Plain Concrete. J. Engrg. Mech., ASCE, 120(9), 1983-2011.

Ju, J. (1990). Consistent tangent moduli for a class of viscoplasticity. J. Engrg. Mech., ASCE, 116(8), 1764-1779.

Perzyna, P. (1966). Thermodynamic theory of viscoplasticity. *Advances in Applied Mech.*, New York, N.Y., Vol. 11.

Sluys, L. (1992). Wave Propagation, Localization and Dispersion in Softening Solids. Ph.D. Dissertation, Technical University Delft, Delft, The Netherlands.

Willam, K., Etse, G. and Muenz, T. (1993). Localized Failure in Elastic-Viscoplastic Materials. Proceeding of the Fifth International RILEM Symposium. Barcelona, Spain.