AN ANALYSIS FOR PIPEWORK SYSTEMS UNDER CREEP CONDITIONS

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ABSTRACT

Little information is available to assist with the design of pipework systems operating in the creep regime. All previous work neglected the effects of the smooth bends, although it is known that they exert a considerable influence on pipeline behaviour. An analysis is presented for uniplanar systems including the effect of straight and curved portions, based on a simple creep constitutive relationship. It is demonstrated that the bends have a governing role in the relaxation characteristics of a system and their neglect leads to results which are grossly conservative. Some simplifications are proposed and more involved situations are discussed.

1. INTRODUCTION

The basic function of pipeline systems dictates that they be designed to cope with expected thermal expansions arising during operation of the plant. Essentially they may be considered as deformation controlled systems. Frequently where anchor points are assumed to be regid the overall displacements are zero. In such a situation creep effects lead to a relaxation with time of the various stresses and loads within the system. It is the purpose of this paper to examine the relaxation characteristics of some simple uniplanar systems with particular reference to the influence of bends on the relaxation characteristics.

It is assumed throughout that the pipeline is at constant temperature and that its creep response may be approximated by a simple n-power steady state constitutive relationship. It is recognised that the steady state law is not particularly good in this respect but, it can be shown to be adequate in some situations, it is the one used by previous investigators and other considerations including simplicity overule. Two previous authors, Robinson [1] and Gorczynski [2, 3] treated simple systems but neglected the bends. Reference [1] deals with symmetrical systems and [2] and [3] which are similar deal with offset loops. The present work will use the same examples for comparison purposes although the cases are not identical.

2. BASIC METHOD AND ASSUMPTIONS

a) The Relaxation Situation

It is assumed that the constrained thermal expansion of pipework can be considered by taking the terminal force actions and using them as the basic actions causing stressing The total displacement due to these actions at time zero is constant. Initially it is an elastic displacement but as time progresses it is made up of a creep component and a reduced elastic component. Thus at any time the relaxation equation becomes

$$\dot{\delta}_e + \dot{\delta}_c = 0 \tag{1}$$

where dots denote differentiation with respect to time.

The pipeline problem is usually simplified by working from the "thrust line" [4] whereupon only a single direct load at some location is required. In all that follows displacements are referred to the thrust line.

b) The Constitutive Relationship and Material Properties

A simple n-power steady state law in non-dimensional form is used

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} = \left(\frac{\sigma}{\sigma_o}\right)^n \tag{2}$$

where σ_o, $\dot{\varepsilon}_o$ and n are material properties. In [1] to [3] a typical pipework material, $2\frac{1}{4}$% Cr - 1% M_o steel was assumed with the following properties at 540°C σ_o = 7 800 lbf/in^2 (53.8 MN/m^2), $\dot{\varepsilon}_o = 10^{-7}$ /hr, n = 4 Young's Modulus E = 23 x 10 lbf/in^2 (159 GN/m^2). In [1] slightly different values were used but they give the same results within 2%. In using a steady state law it is assumed that steady state stresses immediately obtain and no time is allowed for stress redistribution.

c) Load, Displacement Relations

For the straight portions of the pipeline the load, displacement relations are derived from virtual work. In the case of the unit load equation it is

$$\delta = \int_v \sigma^s \, \varepsilon^c \, dv \tag{3}$$

where σ^s is a stress field in equilibrium with the unit load which is applied where the displacement δ is required. The strain field ε^c must be compatible with the displacement.

Since bending is usually the primary loading only bending actions will be considered. For linear elasticity it is not difficult to show that eq. (1) becomes

$$\delta = \int_0^x \frac{M}{EI} \, m \, dx \qquad (4)$$

where m is the bending moment due to the unit load and I is the second moment of area of the cross-section. In the creep case it can be shown that

$$\dot{\delta} = \int_0^x \frac{\dot{\varepsilon}_o}{r} \left(\frac{\sigma_s}{\sigma_o}\right)^n m \, dx \qquad (5)$$

where $\sigma_s = \frac{Mr}{I} \cdot \frac{\pi}{D_o}$ and σ_s is the extreme fibre steady state stress in a straight pipe under bending. $D_o = 4 \int_0^{\pi/2} (\sin\phi)^{(n+1)/n} d\phi$ where ϕ is the cross-sectional angle.

3. PRELIMINARY DATA FOR FORCE ACTIONS ON BENDS

In a general system various force actions occur at the bends. Basic data have been given recently by Spence [5, 6] on the behaviour of smooth curved pipes under in-plane bending in creep and other load action effects can easily be derived.

a) Constant Moment Action

Results for flexibility factors are given in [5] and [6] for lower and upper bound analyses respectively. The flexibility factor K was defined as

$$K_M = \frac{\text{the end rotation rate of a bend under creep}}{\text{the end rotation rate of an equivalent length of straight in creep}}$$

both being subject to the same load M. The results will not be repeated here but from [5] K_M may be approximated to within about 4% in the range $0.05 \leq \lambda \leq 0.5$ where $\lambda = 2hR/r^2$ (Figure 1) by the formula

$$K_M = 1.25/\lambda^{(5n+3)/8} \qquad (6)$$

Once the end rotations are known, displacements at the thrust line are given by multiplying by the offset distance.

b) Direct (F_1) and Shear (F_2) Loads

Figure 1 illustrates the types of load envisaged. Again only end rotation rates due to bending will be considered. Displacement rates at the position of the load will not be included.

The end rotation rate ($\dot{\gamma}_o$) for a straight pipe may easily be derived [5] as

$$\dot{\gamma}_o = \frac{R\alpha}{r}\left[\frac{M\dot{\varepsilon}_o^{1/n}}{2hr^2\sigma_o}\right]^n \frac{1}{D_o^{\,n}} \qquad (7)$$

For the constant moment case $\dot{\gamma} = K_M \dot{\gamma}_o$ and it is easily shown that due to the force F_1 the end rotation rate is

$$\dot{\gamma}_{F1} = \frac{R\alpha}{r}\left\{\frac{F_1 R \dot{\varepsilon}_o^{1/n}}{2hr^2 \sigma_o}\right\}^n \frac{K_M D_1}{D_o^n} \tag{8}$$

where $D_1 = \frac{1}{\alpha}\int_0^\alpha (1 - \cos\theta)^n \, d\theta$

Similarly the shear load gives rise to an end rotation rate

$$\dot{\gamma}_{F2} = \frac{R\alpha}{r}\left(\frac{F_2 R \varepsilon_o^{1/n}}{2hr^2 \sigma_o}\right)^n \frac{K_M D_2}{D_o^n} \tag{9}$$

where $D_2 = \frac{1}{\alpha}\int_0^\alpha (\sin\theta)^n \, d\theta$

For convenience values of D_1 and D_2 have been evaluated and are given in Table I. Also $K_{F2} = K_M D_2$ etc.

4. AN ANALYSIS FOR SIMPLE PIPING LAYOUTS

The analysis is similar to that of Robinson and Gorczynski except for the inclusion of the terms due to the bends.

a) A symmetrical System

Figure 2(a) illustrates a symmetrical system typical of the type analysed in [1]. Using eq. (4) the elastic displacement rate is found for each part of the loop to give at the thrust line.

$$\dot{\delta}_e = \frac{2\dot{\sigma}_b}{Er}\{\Delta\} \tag{10}$$

where $\Delta = \left[\left(\frac{X_1^3}{3} + X_1^2 R + X_1 R^2\right) + (X_1 + 2R)^2 X_2 + (X_1 + R)^3/3. + 2K_M (X_1 + R)^2 R\alpha + 2K_{F2}(X_1 + R)R^2\alpha\right] \Big/ (X_1 + 2R)$

The influence of the F_1 displacement is negligible being so near to the thrust line. In eq. (10) all the displacement functions have been expressed by simple proportion in terms of the maximum nominal elastic bending stress σ_b which is located at the top of the loop and given by $\sigma_b = F(X_1 + 2R)r/I$.

The creep displacement rates can be found in exactly the same manner starting from eq. (5). Again all displacement functions have been expressed in terms of σ_b

$$\dot{\delta}_c = \frac{2\dot{\varepsilon}_o}{r} \left(\frac{\pi \sigma_b}{D_o \sigma_o}\right)^n \quad \Phi \tag{11}$$

where $$\Phi = \left[\int_0^{X_1} (R + u)^{(n+1)} du + X_2(X_1 + 2R)^{(n+1)} + (X_1 + R)^{(n+2)}/(n+2) \right.$$

$$\left. + 2K_M R\alpha (X_1 + R)^{(n+1)} + 2K_{F2}\alpha R^{(n+1)}(X_1 + R)\right] \Big/ (X_1 + 2R)^n$$

Substitution of eqs. (10) and (11) into eq. (1) gives the relaxation equation

$$- \int_{\sigma_{bi}}^{\sigma_b} \frac{d\sigma_b}{\sigma_b^n} = \frac{\Phi}{\Delta} \frac{\dot{\varepsilon}_o E}{\sigma_o^n} \left(\frac{\pi}{D_o}\right)^n \int_0^t dt \tag{12}$$

where σ_{bi} is the value of the nominal linear elastic beam bending stress at time zero. At any time t the stress is σ_b Integrating gives

$$\left(\frac{\sigma_b}{\sigma_{bi}}\right) = \frac{1}{(Nt+1)}^{1/(n-1)} \tag{13}$$

where $N = \frac{\Phi}{\Delta} \frac{\dot{\varepsilon}_o E}{\sigma_o^n} \left(\frac{\pi}{D_o}\right)^n (n-1)\ \sigma_{bi}^{(n-1)}$

Various relaxation curves for the symmetrical loop are shown in Figure 3 with and without bends. Different R/L values are shown, all for a pipe factor (λ) value of 0.1. To show the effect of maintaining the same system but altering the size of the bend radius R, consider changing from L = 10R to L = 4R which necessitates a change in λ from 0.1 to 0.25. The result is illustrated in Figure 3. Rather conservative values of K_M have been chosen deliberately to avoid giving too optimistic a view of the results. Table II shows some intermediate evaluation of the displacement functions which allows some assessment of the relative importance of the different portions of the pipeline. An arbitrary value of 15,000 lbf/in^2 (103.6 MN/m^2) has been selected for σ_{bi} although the graph is shown in non-dimensional form. If maximum stresses are of interest the stress concentration factor for the bend must be included.

b) An Offset Loop System

In Figure 2(b) a typical system considered by Gorczynski is shown. For the sake of simplicity it is assumed that the thrust line position is not altered by the inclusion of the bends. The analysis is exactly similar to that above. Again the functions are all referred to the stress σ_b at the top of the loop. It can be shown that

$$\Phi = \left[\frac{(X_2 - R)^{n+2)}}{(n+2)X_2^{\,n}} + \frac{X_1^{(n+2)}}{(n+2)X_2^{\,n}} + \frac{X_2}{2}(X_3 - 2R) \right.$$

$$\left. + \frac{K_M R\alpha (X_2 - R)^{(n+1)}}{X_2^{\,n}} + \frac{K_{F2}\alpha R^{(n+1)}(X_2 - R)}{X_2^{\,n}} \right]$$

and $\Delta = (\Phi)_{n=1}$.

Tables III and IV detail the various functions for two sizes of loop one being twice the size of the other. Eq. (13) is relevant and the relaxation characteristics are given in Figure 4. Both systems have been assumed to have an initial stress of 7,500 lbf/in^2 (51.8 MN/m^2) at the top of the loop. The reason for the choice of location is that most design criteria boil down eventually to an allowable maximum stress and the actual maximum stress (including concentration effects) will be in the bends at the top of the loops.

5. SIMPLIFICATIONS FOR SPECIAL CASES

Evidently relaxation assessment of actual pipework systems will be rather complex particularly if several alternative layouts have to be evaluated. It seems worthwhile examining possible simplifications:

a) Neglect of Straight Portions

In contradistinction to previous analyses which considered only straight sections it seems possible that in many applications the straights may be neglected in the creep analysis by comparison with the bends. The effects of the bends only dominate as the creep index increases and the full elastic analysis is always required. Tables II to IV show the effect. For the elastic case the straights may contribute half the flexibility but when n=4 their contribution becomes negligible or at least small. When $n>3$ the straights could probably safely be neglected. For $1<n<3$ they could possibly be neglected depending on the system. The creep part of the analysis then becomes almost trivial since the basic data required is available in readily usable form [5, 6].

b) Neglect of Pressure Effects

So far the effect of internal pressure has not been mentioned. Inclusion of pressure (constant in time) in the relaxation problem constitutes a serious complication. However some unpublished work by the author indicates that for many typical pipeline systems the pressure will be of such a magnitude that its influence on the relaxation characteristics can be neglected without significant loss of accuracy. This will not be discussed further here.

6. DISCUSSION

A fairly general analysis for the relaxation behaviour of pipework has been outlined and some results presented for special cases. It will be apparent from Figures 3 and 4 that pipe bends exert considerable influence on the system characteristics. The assumption of steady state behaviour is expected to lead to conservative estimates of relaxation time. Since actual creep rates might be higher, the relaxation rates are expected to be greater than calculated.

Comparisons with the work of previous investigators who neglected the bends ($R = 0$), show that such a simplification can lead to grossly conservative results. For example differences of an order of magnitude in the time for reduction of the stress by half its initial value can easily occur.

It is evident that maximum flexibility may be obtained from a pipe bend by making the pipe bend factor λ as low as possible. While an optimisation study is beyond the scope of this paper it seems worth pointing out that there is some merit in efficiently selecting the size of bend within a given overall size of loop. The relevant displacement function is given in Figure 6 for a fixed size of system and shows maxima for different n. The graph has the weakness that it does not take into account a change in λ consequent upon a change in ℓ/R. Nevertheless the trends are of interest and could give a designer some initial guidance.

Only uniplanar piping and force systems have been considered. The basic data on out-of-plane actions on bends are not yet available for creep conditions. The situation is more complicated than in the elastic case where in and out-of-plane bending lead to the same flexibility factors. Due to the combined loading arising from the torque action associated with out-of-plane bending the flexibility will be increased compared with the in-plane case. Use of the data in [5, 6] for out-of-plane three dimensional piping will be conservative but it is not known how conservative.

It has been necessary herein to use specific material properties and to select an initial stress for comparative purposes. In general, results can be presented in a truly non-dimensional fashion but they may not be as meaningful. It is possible to make some even greater simplifications which remove the restriction of the steady state law and the necessity for knowing a unique value of the stress index [7] but this cannot be detailed here.

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[1] ROBINSON, E.L., "Steam piping design to minimise creep concentrations", Trans. ASME 77, 1147 (1955)

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[4] KELLOG, M.W., "Design of piping systems", J. Wiley & Son, N.Y. 1956

[5] SPENCE, J., "The creep behaviour of smooth curved pipes under bending", 1st Int. Conf. on Pressure Vessel Technology, Delft, Part I 309 (1969)

[6] SPENCE, J., "An upper bound analysis for the deformation of smooth pipe bends in creep", 2nd IUTAM Sym. Creep in Structures, Gothenburg (1970)

[7] SPENCE, J., "An approach to the creep relaxation of structures", to be published.

Table I

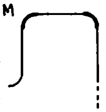
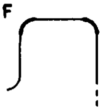
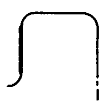
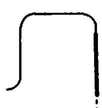
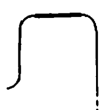
Values of the integrals D_1 and D_2

n	1	1.5	3	4	5	7
D_1	0.3634	0.2788	0.1657	0.1309	0.1082	0.0805
D_2	0.6366	0.5564	0.4244	0.3750	0.3395	0.2910

Table II

Displacement Function for a Symmetrical Loop

$\lambda = 0.1$, $\alpha = \pi/2$

Relevant Section	Function	Elastic n = 1			Creep n = 4		
		$R = \frac{L}{10}$	$R = \frac{L}{4}$	$R = \frac{L}{2}$	$R = \frac{L}{10}$	$R = \frac{L}{4}$	$R = \frac{L}{2}$
M 	$\frac{2K_M R\alpha(X_1 + R)^{(n+1)}}{(X_1 + 2R)^{(n+1)} L^2}$	3.18	5.53	4.90	147.0	147.8	27.7
F 	$\frac{2K_{F2} R^{(n+1)}\alpha(X_1 + R)}{(X_1 + 2R)^n L^2}$	0.226	1.18	3.14	0.143	1.19	17.6
	$\int_o^{X_1} \frac{(R + u)^{(n+1)} du}{(X_1 + 2R)^n L^2}$	0.243	0.136	0	0.088	0.029	0
	$\frac{(X_1 + R)^{(n+2)}}{(n + 2)(X_1 + 2R)^n L^2}$	0.243	0.14	0.04	0.088	0.029	.002
	$\frac{(X_1 + 2R)^{(n+1)} X_2}{(X_1 + 2R)^n L^2}$	0.80	0.50	0	0.80	0.50	0
Approximate totals	Bends	3.406	6.71	8.04	147.14	149.0	45.3
	Straights	1.285	0.776	0.04	0.976	0.56	-
	Total	4.69	7.476	8.08	148.0	149.56	45.3

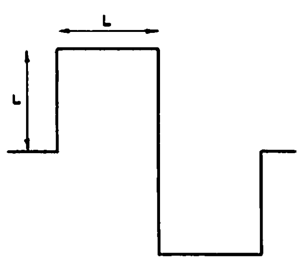
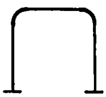
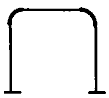
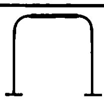
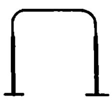
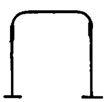


Table III

Displacement Functions for an Offset Loop

$X_1 = \frac{2L}{3},\ X_2 = \frac{L}{3},\ X_3 = L,\ \lambda = 0.1\ ,\ \alpha = \frac{\pi}{2}$

Relevant Section	Function	Elastic $n = 1$		Creek $n = 4$	
		$R = \frac{L}{10}$	$R = 0$	$R = \frac{L}{10}$	$R = 0$
	$\frac{K_M\ R\alpha(X_2 - R)^{(n+1)}}{X_2^n\ L^2}$	0.318	-	6.95	-
	$\frac{K_{F2}\ R^{(n+1)}\alpha(X_2 - R)}{X_2^n\ L^2}$	0.087	-	0.088	-
	$\frac{X_2}{2L^2}(X_3 - 2R)$	0.127	0.167	1.127	0.167
	$\frac{X_1^{(n+2)}}{(n + 2)\ X_2^n\ L^2}$	0.296	0.296	1.19	1.19
	$\frac{(X_2 - R)^{(n+2)}}{(X + 2)\ X_2^n\ L^2}$	0.012	0.037	-	0.014
Approximate totals	Bends	0.405	-	7.04	-
	Straights	0.435	0.49	1.32	1.35
	Total	0.84	-	8.36	-

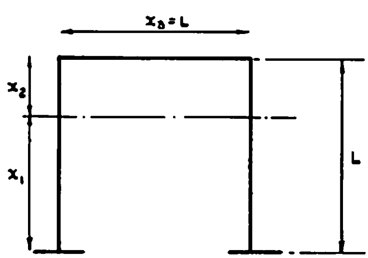
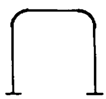
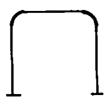
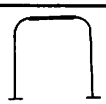
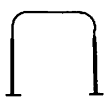
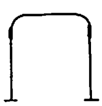
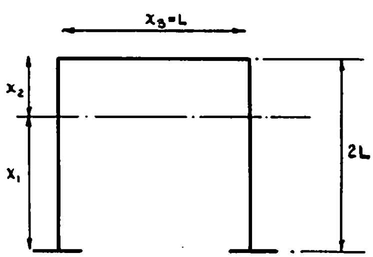


Table IV

Displacement Functions for an Offset Loop

$X_1 = \frac{6L}{5}$, $X_2 = \frac{4L}{5}$, $X_3 = L$, $\lambda = 0.1$ $\alpha = \pi/2$

Relevant Section	Function	Elastic $n = 1$		Creep $n = 4$	
		$R = \frac{L}{10}$	$R = 0$	$R = \frac{L}{10}$	$R = 0$
	$\frac{K_M R\alpha(X_2 - R)^{(n+1)}}{X_2^n L^2}$	1.19	-	51.0	-
	$\frac{K_{F2} R^{(n+1)}\alpha(X_2 - R)}{X_2^n L^2}$	0.11	-	0.02	-
	$\frac{X_2^2}{2L^2}(X_3 - 2R)$	0.32	0.40	0.32	0.40
	$\frac{X_1^{(n+2)}}{(n + 2)\ X_2^n L^2}$	0.72	0.72	1.215	1.215
	$\frac{(X_2 - R)^{n+2}}{(n + 2)\ X_2^n L^2}$	0.143	0.213	0.048	0.106
Approximate totals	Bends	1.30	-	51.02	
	Straights	1.183	1.333	1.583	1.721
	Total	2.48	-	52.60	



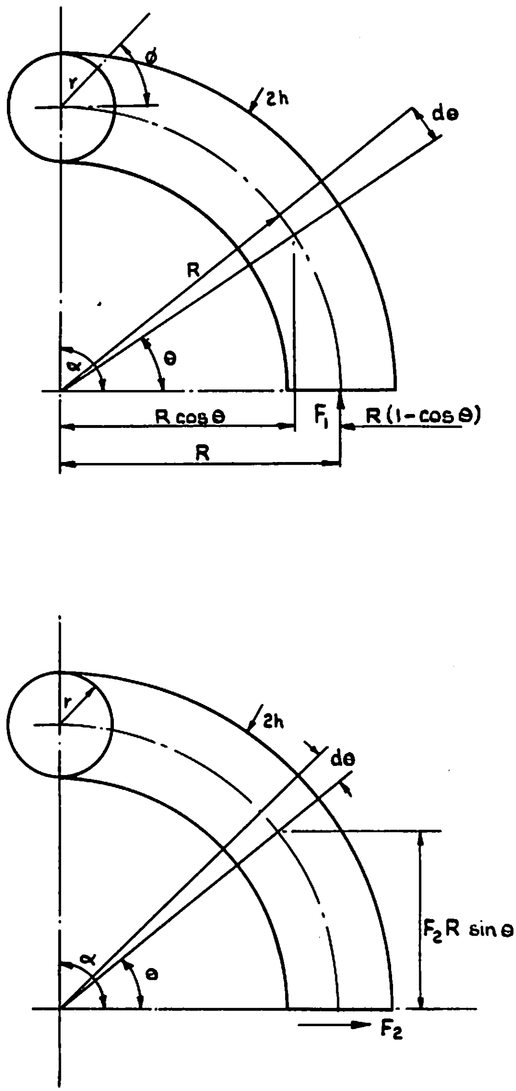


Fig. 1 Force Actions on Bends

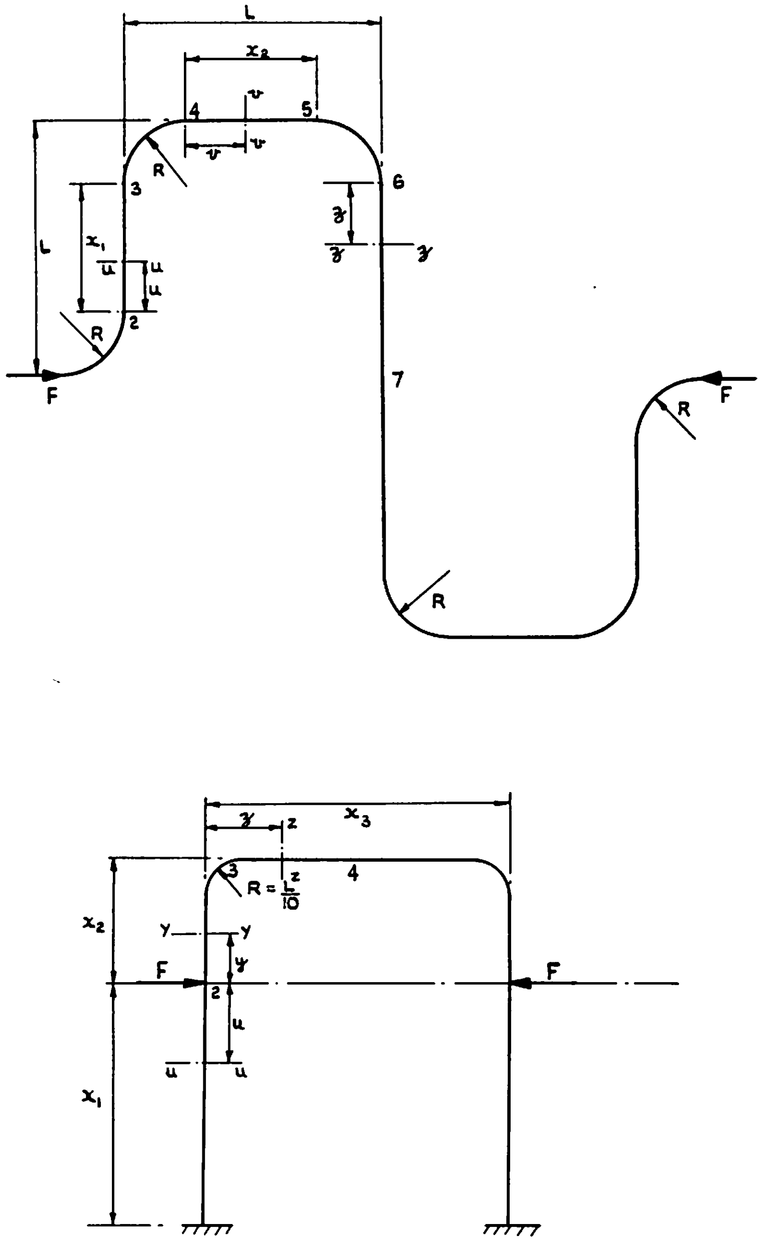


Fig. 2 Simple Pipework Systems

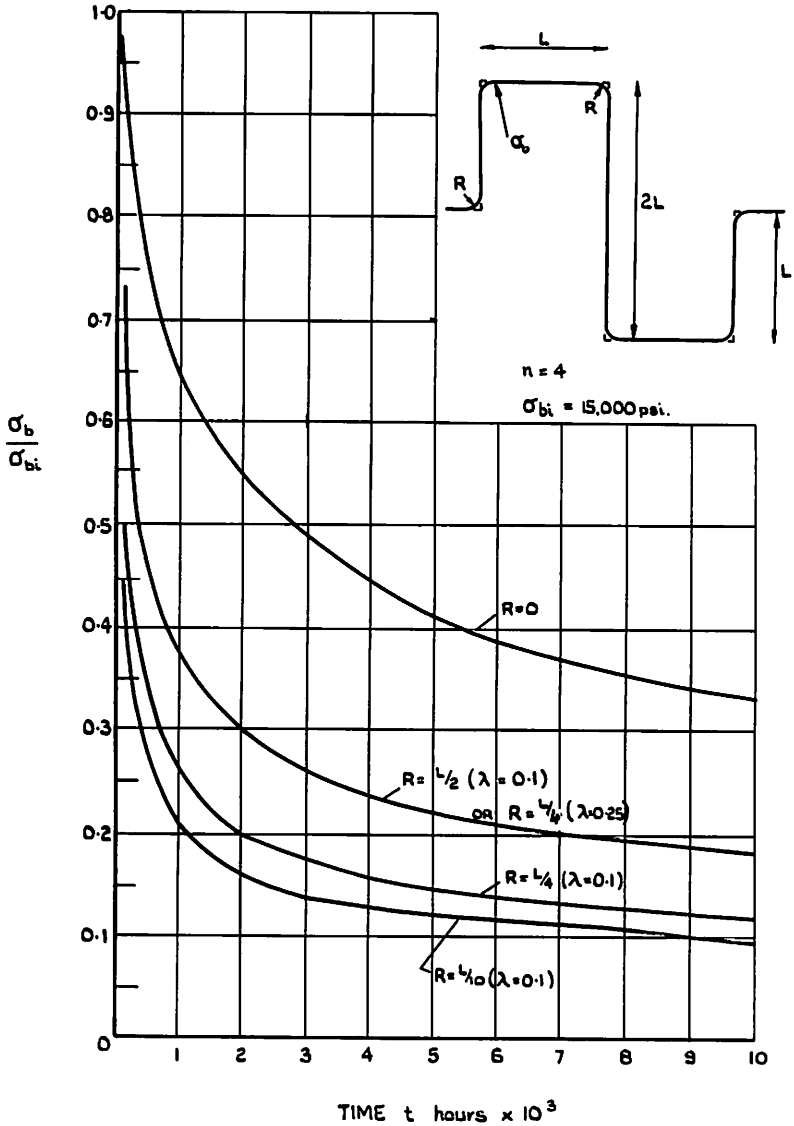


Fig. 3 Maximum Nominal Bending Stress Relaxation

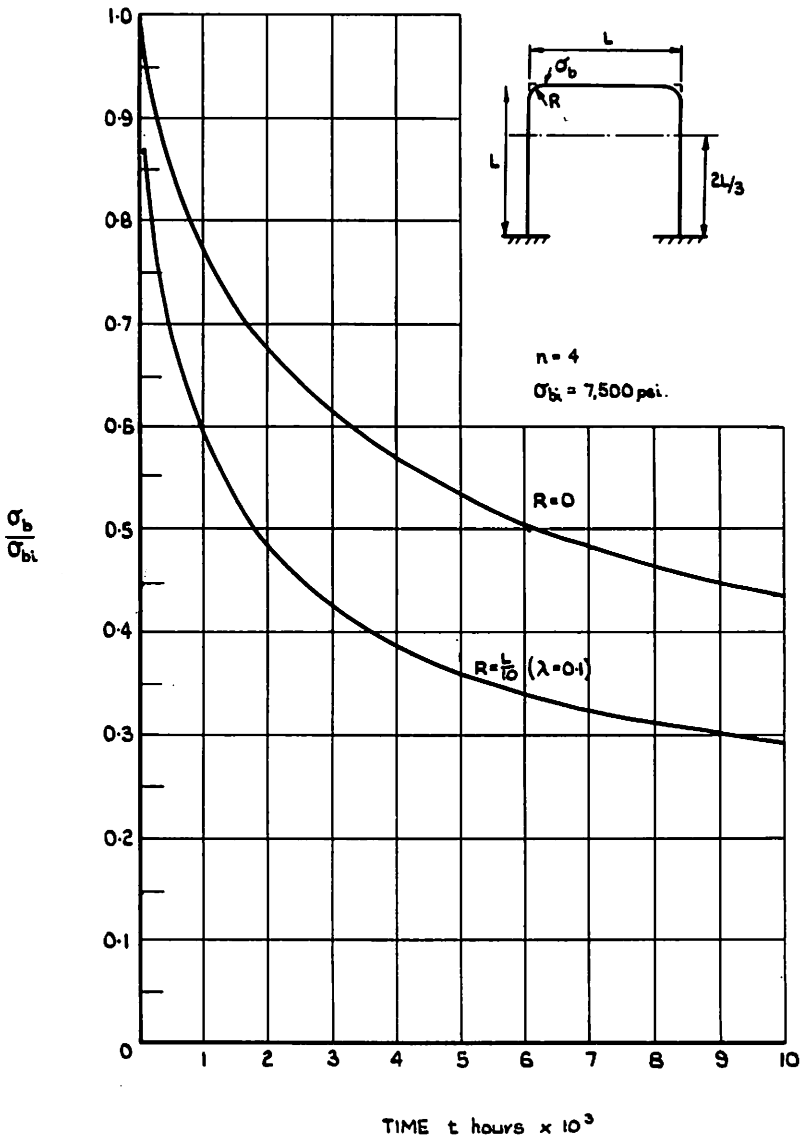


Fig. 4 Maximum Nominal Bending Stress Relaxation

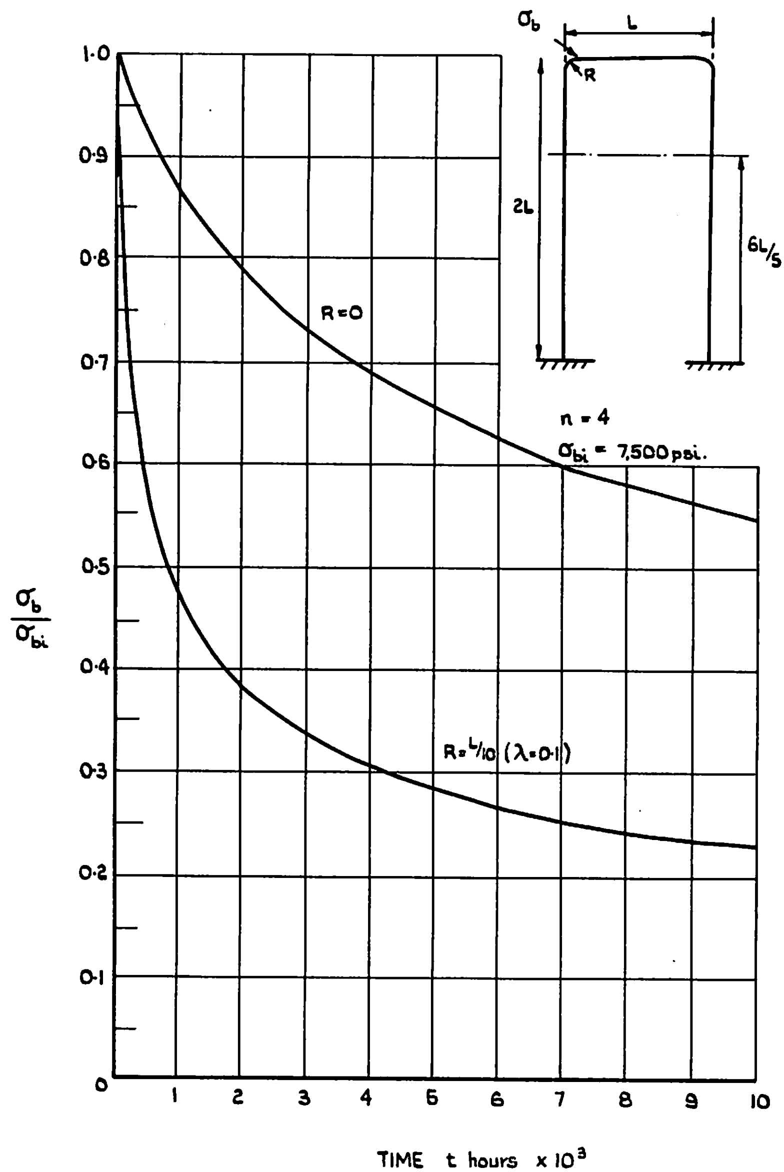


Fig. 5 Maximum Nominal Bending Stress Relaxation

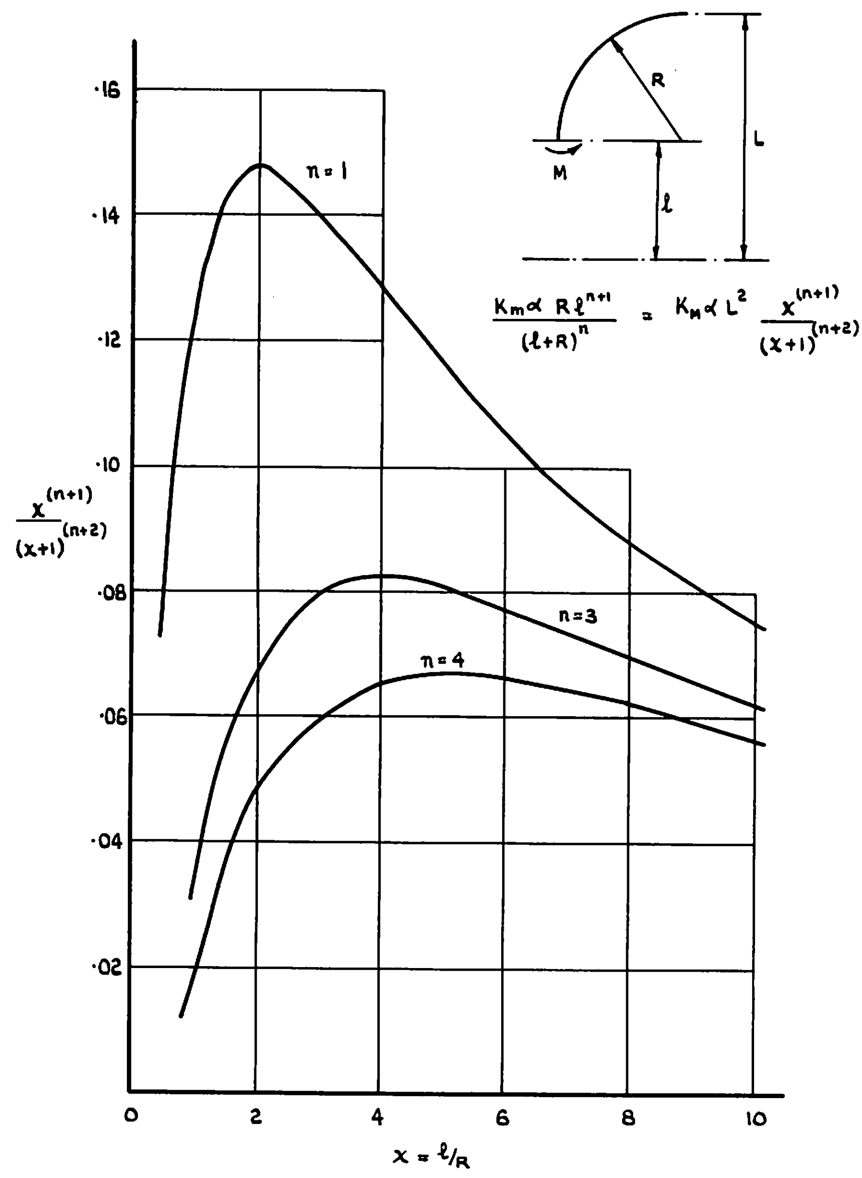


Fig. 6 Influence of Bend Location