Transactions of the **14th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 14), Lyon, France, August 17-22, 1997 **K05/1**

System identification of structural characteristics for ground motion using forced vibration test results

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ABSTRACT : Resonance curves from forced vibration tests on buildings with soil-structure interaction using an exciter at the upper floor have been generally regarded to be different from responses to ground motions such as earthquakes. This paper presents a mathematical method for converting forced vibration test results to transfer functions of buildings for ground motion with rigid ground conditions. Results of analytical parametric studies for base deformation effect etc. are also shown.

1. INTRODUCTION

Forced vibration tests with an exciter at the upper floor have been performed to obtain dynamic characteristics of nuclear reactor buildings, but few studies have been performed concerning the relation between the obtained characteristics and dynamic characteristics for ground motions, such as earthquakes and explosion tests. This paper presents a method for determining dynamic characteristics (transfer functions) of structures for ground motions with rigid ground conditions, using results of forced vibration tests with upper-position exciters and for structures with soil-structure interaction. Results of analytical parametric studies for base deformation effects etc. are also shown.

The authors propose a method for eliminating base rocking effect on transfer functions from observation records of buildings to obtain the dynamic characteristics of buildings themselves [1]. They have applied the method to cases with base deformation as well as soil-structure interaction, using multi-particle and FE models [2, 3]. This paper also presents application limits and application of a simpler method using the response at equivalent height.

2. SYSTEM IDENTIFICATION METHOD

(1) *Fundamental equations*

The equation of motion for a superstructure with a rigid base, sway-rocking motion of the base and excitation by an exciter is given by :

$$[M](\{\ddot{x}\}+\{H\}\ddot{\theta}+\{1\}\ddot{y}_0)+[C]\{\dot{x}\}+[K]\{x\}=[M]\{E\}\ddot{\alpha} \tag{1}$$

where

- $[M]$, $[C]$, $[K]$: mass, damping and stiffness matrices of the superstructure
- $\{x\}$: relative horizontal displacement due to deformation of the superstructure, base sway-rocking motion is subtracted
- $\{H\}$: heights of particles
- y_0, θ : absolute horizontal and rocking displacement of the base

$\{E\}$: vector, the component for the exciter-set particle is equal to 1, and others are 0

$\ddot{\alpha}$: $\ddot{\alpha} = F / m_i$(exciting force divided by the mass of the exciter-set particle)

Transposing some terms, Eq.(1)gives :

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{H\}\ddot{\theta} - [M]\{1\}\ddot{y}_0 + [M]\{E\}\ddot{\alpha} \tag{2}$$

Substituting $[U]\{q\} = \sum_{s=1}^{n} \{_s u\}_s q = \{x\}$ for $\{x\}$ and multiplying eq. (2) by $\{_s u\}^T$ on the left, orthogonality gives :

$${}_sM \cdot {}_s\ddot{q} + {}_sC \cdot {}_s\dot{q} + {}_sK \cdot {}_sq = -\{_s u\}^T[M]\{H\}\ddot{\theta} - \{_s u\}^T[M]\{1\}\ddot{y}_0 + \{_s u\}^T[M]\{E\}\ddot{\alpha} \tag{3}$$

where $\{_s u\}$ is the sth eigenvector of the superstructure for rigid ground condition; ${}_sM$, ${}_sC$, ${}_sK$ are the sth generalized mass, damping and stiffness, respectively. If we use the notations below, ${}_s\beta$ for the sth modal participation factor, ${}_s\gamma$ for excitation factor and ${}_s\bar{H}$ for equivalent height :

$${}_s\beta = \frac{\{_s u\}^T[M]\{1\}}{\{_s u\}^T[M]\{_s u\}}, \quad {}_s\gamma = \frac{\{_s u\}^T[M]\{E\}}{\{_s u\}^T[M]\{_s u\}}, \quad {}_s\bar{H} = \frac{\{_s u\}^T[M]\{H\}}{\{_s u\}^T[M]\{1\}} \tag{4}$$

and notations ${}_s\omega$ for $\sqrt{{}_sK / {}_sM}$, ${}_sh$ for ${}_sC / 2{}_sM{}_s\omega$ and ${}_sq_0$ for ${}_sq / {}_s\beta$, we have :

$${}_s\ddot{q}_0 + 2{}_sh{}_s\omega{}_s\dot{q}_0 + {}_s\omega^2{}_sq_0 = -{}_s\bar{H}\ddot{\theta} - \ddot{y}_0 + \frac{{}_s\gamma}{{}_s\beta}\ddot{\alpha} \tag{5}$$

If ${}_sq_0 = ae^{i\omega t}$, eq. (5) gives :

$$(-\omega^2 + 2i{}_sh{}_s\omega\omega + {}_s\omega^2)ae^{i\omega t} = -{}_s\bar{H}\ddot{\theta} - \ddot{y}_0 + \frac{{}_s\gamma}{{}_s\beta}\ddot{\alpha} \tag{6}$$

$$\begin{aligned} {}_s\ddot{q}_0 + {}_s\bar{H}\ddot{\theta} + \ddot{y}_0 - \frac{{}_s\gamma}{{}_s\beta}\ddot{\alpha} &= -{}_s\omega^2{}_sq_0 - 2{}_sh{}_s\omega{}_s\dot{q}_0 \\ &= -({}_s\omega^2 + 2i{}_sh{}_s\omega\omega)ae^{i\omega t} \end{aligned} \tag{7}$$

Eqs. (6) and (7) give :

$$\frac{{}_s\omega^2 + 2i{}_sh{}_s\omega\omega}{{}_s\omega^2 - \omega^2 + 2i{}_sh{}_s\omega\omega} = \frac{{}_s\ddot{q}_0 + {}_s\bar{H}\ddot{\theta} + \ddot{y}_0 - \frac{{}_s\gamma}{{}_s\beta}\ddot{\alpha}}{{}_s\bar{H}\ddot{\theta} + \ddot{y}_0 - \frac{{}_s\gamma}{{}_s\beta}\ddot{\alpha}} \tag{8}$$

The left side of eq. (8) is the sth modal transfer function (${}_sH_{fix}$) of the superstructure for ground motion and for rigid ground conditions. Therefore, eq. (8) can give the dynamic characteristics (transfer function) of superstructures for ground motion and rigid ground condition using forced vibration test results with upper-position exciters and for structures with soil-structure interaction.

(2) *Equivalent height response method (simpler method)*

Eq. (8) has the weak point in its application that the eigenvector and modal partiticipation factor are necessary to obtain ${}_s\ddot{q}_0$. Therefore, a more practical method is proposed to identify the dynamic characteristics around the 1st peak.

The response around the 1st peak is given approximately using only the 1st modal response. Hence :

$$\ddot{x}_i = {}_1\ddot{q}_0{}_1\beta_1u_i \quad \text{or} \quad {}_1\ddot{q}_0 = \frac{\ddot{x}_i}{{}_1\beta_1u_i} \tag{9}$$

If the component of the mode vector for the top particle (${}_1u_1$) is equal to 1, eq. (9) gives :

$$ {}_1\ddot{q}_0 = \frac{\ddot{x}_1}{{}_1\beta} \tag{10} $$

The proposed method for approximation in this chapter is to substitute the relative response at the equivalent height for the right side of eq. (10). The error by this approximation is evaluated as follows.

The 1st mode shape of a PWR building for rigid ground conditions can be considered to range between an inverted triangular shape and a quarter sinusoidal wave shape. Therefore, the error for these two cases is evaluated. If the mode shape is an inverted triangle and the masses of all the particles are equal, ${}_1\beta = 3/2$(vid. eq. (4)) and ${}_1\ddot{q}_0 = (2/3)\ddot{x}_1$, (vid. eq. (10)). On the other hand, the relative amplitude at the equivalent height is $(2/3)\ddot{x}_1$,because the equivalent height is 2/3 of the total height H_1 (vid. eq. (4)). Thus, the error is zero for this case. Secondly, if the mode shape is a quarter sinusoid and the masses are equal, ${}_1\beta = 4/\pi$ and ${}_1\ddot{q}_0 = (\pi/4)\ddot{x}_1 = 0.79\ddot{x}_1$. The relative amplitude at the equivalent height is sin $((\pi/2)\times(2/\pi)) = 0.84$ because the equivalent height is $(2/\pi)H_1$. Thus the error is 0.07 for this case. The above discussion is the same when ${}_1u_1$ is not equal to 1.

Eq. (8) then gives :

$$ {}_1H_{fix} = \frac{{}_1\omega^2 + 2i_1h_1\omega\omega}{{}_1\omega^2 - \omega^2 + 2i_1h_1\omega\omega} = \frac{\ddot{x}_E - \frac{{}_1\gamma}{{}_1\beta}\ddot{\alpha}}{{}_1\bar{H}\ddot{\theta} + \ddot{y}_0 - \frac{{}_1\gamma}{{}_1\beta}\ddot{\alpha}} \tag{11} $$

where :

${}_1H_{fix}$: 1st modal transfer function of the superstructure for ground motion and rigid ground condition

$\ddot{x}_E$: absolute response at the equivalent height

3. PARAMETRIC STUDY FOR APPLICATION LIMIT

Eqs. (8) and (11) are applied to analytical model responses, and the effect of base flexure on system identification is examined. This effect is neglected in Eqs. (8) and (11).

(1) *Analysis model*

The model comprises a reactor building (outer shell), a base mat and soil springs. The dimensions of the structure are those of typical PWR reactor buildings in Japan. A shear wave velocity of 1000m/sec is assumed for the soil. The model is shown in Fig. 1. The dimensions and properties of the structure are shown in Table 1 and those of the soil are shown in Table 2. ${}_1\beta$, ${}_1\gamma$ and ${}_1\bar{H}$ for the 1st mode of the structure under rigid soil condition are shown in Table3.

(2) *Analysis case*

The flexural rigidity of the base mat is varied as a parameter to study the base flexure effect on the identification accuracy. For this purpose, Young's modulus for property number I in Table 1 is varied as 0.1(Case 1), 1(Case 2) and 10000(Case 3) times as large as the 230 ton / cm^2 in Table 1. The vertical soil springs are also varied (vid. Table 4). The rotational angle θ of the base mat is evaluated as the vertical displacement at the bottom of the shell in Fig. 1 divided by its distance from the center (vid. Fig. 2). For upper-position exciter cases, the horizontal force is applied at the 47m level. Some additional cases with ground motion are calculated for reference.

(3) *Frequency response*

1) *resonance curve*

Fig. 3 shows the responses of the top nodal point to 1 Gal sinusoidal ground motion. Fig.4 shows the responses of the same point to a 1ton·f exciter force. There is a large difference between the response to the ground motion and that to the exciter force.

2) *tranfer function*

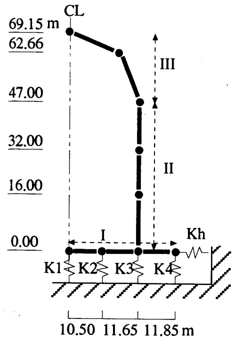

Fig.1 Analysis Model

Table 1 Coefficient for Structural Model

material number	thickness (m)	weight per unit volume (t/m³)	Young's modulus (t/cm²)	Poisson's ratio	damping ratio
I	8	3.4	230	0.1667	0.02
II	1.3	2.6	300	0.1667	0.02
III	1.3	2.1	230	0.1667	0.02

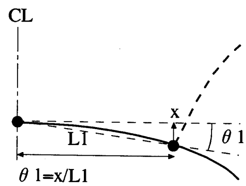

Fig.2 Definition of Rotational Angle of Bases

Table 2 Coefficient for Model Soil and Soil Springs

model soil	shear wave velocity (m/sec)	weight per unit volume (t/m³)	Poisson's ratio	Young's modulus (t/m²)	shear rigidity (t/m²)
soil property	1000	2.0	0.4	57.10×10^4	20.39×10^4

soil spring	Kh	K1	K2	K3	K4
spring constant (ton/cm)	3.78×10^5	8.60×10^2	1.377×10^4	3.306×10^4	5.175×10^4
damping ratio	0.2	0.075	0.075	0.075	0.075

Table 3 Participation Factor and Equivalent Height

participation factor $_1\beta$	participation factor $_1\gamma$	equivalent height $_1\bar{H}$
-1.465	-0.439	46.522 (m)

Table 4 Analysis Cases (1)

	base rigidity ratio	vertical soil spring ratio
CASE 1	0.1	0.2
CASE 2	1	1
CASE 3	10000	1

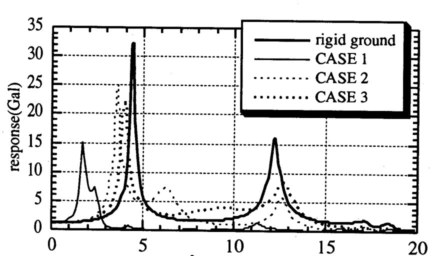

Fig.3 Response of the Tops
(for Ground Motion, Case 1～3)

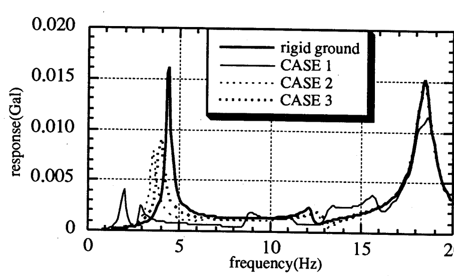

Fig.4 Response of the Tops
(for Excitation, Case 1～3)

Fig. 5 shows the transfer functions for the ground motion cases given by eq. (8) and that for the rigid ground condition. In the equation, the 1st mode is assumed and α is made equal to zero. For rigid ground conditions, the peak frequency is 4.4Hz and the peak response ratio is 22.00. For Case 1, the frequency is 4.0Hz and the ratio is 3.81. For case 2, they are 4.4Hz and 26.29. For Case 3, 4.4Hz and 23.01. Case 2 and Case 3 give good approximations for the 1st peak frequencies and response ratios. Some differences in the ratio are due simply to the frequency resolution. However, in Case 1, eq (8) does not give the correct peak frequency or ratio, and the phase lag is different from those of Case 2 and Case 3.

Fig. 6 shows the transfer functions for the exciter cases given by eq. (8), and that for ground motion with rigid ground condition. The 1st mode $\{{}_1u\}$ is assumed for eq. (8). The peak frequency and response ratio for rigid ground condition are the same as in Fig. 5. For Case 1, the peak frequency is 4.4Hz and the peak ratio is 3.88. For Case 2, they are 4.4Hz and 25.35. For Case 3, they are 4.5Hz and 23.57. Case 2 and Case 3 give good approximations. Case 1 gives the correct peak frequency but does not give the correct peak ratio. Also, Case 1 gives a different phase lag from the others.

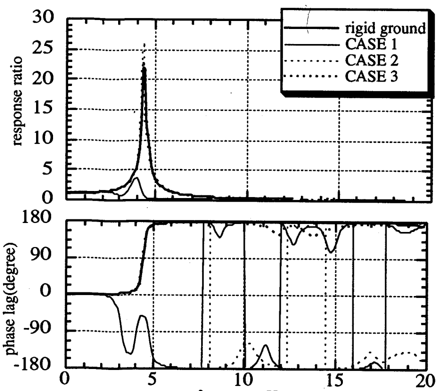

Fig.5 Transfer Function
(for Ground Motion, Case 1～3)

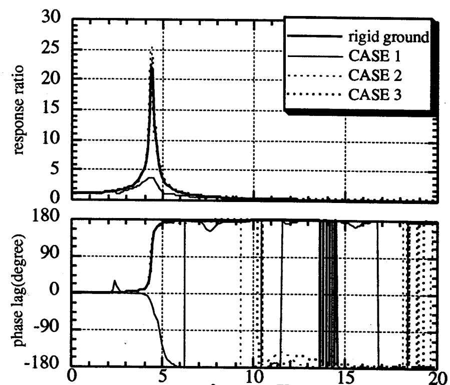

Fig.6 Transfer Function
(for Excitation, Case 1～3)

3) *detailed study on base flexure effect*

As stated above, in the small flexural rigidity case (Case 1), eq. (8) does not give the correct transfer function. This chapter describes more cases between Case 1 and Case 2 to find the application limit of eq. (8). These cases, Case 4 ~ Case 7, are shown in Table 5. The base rigidity ratios range from 0.2 to 0.5 times those of the fundamental case, Case 2, where the ratio is 1.0. The ratio is 0.2 for Case 4 and 0.5 for Case 7.

Fig. 7 shows the top nodal responses. A smaller flexure rigidity gives a smaller first-peak frequency. Fig. 8 shows the transfer functions. In Case 4, where the rigidity ratio is the smallest, a large (more than 40) peak occurs at 4.7Hz and a small peak occurs at 3.2Hz. In Case 5, the large peak at 4.7Hz yields a ratio of approximately 21, and the small-peak frequency yields 3.7Hz. In Case 6, the ratios of those two peaks become nearly equal and the frequencies are

Table 5 Analysis Cases (2)

	base rigidity ratio	vertical soil spring ratio
CASE 4	0.2	0.2
CASE 5	0.3	0.2
CASE 6	0.4	0.2
CASE 7	0.5	0.2

Table 6 Peak Frequency Averages for Transfer Function

	1st pesk		2nd peak		weighted mean of peak frequency
	peak frequency	response ratio	peak frequency	response ratio	
CASE 1	2.3 Hz	1.735	4.4 Hz	3.884	3.752 Hz
CASE 4	3.2 Hz	2.745	4.7 Hz	42.85	4.610 Hz
CASE 5	3.7 Hz	4.619	4.7 Hz	21.36	4.522 Hz
CASE 6	4.1 Hz	9.312	4.8 Hz	8.631	4.437 Hz
CASE 7	4.3 Hz	16.06	5.0 Hz	4.170	4.444 Hz

4.1Hz and 4.8Hz. In Case 7, the first peak becomes larger, 4.3Hz, and the ratio becomes approximately 16, and the second peak is 5.0Hz and ratio is approximately 4. These results mean that Case 6 is a kind of boundary. If the rigidity ratio is smaller than that of Case 6, the second peak is very large in some cases. If the rigidity ratio is larger than that of Case 6, the first peak approaches the peak for the rigid ground case. Those from Case 4 to Case 6 have larger errors or fluctuations. Case 7 has an unnegligible error. Therefore, those cases from Case 4 to Case 7 prove to have unnegligible error, and the application limit for base rigidity is Case 2.

Taking note of the fact that the 4.4Hz peak for the rigid ground case is between the two peaks for each case in Fig. 8, a weighted mean method for obtaining the real peak frequency is tried. Two peak frequencies are averaged, being weighted based on the peak ratios. Results are shown in Table 6. Except for Case 1, this averaging gives a good approximation. Therefore, the weighted mean of two peaks for small-base-rigidity cases may give a good approximation of the real peak

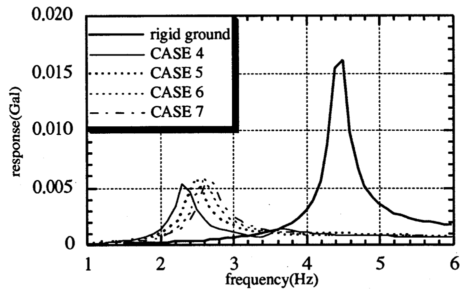

Fig.7 Response of the Tops (for Excitation, Case 4~7)

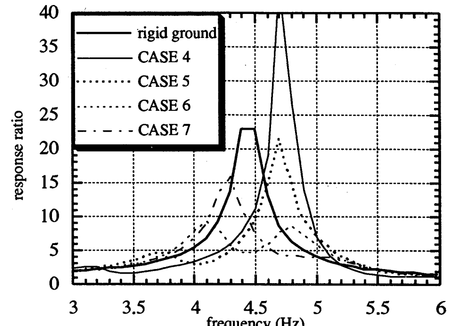

Fig.8 Transfer Function (for Excitation, Case 4~7)

frequency.

4) *effect by error in equivalent height*

For the proposed method, the equivalent height is the most probable numerical value to be evaluated with some error. Considering that the maximum error in section 2-(2) is 7%, ± 2%, ± 4% and ± 8% errors for the equivalent height are given to the result from Case 2 with an exciter. Fig. 9 shows the transfer functions given by eq. (8) for these cases. Error in frequency and peak ratio are both small.

5) *result by equivalent height method*

Fig. 10 shows the transfer function for Case 2 with an exciter given by eq. (11). The first peak of the superstructure is evaluated well.

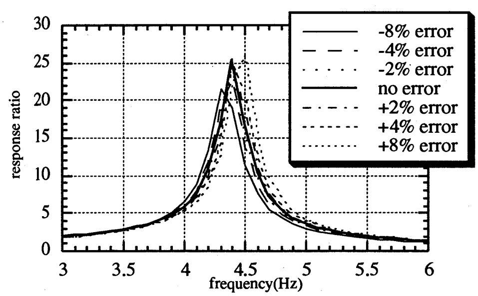

Fig.9 Error in the Equivalent Height and Transfer Function

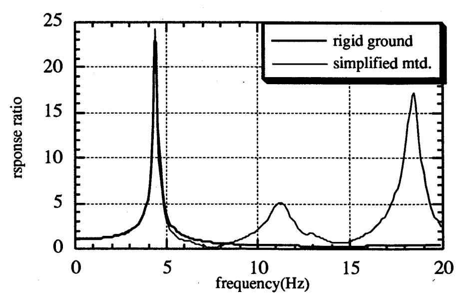

Fig.10 Transfer Function by Simplified Method

4. RESPONSE OF THE BASE AROUND THE FIRST PEAK

Fig. 3, Fig. 4 and Fig. 7 indicate that in the frequency range around the first peak of the transfer function and when the base and soil rigidities are small, the response of the top nodal point is comparatively constant and the resonance curve is flat. Therefore, the peak of the transfer function is given by the trough in the denominator of eq. (8) or eq. (11). In this chapter, for Case 4 to Case 7, the relation between ratio of transfer function, the denominator, horizontal and rocking motion of the base, and excitation force term is studied. Fig. 11 shows these quantities in the

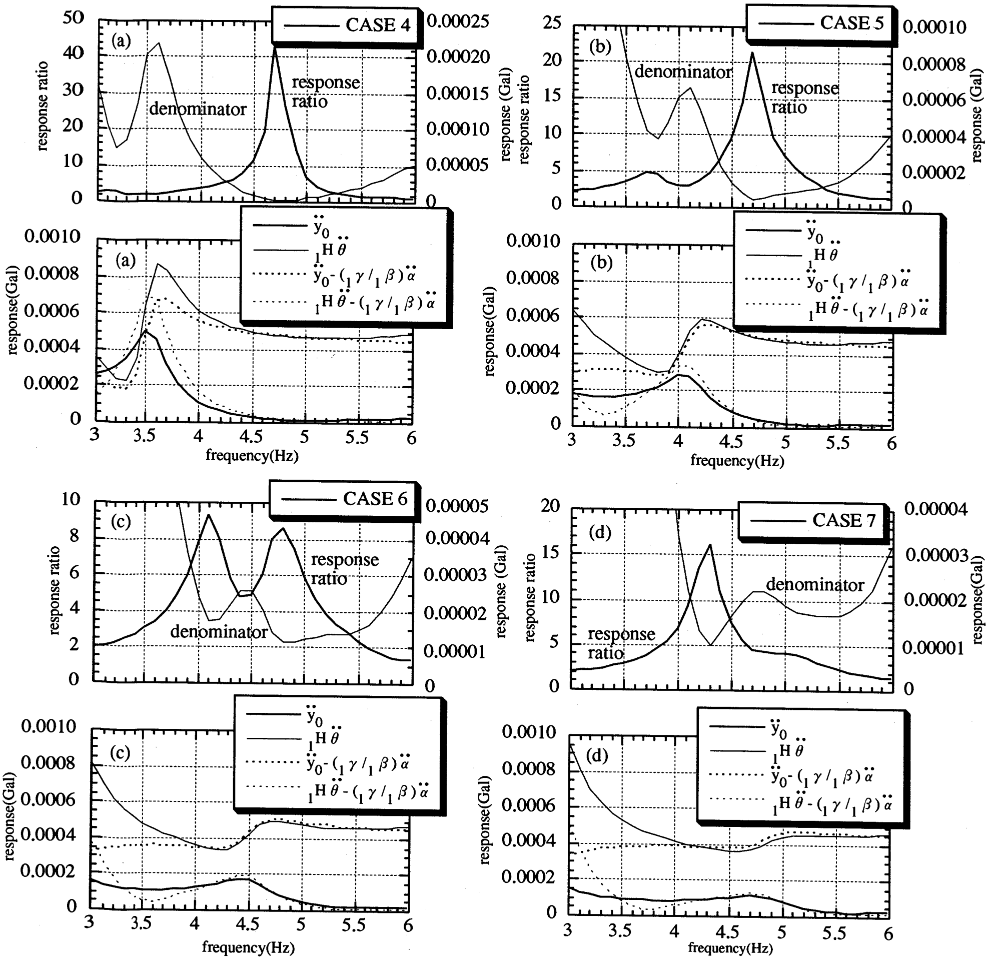

Fig.11 Transfer Function and Base Response of Horizontal and Rocking Components

frequency range from 3Hz to 6Hz. The following can be stated:

1) The ratio of the transfer function (response ratio) and the denominator are in approximately inverse proportion. Therefore, the response ratio peak is given by the trough in the denominator.

2) ${}_1\bar{H}\ddot{\theta}$ and $\ddot{y}_0-({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$ are large, and $\ddot{y}_0$ and ${}_1\bar{H}\ddot{\theta}-({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$ are small. Considering that the denominator is ${}_1\bar{H}\ddot{\theta}+\ddot{y}_0-({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$, it can be said that ${}_1\bar{H}\ddot{\theta}$ and $({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$ cancel each other out because of their phase difference around the response ratio peak, and $\ddot{y}_0$ itself is small.

3) The peak magnitude of the response ratio correlates with the magnitude of $\ddot{y}_0$ and ${}_1\bar{H}\ddot{\theta}-({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$.

4) $\ddot{y}_0$and ${}_1\bar{H}\ddot{\theta}-({}_1\gamma/\,{}_1\beta)\ddot{\alpha}$ are comparatively flat in the range of approximately 0.5Hz around the response ratio peak. The sharp trough in the denominator is given by their steep phase difference change around the peak.

5. CONCLUSION

A method for identifing dynamic characteristics of superstructures from forced vibration test results is proposed and applied to calculated responses of axisymmetric FE models. The follow-

ing conclusions can be stated:
1) The suitability of the method and its simplified version is verified.
2) The application limit for base flexure rigidity is obtained. Smaller rigidities than Case 2 cause unnegligible errors.
3) Identification errors caused by equivalent height errors are studied. These errors are small.
4) For the frequency range around the first peak of the transfer function, the relation between the response ratio and its denominator, horizontal and rocking motion of the base and exciting force is studied.
5) The weighted mean method for double peaks is suitable for small base rigidity cases.
6) A more strict identification method for base flexure cases and a theoretical basis for the weighted mean method are future problems to be solved.

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