# A Simple Model for the Dynamic Analysis of Deteriorating Structures

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Abstract

A simple model exhibiting a multi-linear constitutive law is presented which describes the behaviour of structural members and subassemblages under severe cyclic loading.

The proposed model allows for:

1) pinched form of force-displacement diagrams due to, e.g., cracks in reinforced concrete members and masonry panels;
2) slippage effects due to lack of bond of steel bars in reinforced concrete and clearances in steel bolted connections;
3) post-buckling behaviour of subassemblages with unstable members;
4) cumulative damage affecting strenght and/or stiffness at low cycle fatigue.

The parameters governing the model behaviour have to be estimated on the basis of experimental results.

The model is well suitable for analysis under statically applied cyclic displacements and forces, and under earthquake excitation.

An X-type bracing system is then worked out where the member behaviour is schematized according to the proposed model.

## 1. Introduction

In the last years progress in the studies about the ultimate behaviour of structures under severe cyclic loadings has pointed out the requirement of constitutive laws more comprehensive than usual elastic-plastic strain hardening relations.

Namely in steel structures, clearances occurring in bolted connections and buckling of structural members, or of their parts, require introduction of locking effects into constitutive relations. The same problem occurs in reinforced concrete members due to cracking and lack of bond of steel bars, and in masonry panels too.

Such structural behaviours have been shown in numerous experiments at low cycle fatigue worked out in order to investigate seismic behaviour of structures. Four typical generalized force-displacement diagrams are shown in Fig. 1. The hysteretic moment-curvature diagram of Fig. 1a, [1], is typical of a bolted moment-resisting connection; the slippage at the faying surfaces causes its characteristic shape. The cyclic behaviour of an individual steel brace is represented in Fig. 1b, [2]; the loop shape is due to its post-buckling behaviour. In Fig. 1c, [3], the force-displacement diagram of a reinforced concrete cantilever shows the pinching effect due to the influence of bond deterioration. Typical shear deformation diagram under alternate shear force and constant normal stress in a brick masonry is presented in Fig. 1d, [4], where the tensile cracking of the mortar produces the characteristic shape.

Furthermore the above mentioned experiences have shown the importance of effects of strenght and stiffness deterioration in structural members; therefore constitutive laws ruled by parameters whose values vary according to number of cycles and to magnitude of deformation need to be assumed, [5, 6, 7].

The aim of the present paper is modelling the behaviour of deteriorating structural members from a synthetic point of view, that is by means of a restricted number of parameters which allows for the analysis of structures at low computational effort.

On the basis of previous studies, [8, 9, 10], where reological models are proposed, herein a simple model of member behaviour exhibiting a multi-linear constitutive law is presented, which allows for:

1) pinched form of generalized force-displacement diagrams due to, e.g., cracks in reinforced concrete members and in masonry panels;
2) slippage effects due to lack of bond of steel bars in reinforced concrete members and clearances in steel bolted connections;
3) post-buckling behaviour of subassemblages with unstable members;
4) cumulative damage affecting strenght and stiffness of structural members at low cycle fatigue, where the deterioration law of each parameter governing the model behaviour depends upon both the maximum values of strains attained during the loading history and the total energy dissipated from the beginning of the loading history up to the present time.

## 2. Description of the model

The multi-linear constitutive law governing the model behaviour is shown in Fig. 2. It is defined through the slopes of its 8 linear branches ($k_1$, $k_2$, ..., $k_8$) corresponding to the different stiffnesses exhibited by the member during each cycle, and through the 4 values of generalized force ($f_2$, $f_4$, $f_6$, $f_8$) associated to the intersection points between the elastic branches 1 and 5, and the hardening or softening branches 2, 4, 6 and 8. The branches 1, 3, 5 and 7 may be covered in both ways (elastic behaviour), whereas the branches 2, 4, 6 and 8 can be covered in the cycle direction only (plastic behaviour).

The f-$\delta$ relationships characterizing each branch of the cycle read as follows, where those of the branches 3 and 7, whose position may vary from cycle to cycle, depend on the current point ($\tilde{\delta}$, $\tilde{f}$):

$$
\begin{array}{lll}
① & f = k_1\ \delta; & \\
② & f = k_2\ \delta + f_2 \left(1 - \frac{k_2}{k_1}\right), & \dot{f}\ k_2 > 0; \\
③ & f = k_3\ \delta - k_3\ \tilde{\delta} + \tilde{f}; & \\
④ & f = k_4\ \delta - f_4 \left(1 - \frac{k_4}{k_5}\right), & \dot{f}\ k_4 < 0; \\
⑤ & f = k_5\ \delta; & \\
⑥ & f = k_6\ \delta - f_6 \left(1 - \frac{k_6}{k_5}\right), & \dot{f}\ k_6 < 0; \\
⑦ & f = k_7\ \delta - k_7\ \tilde{\delta} + \tilde{f}; & \\
⑧ & f = k_8\ \delta + f_8 \left(1 - \frac{k_8}{k_1}\right), & \dot{f}\ k_8 > 0.
\end{array}
\qquad (1)
$$

The dashed lines in Fig. 2 mean that the branches 4 and 8 are not necessarily bounded by the branches 5 and 1 respectively in the cycles following the first one, but they go on as far as they meet the branches 3 and 7, whose position depends on the maximum deformation attained in previous loading cycles. Namely the projection $\bar{\delta}$ on the $\delta$-axis of the 4 or 8 branch section, which must be furthermore covered after the intersection with the branches 5 or 1, has been related to the maximum plastic deformation $\delta^*$ previously attained beyond a suitable treshold $f_L$ on the branches 6 or 2 respectively by means of a multiplier $c_L$. Thus the above rule requires two more parameters, $f_L$ and $c_L$, for each of the branches 4 and 8, as shown in Fig. 3 for the branch 8 only.

So far the model behaviour is governed by 15 parameters; it is worth to notice, however, that the parameters strictly required for schematizing the single structural behaviours are fewer and they differ according to the structural type under investigation and they should be estimated on the basis of experimental results.

## 3. Deterioration effects

In the model behaviour up to now described the cycle shape can be changed during the loading history only by the slippage affecting the branches 4 and 8.

On the contrary, the actual behaviour of real structures exhibits deterioration phenomena affecting strenght (f-type parameters) and stiffness (k-type parameters).

Deterioration phenomena seem to depend mainly on two quantities, [5, 6, 7]:

I) the maximum values of positive and negative deformation $\delta^+$ and $\delta^-$ attained from the beginning of the loading history up to the current time;

II) the energy $\Omega$ dissipated during the loading history preceding the current time.

The degradation law of the generic strenght or stiffness parameter "p" takes the following form:

$$p = p_0 - \Delta p \left[ \alpha \left( \frac{\delta^+}{\delta^+_{lim}} \right)^m + (1 - \alpha) \left( \frac{\delta^-}{\delta^-_{lim}} \right)^m + \left( \frac{\Omega}{\Omega_{lim}} \right)^n \right] , \qquad (2)$$

where:

$p_0$ = initial value of the parameter "p",

$\Delta p$ = characteristic value of the parameter decrement,

$\delta^+_{lim}$, $\delta^-_{lim}$ = suitable limit values of the deformation,

$\Omega_{lim}$ = suitable limit value of the dissipated energy.

The exponents "m" and "n", and the coefficient "$\alpha$" should be estimated on the basis of experimental results.

The degradation law (2) allows for modelling on one hand the behaviour at low cycle fatigue and even at monotonically increasing loads, and on the other hand the behaviour at high cycle fatigue.

In the former case the first two terms within square brackets in (2), or even only one of them, become significant; the coefficients "$\alpha$" and "$1 - \alpha$" represent the influence of the positive deformation $\delta^+$ on the degradation of the parameters ruling strenght and stiffness in the half-plane of negative generalized forces and vice versa.

In the latter case (high cycle fatigue) the influence of $\delta^+$ and $\delta^-$ becomes negligible and then the third term within square brackets in (2) plays an essential role, since the dissipated energy increases in proportion to the number of cycles; for $n = 1$ Eq. (2) seems to be related to the Palmgren-Miner law, [6] .

Figures 4a and 4b show two possible generalized force-displacement diagrams degrading according to the proposed law (2), which schematize typical behaviours respectively of a reinforced concrete member and of an individual steel brace.

## 4. Numerical application

The proposed model can synthetically represent the behaviour of simple structures subjected to loads depending on a single parameter.

In order to perform more refined analyses a structure can be discretized into a finite number of members, each one of them is constituted of one or more elements connected in different manners.

Namely as far as truss structures are concerned, the mechanical behaviour of each truss can be represented by a single member, even in the post-buckling range, as already noted in Sec. 1 (see Fig. 4b).

As a simple application of the above outlined model the dynamic, geometrically non linear, analysis of an X-type bracing system was performed under dead loads and seismic excitation (1976 Tolmezzo E-W earthquake).

The structural scheme is shown in Fig. 5a, while Figs. 5b,c represent the behaviour of the members 1 and 2 respectively, and Figs. 5d,e,f represent three different models assumed for the bracing members 3 as well as the relevant values of the governing parameters.

Figure 6 shows the time history of the horizontal displacement of the joint 2 for the three different behaviours assumed for the bracing members during the first 5 seconds of the accelerogram.

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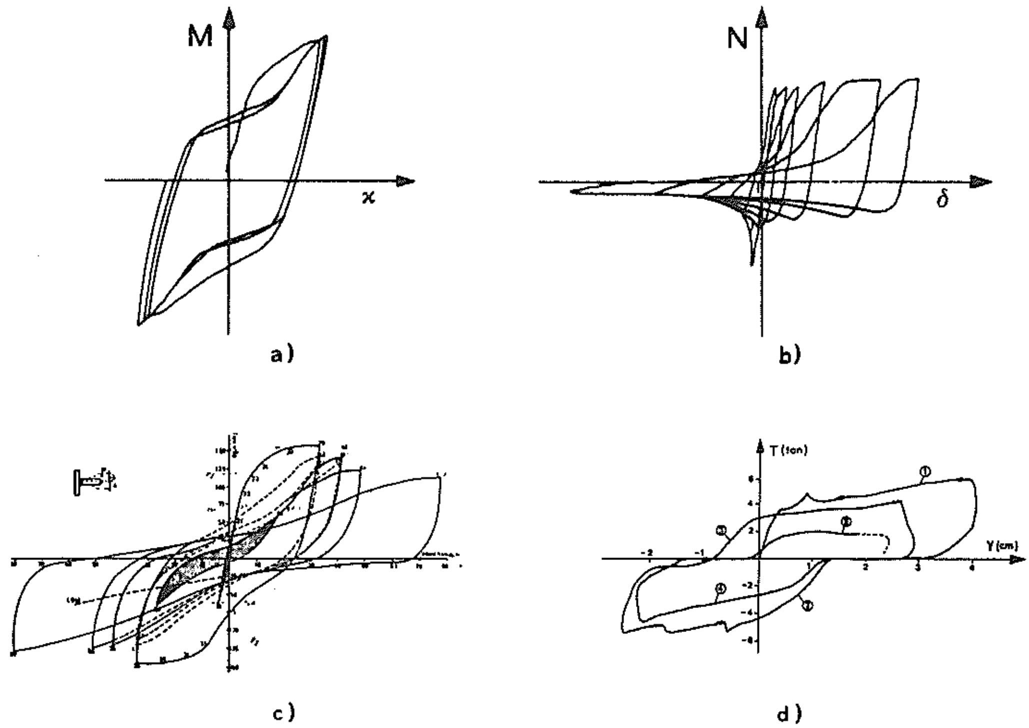



Fig. 1

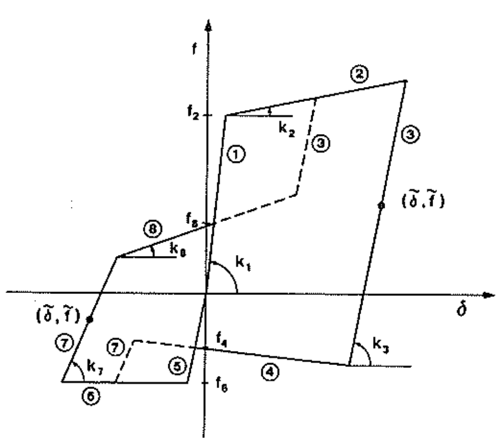



Fig. 2

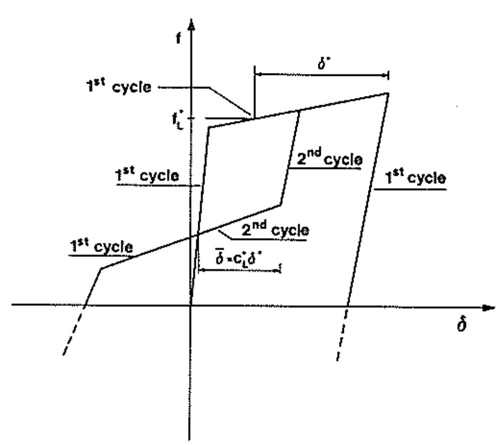
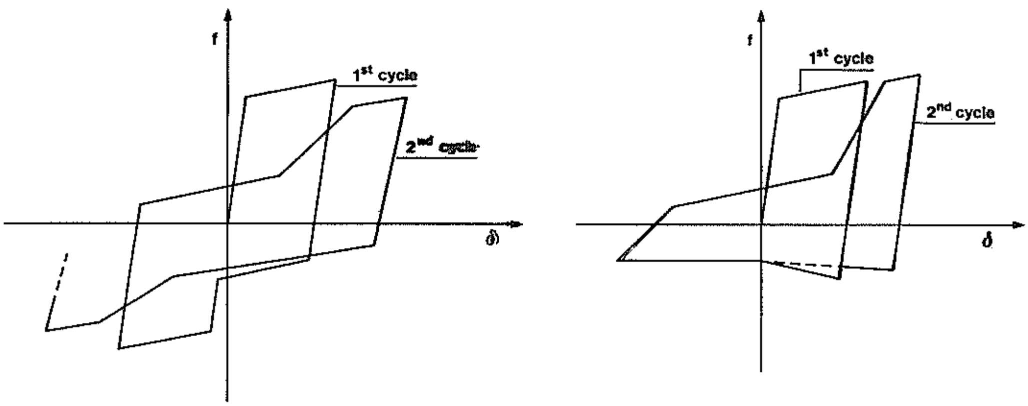
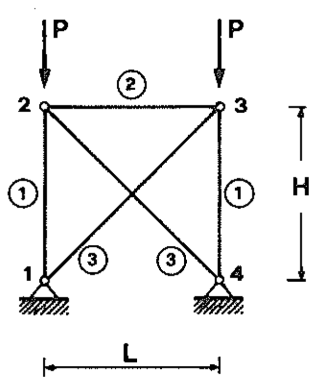



Fig. 3



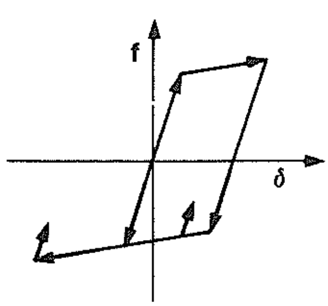

a) b)

Fig. 4




H = L = 300 cm,
$m_2 = m_3 =$ 80 kg cm$^{-1}$ sec$^2$,
P = 5,100 kg.

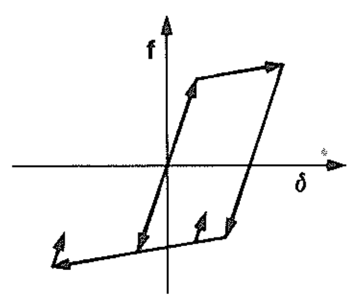
Fig. 5a




$f_2 = f_4 = f_6 = f_8 =$ 156,720 kg,

$k_1 = k_3 = k_5 = k_7 =$ 457,100 kg/cm,
$k_2 = k_4 = k_6 = k_8 =$ 4,571 "

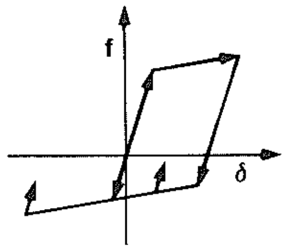
Fig. 5b




$f_2 = f_4 = f_6 = f_8 =$ 150,240 kg,

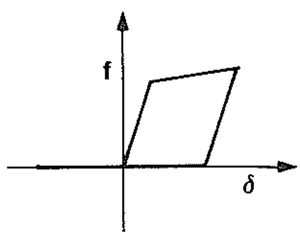
$k_1 = k_3 = k_5 = k_7 =$ 438,200 kg/cm,
$k_2 = k_4 = k_6 = k_8 =$ 4,382 " .

Fig. 5c



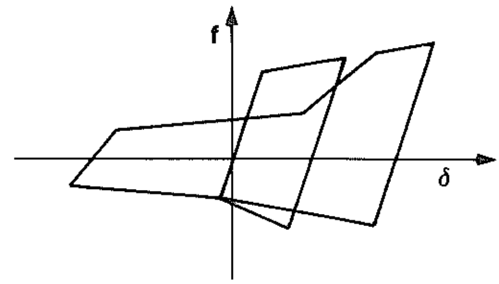

$f_2 = f_8 = 37{,}200$ kg,
$f_4 = f_6 = 11{,}160$ ",
$k_1 = k_3 = k_5 = k_7 = 76{,}721$ kg/cm,
$k_2 = k_4 = k_6 = k_8 = 767$ kg/cm.

Fig. 5d




$f_2 = 37{,}200$ kg,
$f_4 = f_6 = f_8 = 0.$ kg,
$k_1 = k_3 = 76{,}721$ kg/cm,
$k_2 = 767$ kg/cm,
$k_4 = k_5 = k_6 = k_7 = k_8 = 0.$ kg/cm

Fig. 5e




$f_2 = 37{,}200$ kg,
$f_4 = f_6 = f_8 = 11{,}160$ kg,
$k_1 = k_5 = 76{,}721$ kg/cm,
$k_2 = 767$ kg/cm,

$$k_3 \begin{cases} \text{loading} = 76{,}721 \left(1 - \frac{\delta^-}{12.}\right) \text{ kg/cm}, \\ \text{unloading} = 76{,}721 \quad ", \end{cases}$$

$$k_4 = -\,4{,}650 \left\{1 - \left[1 - \left(\frac{\delta^+}{1.45}\right)^{-1}\right]\right\} \quad ",$$

$k_6 = -\,548$ ",
$k_7 = 7{,}672$ ",
$k_8 = 1{,}096$ ",

$f_L^+ = f_2, \quad c_L^+ = (1.07)^{-1}.$

Fig. 5f

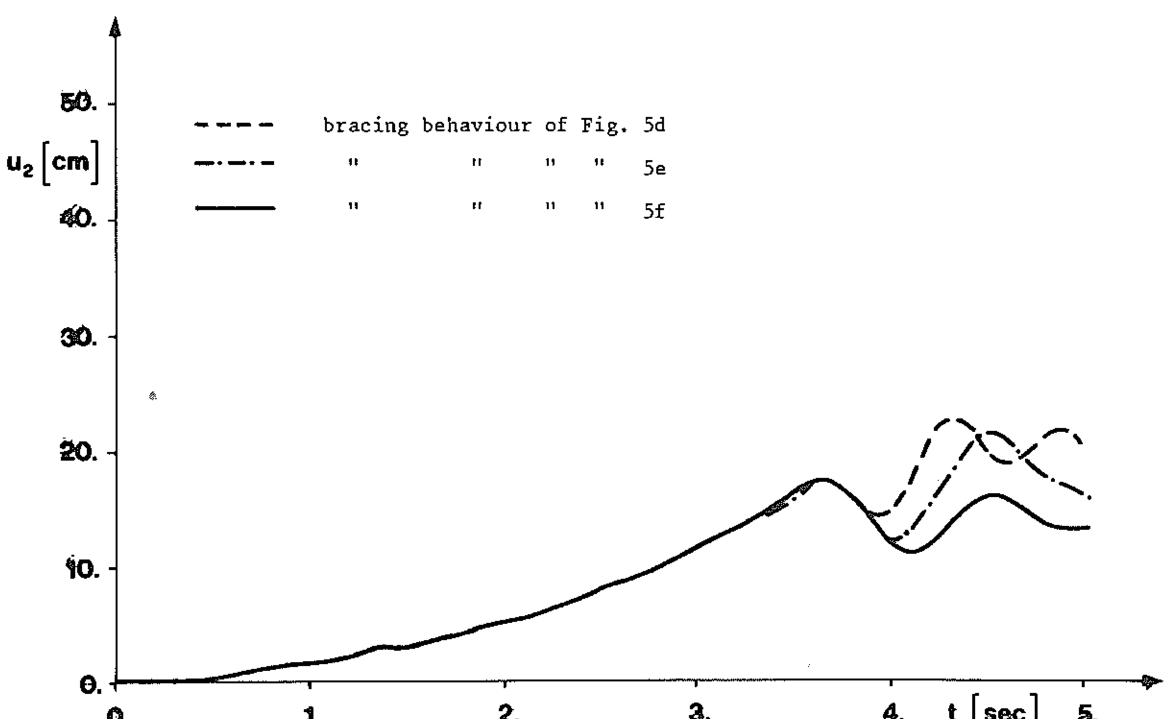



Fig. 6