

DYNAMIC BUCKLING OF SHELLS: EVALUATION OF VARIOUS METHODS

V. SVALBONAS

*Engineering Department,
The Franklin Institute Research Laboratories, Philadelphia, Pennsylvania, 19103, U.S.A.*

A. KALNINS

*Department of Mechanical Engineering and Mechanics,
Lehigh University, Bethlehem, Pennsylvania 18015, U.S.A.*

SUMMARY

The problem of dynamic stability is substantially more complex than the buckling analysis of a shell subjected to static loads. Even at this date suitable criteria for dynamic buckling of shells, which are both logically sound and practically applicable, are not easily available. Thus, a variety of analyses are available to the user, encompassing various degrees of complexity, and involving a range of simplifying assumptions. The purpose of this paper is to compare and evaluate some of these solutions by applying them to a specific problem. A shallow spherical cap, subjected to an axisymmetric, uniform-pressure, step loading, is used as the structural example. The predictions, by various methods, of the dynamic buckling of this shell into unsymmetric modes, are then investigated and compared.

The approximate methods used by Akkas are compared to the more rigorous and general solutions of the KSHEL, STARS, DYNASOR, and SATANS computer programs, and the various simplifying assumptions utilized are evaluated. Also included in the comparisons, are the predictions of the relatively simple "dynamic buckling model" approach of Budiansky and Hutchinson.

The approaches utilized by the more complex programs [KSHEL (spatial integration, modal superposition, perturbation approach), DYNASOR (finite elements, time integration of non-linear dynamic equilibrium equations), SATANS (finite differences, pseudo load method, time integration), STARS (spatial and time integration, non-linear equilibrium or perturbation approaches)] will in turn be compared in terms of accuracy, idealization complexity, ease of use, and user expertise and experience required for analysis.

The comparisons show that the more approximate methods underpredict the dynamic buckling loads for this problem. In addition, the basic assumptions of the simpler methods are found to be invalid.

1. Introduction

The stability of a transient response solution for a shell under dynamic loading is an important phenomena. It is just as important to know, for instance, whether a shell, under a certain impulsive loading, will vibrate itself to collapse after a period of time as it is to know if the shell will collapse under a certain magnitude of static load. The whole area of dynamic stability, however, has received much less attention than the corresponding static stability area. Even recent literature contains such quotes [1]: "The determination of dynamic buckling loads for structures is still a highly undeveloped area, even to the definition of reasonable failure criteria."

Extensive work in the area of dynamic stability was done by V. Bolotin [2] involving cases of parametric resonance. In Reference 2 Bolotin gives two definitions for the stability of the response of an elastic system subjected to periodic loads. The system is said to become unstable if a perturbation superimposed on its response due to the periodic loads results in a boundless increase of the components of displacement. If the superimposed perturbation involves unit elongations, shears and rotations, negligibly small as compared to unity, then the system exhibits instability in the small. If the superimposed perturbation involves unit elongations, shears, and rotations, which are large enough not to be negligible compared to unity, then the system exhibits instability in the large. The study of stability in the small is roughly analogous to the analysis for establishing the classical static buckling load. The study of stability in the large is more analogous to the postbuckling analysis. An appropriate definition of dynamic stability was also proposed in Reference [3], and is quoted herein.

"A structure is in a stable state if admissible finite disturbances of its initial state of static or dynamic equilibrium are followed by displacements whose magnitude remains within allowable bounds during the required lifetime of the structure."

This definition encompasses both of the Bolotin definitions. In the present study only infinitesimal perturbations will be considered.

The dynamic stability of spherical caps, subjected to uniform step loadings, was pursued by various researchers and thus presents an ideal basis for method comparison. The method of predicting dynamic instability from the study of a nonlinear response solution was well defined by Fulton and Barton [1]. It involves the examination of the peak values during the first cycle of oscillation of the average displacement of the structure for different values of the applied load. The value of the load for which the peak value of the average displacement exhibits a sudden increase for a very small increase in the load is defined as the dynamic buckling load. Such criteria were carefully studied by Stephens and Fulton [4] for spherical caps using the finite difference method. They found that the "inflection point" of the average displacement versus load curve may not always be a suitable criterion for the establishment of the buckling load [5]. Stephens and Fulton proposed a criterion which is more conservative for some cases. This criterion corresponds to the point where the displacement begins to change rapidly near the knee of the curve below the inflection point. Thus, these criteria are similar to the static "top or bottom of the knee" criteria used to find buckling loads from nonlinear equilibrium analyses.

The necessity of considering more than one oscillation prior to dynamic buckling was also noted in Reference 4, where only axisymmetric buckling modes caused by axisymmetric loads were analyzed.

Unsymmetric loads and unsymmetric dynamic buckling modes were considered by Ball [6] using the finite difference method. However, the use of the "pseudo-load" method without any equilibrium corrections to solve the nonlinear response equations, apparently can give rise to convergence problems [6,7].

A drawback of the method of obtaining dynamic buckling loads for shells of revolution from the study of their nonlinear response directly is that, for an applied axisymmetric dynamic load, only the axisymmetric response can be considered, whereas in many cases unsymmetric buckling has been observed experimentally to be critical in shells subjected to symmetric loading. This problem was overcome for arches by allowing the asymmetric response to be generated by using the computer numerical round-off as an unsymmetric disturbance [4]. For spherical caps Stricklin, et. al. [8] superimposed a small unsymmetric load distribution in order to excite unsymmetric modes, obtaining the solution by utilizing revolved finite elements.

Besides the more general solutions mentioned above, there exist a series of "quick" solutions based upon simplified modeling [9-11], as well as simplifying assumptions [12]. Because of the difference in scope and procedure of the various solutions, the problem to be answered may be posed as finding "the most economical satisfactory analysis" rather than "the best analysis."

2. New Approaches

2.1 KSHEL Solution

It has been assumed from some time [13] that modal methods would not be useful in the solution of nonlinear dynamics (or stability) problems. However work has continued in adapting these methods, [14,15], and such a solution has been proposed [15] using the KSHEL codes, and is extended below for finding unsymmetric buckling modes.

The solution proposed in [15] for axisymmetric shells is based on the modal expansion of the nonsymmetric disturbances. Any variable of the disturbance is expanded in the series

$$V(x,n,t) = \sum_i V_i(x,n) q_i(t) \quad (1)$$

where x is the meridional coordinate, t is time, V denotes any variable of the disturbance (expanded in Fourier series) and the subscript i refers to the i -th mode of free vibration for a given wave number n and $V_i(x,n)$ denotes the value of the variable in the i -th mode. As shown in [15], the principal parametric resonances of the shell determine the state of dynamic buckling. The criterion of dynamic buckling is expressed by the solutions of

$$\ddot{q}_i(t) + [\omega_i^2 - \lambda F(t) K_{ii} / M_i] q_i(t) = 0 \quad (2)$$

where λ is a common multiplier for all loads,

$$K_{ii} = -\pi \int_{x_1}^{x_2} [N_x(x) k_x^i(x,n) + N_\theta(x) k_\theta^i(x,n)] w_i(x,n) r dx \quad (3)$$

$$M_i = \pi \int_{x_1}^{x_2} \rho h u_i(x, n) \cdot u_i(x, n) r dx \quad (4)$$

and k_x^i and k_θ^i denote the curvature changes and $w_i(x, n)$ is the normal deflection in the i -th mode; r denotes the distance of a point on the reference surface to the axis of symmetry; x_1, x_2 denote the endpoints of the meridional arclength of the shell; ρ is mass density; h is thickness; and $u_i(x, n)$ is the displacement vector in the i -th mode.

The criterion of dynamic buckling is obtained from eq. (2). When λ in eq. (2) is zero, the solution for all $q_i(t)$ is oscillatory with a constant amplitude. This means that the disturbance, as given by eq. (1), started with some initial conditions, will consist of the superposition of constant-amplitude oscillatory responses. As λ is increased, a value can be reached at which the solution for one $q_i(t)$, with a specific value of i , becomes unstable; that is, its amplitude increases exponentially. The lowest value of λ which produces such a solution for any one $q_i(t)$ for all possible wave numbers n is designated as λ_{cr} . The critical initial stress state is then given by

$$p(x, t) = \lambda_{cr} P(n) f(t) \quad (5)$$

Formally, the solution to the dynamic buckling problem can be regarded as given by eq. (2). However, the determination of the values of those λ 's for which the amplitude grows, for an arbitrary $F(t)$, can involve a considerable computational effort. This difficulty can be avoided if time-dependence of the initial response variables can be assumed in the form

$$F(t) = A + b \cos \Omega t \quad (6)$$

because then specific charts are available from which the instability regions for $q_i(t)$ in eq. (2) can be simply read off (see, for example, Figs. 1-3 in [15], or their original sources given in [15]). The use of the charts of [15] is applicable only when the time-dependence of the initial stresses is that of eq. (6). When the time-dependence is not of that form, then either a numerical solution of eq. (2) must be obtained or the initial stresses must be simulated by a less accurate time-dependence that is in the form of eq. (6). The latter approach will be followed for the subject example.

2.1.1 Example: Spherical Cap

For the purpose of comparison with the other methods of dynamic buckling analysis, a spherical cap with clamped edge is considered that is subjected to a uniform, suddenly applied pressure. The first step in the analysis is the determination of the initial stress state. Its solution can be obtained from the axisymmetric modal superposition. Such a solution is given by the series

$$\begin{Bmatrix} N_x(x, t) \\ N_\theta(x, t) \end{Bmatrix} = \lambda \sum_i \begin{Bmatrix} N_{xi}(x) \\ N_{\theta i}(x) \end{Bmatrix} (1 - \cos \Omega_i t) \quad (7)$$

where $N_{xi}(x)$ and $N_{\theta i}(x)$ are the stress resultant distributions in the i -th axisymmetric mode and Ω_i is the natural frequency. The multiplier λ is proportional to pressure.

As it is evident from eq. (7), the initial stresses are not in the form of eq. (6), so that an approximate expression must be found. The use of a single mode in eq. (7) was investigated but for this case it was judged too inaccurate. Since the constant term in the series of eq. (7) must add up to the static solution, then the following approximation was used. The initial stresses were approximated by

$$\begin{Bmatrix} N_x(x,t) \\ N_\theta(x,t) \end{Bmatrix} = \lambda \begin{Bmatrix} n_x(x) \\ n_\theta(x) \end{Bmatrix} (1 - \cos \Omega_1 t) \quad (8)$$

where $n_x(x)$ and $n_\theta(x)$ are the membrane stresses obtained from the static solution. The static solution was calculated with the KSHELL computer program. The lowest axisymmetric natural frequency Ω_1 , the natural frequencies for wave numbers $n = 1, 2, 3, 4$ and the integrals for K and M were obtained by the KSHELL3 computer program. The CP computer time on a CDC 6400 required to calculate the critical pressure for one value of Λ was 121 seconds. The results are shown in Table I, where α and β denote the coefficients of the Mathieu equation given in [15]. The critical value of β , β_{cr} , was determined from the charts given in [15]. The final line gives the ratio of the critical pressure divided by its classical value for each geometric parameter Λ . The high value of this present result is currently under investigation.

2.2 STARS-2D Procedures

The STARS-2D code contains two procedures for the analysis of dynamic stability of shells of revolution. Both procedures utilize numerical integration in the spatial and time coordinates. The first technique is identical to the generally available techniques [8] except for the fact that numerical integration is used, rather than finite elements as in [8]. The stability of the dynamic response of multi-branched shells of revolution subjected to arbitrary dynamic loads is obtained by studying their nonlinear response over a period of time. If the loading is unsymmetric, the multi-harmonic response equations are coupled, and multi-harmonic critical modes can be found. If the loading is axisymmetric, small unsymmetric disturbance loads are included in the analysis to generate the unsymmetric response harmonics. The critical modes may thus be symmetric or unsymmetric, but are expected to be describable by a single Fourier harmonic.

The second procedure is more analogous to that used for classic eigenvalue analyses for static stability. First a linear transient response solution is obtained for the problem spanning a satisfactory time interval. The time integration procedure used is the backward difference procedure due to Houbolt [16]. Then various perturbations to this axisymmetric response are studied for periods of time. If the disturbances decrease with time, the motion is deemed stable for that value of load, whereas if the disturbance amplitudes increase or stay constant, the motion is deemed unstable. The lowest value of applied load for which an unstable disturbance may be found is the critical dynamic buckling load.

The dynamic stability equations which describe the disturbances can be obtained from the dynamic equilibrium equations [17] in a similar fashion as those obtained for static eigenvalue analysis [17]. The typical variable in the nonlinear dynamic response equations, denoted by V , may be decomposed into two components

$$V = V_p + v_B \quad (9)$$

where V_p represents the value of the time dependent variable during the prebuckled response, while V_B represents the time dependent change of the variable due to the buckling. The variables V and V_p must satisfy the general nonlinear response equations. Thus, two similar sets of equations are obtained, one for V_p describing the prebuckling response, and another for V describing the unstable response. Subtracting the one set from the other, and neglecting terms nonlinear in V_B (considering only stability in the small), the equations for tracing the time-history of the dynamic perturbations may be obtained.

3. Spherical Cap Study

The spherical cap to be analyzed is shown in Fig. 1. This problem was studied previously using finite elements [8], finite differences [4,5,18,21], finite differences with simplifying assumptions [12], simple models [11], and experimentally [19]. These results which will be the subject of discussion together with the present analyses, are presented in Figs. 2 and 3.

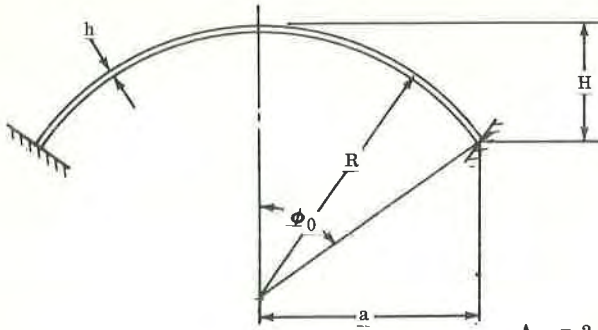
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Table I. Calculation of Critical Dynamic Buckling Pressures by the Modal Method

Λ	6	7	8
ω_1	143.6	175.4	209.8
n	2	3	3
ω_1	157	211	242
K/M	128	144	115
β	0.0062	0.0047	0.0026
α	0.90	1.44	1.33
β_{cr}	0.66	0.85	0.76
$f_{e_{cr}}$	106	181	292
q_0	133	242	404
p_{cr}	0.80	0.75	0.72



$$\Lambda = 2 \left[3 (1 - \nu^2) \right]^{\frac{1}{4}} \left(\frac{H}{h} \right)^{\frac{1}{2}}$$

$$q_0 = \frac{32 E H^3 h}{\Lambda^2 a^4}$$

$$p_{cr} = \frac{f_{cr}}{q_0}$$

$$\tau = \left(\frac{E}{\rho R^2} \right)^{\frac{1}{2}} t$$

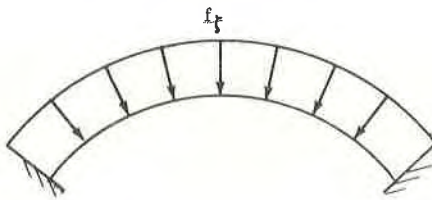


Figure 1. Shallow Spherical Cap

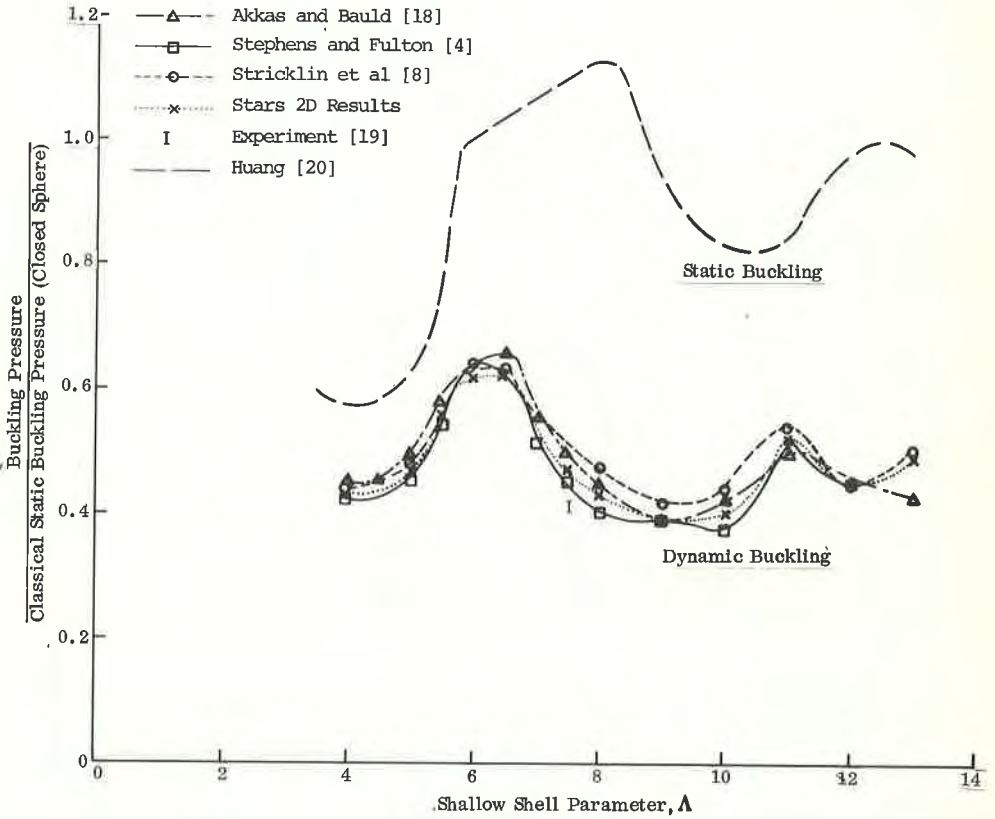


Figure 2. Axisymmetric Buckling of Shallow Caps

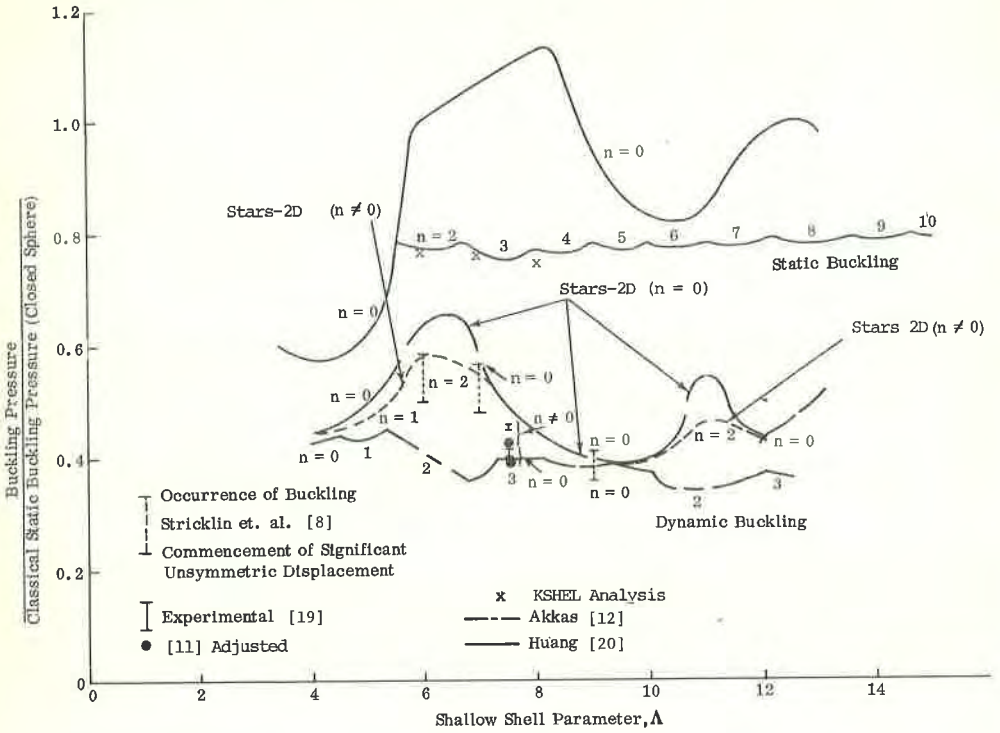


Figure 3. Unsymmetric Buckling of Shallow Caps