Optimization of Procedures for the Experimental Modal Analysis of Fluid/Structure-Interaction Systems

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Summary

The contribution deals with various experimental procedures available to extract the eigenfrequencies, the mode shapes and the critical damping ratios of typical fluid/structure-interaction systems investigated in the nuclear safety domain. It is concluded that the best performance features can be obtained by application of various modifications of the step relaxation technique extracting the modal characteristics of the structure under study from a set of simultaneously measured relaxation response signals. The mathematical background and the most important subroutines of the computer code EVA, developed for this task in the Institute for Reactor Development of the Karlsruhe Nuclear Research Center, are reviewed in the paper.

To verify the code, a comparative experimental modal survey was performed for a vertical circular cylinder Ø 1000 x 3 x 1600 mm, partially filled with water (water level 1350 mm). This coupled structure has 15 eigenmodes (six doublets and one triplet) below 100 Hz, partly with very close eigenfrequencies in the energy spectrum (e.g. 80.26 and 80.81 Hz in the case of the doublet, first axial and seventh circumferential order). This poses high requirements on the frequency resolution of the extraction subroutine. To identify these modes, four different excitation and evaluation techniques were used. They yielded consistent results, confirming the correctness of all four experimental and evaluation procedures compared.

1. Introduction

The experimental modal analysis is one of the few means available to verify the structure dynamic codes or the coupled fluid/structure-interaction codes currently being developed in the reactor safety domain. However, application of this analysis to the real systems occurring in reactor technology is subjected to several severe constraints, which must be taken into account by the choice of a suitable experimental procedure. The most important of these constraints are: a) hostile environmental conditions met when the analysed structures are localized inside of the reactor confinement; b) presence of massive steel or concrete walls surrounding the structure; c) tight time schedule given to perform the necessary measurements. These conditions practically exclude the possible application of the multiple-point sine dwell method developed and widely used in aerospace research [1]. From the point of view of excitation means available, the following modal testing procedures are applicable in one case or the other to perform a modal survey of the mechanical systems studied in the nuclear safety domain:

I. Step relaxation (snapback) method

This method is based on the extraction from the step relaxation response signals of eigenfrequencies, mode shapes and critical damping ratios of the given structure. To obtain the step relaxation response, the structure is usually preloaded by a static load which is abruptly removed (snapback technique). This procedure was successfully applied to examine the modal characteristics of the HDR-core barrel, in a hot water environment (temperature up to 310 ^{o}C, system pressure 110 bar) of the HDR-pressure vessel [2]. The characteristic feature of this method is a broad-band, damped free-vibration-response of the structure providing relaxation signals suitable for the modal extraction up to approx. 1000 Hz (in case of acceleration response).

II. Modified relaxation procedures with preselected excitation spectrum

Sometimes it is desirable to reduce the number of vibration modes participating in the relaxation response of the structure. The simplest way to achive this consists in using an electrodynamic shaker driven in the decay mode. The shaker is fed with a narrow-band input signal which is suddenly removed thus producing a narrow-band relaxation response of the structure. In cases where a shaker cannot be used, pyrotechnical bunkers or rockets, mounted on the outer surface of the structure, can sometimes be applied. They generate a quasi-square-wave, short-duration excitation pulse with a defined spectrum [3]. A similar effect is obtained by use of pressure pulses generated in specially designed blast chambers (with rupture disc and pressurized air) or in the outlet nozzles of the vent pipes and relief valves submerged in the water pool of BWR-pressure suppression systems [4]. Another possibility is offered by the use of stationary, random pressure fluctuations generated in the coolant flow. The corresponding stationary random response signals are auto-correlated and can be evaluated with some suitable modification of Cole's random decrement method and Ibrahim's time domain method [5-7].

III. Single- or multiple-point random excitation methods

These methods generally use stationary random excitation with one or several electrodynamic shakers attached to the structure and extract the modal information from the set of frequency response functions, calculated on-line from simultaneously measured random excitation forces and corresponding response signals of the structure [8, 9]. The frequency bandwidth may vary between several Hz when specially developed zoom-subroutines are used, and several hundreds Hz. The important limiting factor determining the frequency resolution of the procedure and the number of identifiable modes is the size of the direct access memory of the computer used to extract the modal data from the set of frequency response functions.

IV. Impact methods

These procedures rely on the response signals generated by transient impacting of the structure studied. As in the preceding case, the set of frequency response functions interconnecting excitation and response is used to extract the modal characteristics /¯10_7. To generate the required response, various types of instrumented impactors, from light impulse hammer to heavy drop weights, are used.

The application of electrodynamic shakers in the nuclear safety domain is laborious and may lead to considerable time delays in the preparation of the modal experiments. This drawback can be substantially reduced in case of the first two procedures which require only simple and sometimes not even an extra excitation device. The price to be paid for this simplification is the relatively high computational effort necessary to evaluate the response signals.

The reduction of this effort is the purpose of optimizing individual subroutines of the computer code EVA /¯11_7, currently being developed and applied to various modal survey tasks in the nuclear safety domain. The common base of evaluation schemes for procedures I and II allowing to extract from the response signals the eigenfrequencies, the mode shapes and the critical damping ratios consists in the mathematical idealization of the given structural response which is supposed to be a free vibration of a linear, linearly damped multiple degree-of-freedom system, satisfying the equation

$$\chi_r(t) = \sum_{n=1}^{2N} C_{rn} \exp(\lambda_n t), \qquad (1)$$

where

$$\lambda_n = -\zeta_n\omega_n + i\Omega_n = \omega_n(-\zeta_n + i\sqrt{1-\zeta_n^2}) \qquad (2)$$

designates the complex eigenvalue of the n-th vibration mode; ω_n and Ω_n are the corresponding natural angular frequencies of the undamped and damped systems, respectively; ζ_n is the corresponding critical damping ratio, and C_{rn} is the complex initial value of the contribution of the n-th mode to the total response measured at $t = 0$ at the r-th location (the measurement of the time t begins when all excitation forces have already diminished to zero). The initial values C_{rn} are directly proportional to individual n-th mode shapes at the r-th location. Symbol $\chi_r(t)$ designates the original response signal (acceleration, displacement) measured at the r-th location, which serves as a base for the evaluation.

Since the modal characteristics of the structure can already be extracted from the free damped response described by eq. (1) it is not necessary to know the excitation prehistory (for $t < 0$) of the structure. This offers the possibility of utilizing all excitation means available in nuclear technology, which are summarized above under I and II. To perform the modal survey with this method, two important conditions must be fulfilled:

- Simultaneous measurement and acquisition of the structural response at R independent transducer locations, AD-converting and storing of the converted data on conventional data carrier (e.g. magnetic tape) compatible with the computer system used for the evaluation;

- Extraction of the modal characteristics of the structure from the data stored.

The simultaneous measurements of the response data in the hostile environment are nowdays the currently used technique [2]. One possible way to evaluate the data recorded is offered by the computer code EVA whose basic features will be described in the following section 2. Application of this method to a simple fluid/structure-interaction system is illustrated in section 3, dedicated to a comparative modal survey based on four different experimental procedures.

2. Computer Code EVA

The theoretical background of the computer code EVA is described in detail in report [11]. The set of R original response signals $\chi_r(t)$ $(r = 1,2,\ldots,R)$, which serve as an input for the calculations, is simultaneously sampled, digitized, digitally filtered and Fourier-transformed. This provides a set of R input vectors, each of them containing K+1 equidistantly spaced complex values $\hat{\chi}_{rk} = \hat{\chi}_r(\omega_k)$. They are represented mathematically by the integral

$$\hat{\chi}_{rk} = \hat{\chi}_r(\omega_k) = \left[\int_0^{T_{max}} \chi_r(t) e^{-i\omega t} dt \right]_{\omega=\omega_k} = \sum_{n=1}^{2N} \frac{C_{rn}}{i\omega_k - \lambda_n} \left(1 - e^{(\lambda_n - i\omega_k)T_{max}}\right) \tag{3}$$

To extract from the ensemble of R(K+1) values $\hat{\chi}_{rk}$ the set of N natural frequencies $f_n = \Omega_n/2\pi$ and N critical damping ratios ζ_n, the computer code EVA uses two different subroutines, the subroutine EIGEST and the subroutine EIGVAL.

Subroutine EIGEST

This subroutine multiplies the original input signals by an additional damping function $\exp(-\alpha_o t)$ (this allows to neglect the second term in parenthesis in eq. (3); $\alpha_o \equiv$ coefficient of additional damping) and extracts the eigenvalues λ_n from the "mean" spectral density function

$$SD(\omega=\omega_k) = \frac{1}{R} \sum_{r=1}^{R} \hat{\chi}_{rk} \bar{\hat{\chi}}_{rk} g_r \tag{4}$$

calculated from all R input vectors (the bar above the symbol designates the conjugate complex value, g_r is a weighting factor, depending on the type of the measured response signal - e.g. acceleration, displacement, etc.). To obtain the eigenvalues λ_n, the "mean" spectral density function $SD(\omega=\omega_k)$ is resolved into the components α_n, Ω_n, A_n and B_n according to the theoretical model

$$SD(\omega) = \sum_{n=1}^{N} \frac{A_n + B_n(\Omega_n - \omega)}{\alpha_n^2 + (\Omega_n - \omega)^2} + \frac{A_n + B_n(\Omega_n + \omega)}{\alpha_n^2 + (\Omega_n + \omega)^2} \tag{5}$$

The numerical extraction from $SD(\omega=\omega_k)$ of the eigenvalues α_n and Ω_n satisfying eq. (5) is performed with a special iteration procedure in the narrow frequency window, while passing in elementary frequency steps $\Delta\omega = \omega_{k+1} - \omega_k$ through the whole frequency range specified.

Subroutine EIGVAL

This subroutine utilizes the "weighted" spectral density function

$$SW(\omega_s, \omega_k) = \sum_{r=1}^{R} G_{rs} \hat{\chi}_{rk} \tag{6}$$

containing the weighting factor

$$G_{rs} = g_r \sum_{\ell=-L'}^{L} \bar{\hat{\chi}}_r(\omega_s + \ell\Delta\omega). \tag{7}$$

This weighting factor is defined in a narrow frequency window $\omega_s - L'\Delta\omega$, $\omega_s + L'\Delta\omega$ with the center frequency ω_s and half the window width L' on each side. It is assumed that the "weighted" spectral density function $SW(\omega_s, \omega_k)$ can be substituted by a theoretical model function according to equation

$$SW(\omega_s, \omega_k) = \sum_{r=1}^{R} G_{rs} \sum_{n=1}^{2N} \frac{C_{rn}}{i\omega_k - \lambda_n} \left[1 - e^{(\lambda_n - i\omega_k) T_{max}} \right] . \tag{8}$$

The eigenvalues $\lambda_n = -\zeta_n \omega_n + i\Omega_n$ are extracted in a straightforward procedure. The advantage of the subroutine EIGVAL lies in the exploitation of the orthogonality of modes which intensifies the contribution to the "weighted" spectral density function of modes actually being identified.

The subroutine EIGVAL has a very fine frequency resolution (smallest eigenfrequency difference of two identifiable modes) equal to approx. $(\Omega_{n+1} - \Omega_n)/\Omega_n = (\zeta_{n+1} + \zeta_n)/6$. The frequency resolution of the subroutine EIGEST is approx. five times poorer. However, the corresponding computational effort (CPU-time) is approx. twenty times less than with the subroutine EIGVAL. Due to this advantage, the subroutine EIGEST is preferably used to analyse systems with distinctly separated natural frequencies in the spectrum. Systems with closely spaced eigenfrequencies or multiple modes (doublets, triplets), frequently occurring in the shell structures, must be analysed with the subroutine EIGVAL.

Subroutines AMPLIT 1 and AMPLIT 2

These subroutines are used to calculate the initial values C_{rn}. This takes place in a second evaluation step when the set of 2N complex eigenvalues λ_n is already known. The complete set of RK input values $\hat{X}_{rk} = \hat{X}_r(\omega_k)$ is replaced by a reduced set of RN smaller groups, related to N natural frequencies Ω_n. Each group contains 2M values $\hat{X}_r(\omega_{n,m})$ $(m = \pm 1, \pm 2, \ldots, \pm M)$ where M denotes a variable input parameter $(M \geq 2)$. One half of the frequency values $\omega_{n,m}$ is located to the left, the other half to the right of Ω_n on the frequency axis. Substitution of $\omega_{n,m}$ in eq. (3) yields the system of 2RNM equations

$$\hat{X}_r(\omega_{\eta,m}, T_{max}) = \sum_{n=1}^{2N} \frac{C_{rn}}{i\omega_{\eta,m} - \lambda_n} \left[1 - e^{(\lambda_n - i\omega_{\eta,m}) T_{max}} \right] \tag{9}$$

where $\omega_{n,m}$ has been exchanged for $\omega_{\eta,m}$ to indicate that $\omega_{\eta,m}$ is fixed (η corresponds to n, but does not take part in the summation). Application of the least square criterion to $\hat{X}_r(\omega_{\eta,m}, T_{max})$ -estimates yields a system of matrix equations used to calculate the unknown quantities C_{rn}. The subroutine AMPLIT1 is adapted to the subroutine EIGEST and AMPLIT 2 to EIGVAL. The combination of the modified subroutines EIGEST and EIGVAL offers the possibility of extracting the eigenvalues λ_n from the set of R stationary random response signals obtained as a response of the structure to the stationary random pressure field generated, e.g. in the flow of coolant.

Auxiliary subroutines

The computer code EVA is written in the PL1-computer language and utilizes the PL-MATH procedures and subroutines described in the procedure library /_12_7. In addition to these procedures and to the four extraction subroutines summarized above, several additional auxiliary subroutines are used. The subroutine DAT3PUT is used to filter and reduce the original input data sets before they are stored in the working area of the computer memory. The subroutine REKON is used to reconstruct the input signals $x_r(t)$ from the extracted λ_n- and C_{rn}-values;

the comparison of the original with the reconstructed input signals is used to assess the achieved grade of system identification. The subroutine MODPLT is used to approximate the subset of C_{rn}-values, related to measuring positions arranged on a fixed perimeter of axi-symmetric structures (shells), by an auxiliary harmonic function having a finite number of simple cosine waves on the given perimeter.

3. Comparative modal survey of a vertical cylindrical shell partly filled with water

Within the framework of experimental verification of the coupled fluid/structure-interaction code SING-S [13] a detailed modal survey of the vertical cylindrical shell was performed [14]. The sectional view of the stainless-steel test cylinder used to perform the corresponding experiments and its principal dimensions are given in fig. 1. Four lead blocks weighting 47 kg each were attached on the upper flange to increase the inertial mass of the system. First the test cylinder was empty and later it was partly filled with water; the corresponding water level is 1350 mm. The cylinder was installed on a fastening plate approx. 3.3 tons in weight and instrumented with piezoresistive accelerometers fed by 5 kHz-carrier amplifiers. A complete modal survey of the empty test cylinder performed in the frequency range 0-355 Hz yielded 38 modes identified. The modal survey of the test cylinder partly filled with water revealed 55 modes. The complete set of the corresponding eigenfrequencies, mode shapes and critical damping ratios are documented in report [14]. Within the framework of these experiments, a comparative modal survey of the test cylinder partly filled with water was performed in the frequency range 0-100 Hz. In this survey the following four experimental procedures were used:

a) step excitation of the structure with a snapback device, evaluation of the transient response signals with the computer code EVA;
b) harmonic excitation of the structure with an electrodynamic shaker in a decay mode operation, evaluation of the transient response signals with the computer code EVA;
c) stationary random excitation of the structure with an electrodynamic shaker, evaluation of the stationary response signals with the computer code MODAMS [8];
d) impulse excitation of the structure with an impulse hammer, evaluation of the transient response signals with the computer code MODAMS.

The methods above refer to four experimental procedures, type I through IV, discussed in the Introduction. The results of the comparative modal survey are illustrated in figs. 2 to 4.

Figure 2 shows identified natural frequencies of the test cylinder plotted over the circumferential order ν of the corresponding modes. A total of 15 modes were identified in the given frequency range: three variants (triplet) of the fundamental beam mode as well as six shell mode doublets. The three variants of the fundamental beam mode triplet have the eigenfrequencies (mean values) $\overline{f}$ = 20.4, 25.8 and 31.0 Hz; the critical damping ratio $\overline{\zeta}$ = 3.8, 1.9 and 1.93 % and the azimuthal angle $\overline{\psi}$ (measured clockwise from the excitation plane) = 3.0, 45.0 and -47.5°, respectively. The eigenfrequency values identified by the four experimental procedures mentioned above are designated by the symbols ○, ◇, □ and △ . Figure 2 is an illustration of the excellent mutual agreement of these eigenfrequency estimates; the maximum scatter found in the case of the 31 Hz-mode equals $\pm$1.8 %. The shell mode doublets identified have the axial order μ = 1 and the circumferential orders ν from 2 to 7. The agreement between individual eigenfrequency estimates obtained by different experimental procedures is even better here than in case of the fundamental beam mode triplet; the corresponding scatter is limited to ±0.4 %. The individual eigenfrequencies of several doublets are extremely closely spaced in the frequency spectrum. The smallest relative eigenfrequency difference of approx. 0.1 % has the shell mode doublet of first axial and seventh circumferential order. This phenomenon necessitated the use of the zoom-subroutine when procedure IV was applied. Theoretical natural frequency estimates calculated with the computer code SINGS [13] are designated by the symbols -●-. Generally, a good agreement can be observed between theoretical and experimental values; substantial deviations occur for the eigenfrequencies of the fundamental beam mode triplet. It was found that the experimental value of the fundamental frequency strongly depends upon the rigidity of fastening the bottom plate of the test cylinder to the corresponding base. An older modal survey of the test cylinder whose bottom plate had been embedded into a massive concrete base [13] yielded the fundamental frequency value equal to 45.5 Hz (symbol x in fig. 2); this value is considerably closer to the calculated value of the 49.7 Hz than the corresponding experimental values in fig. 2. On the other hand, the influence of this boundary condition on the eigenfrequency values of the shell modes is considerably smaller and practically vanishes in case of shell modes of higher circumferential orders ($\nu \geq 5$).

Experimental values of the critical damping ratio ζ are plotted over the circumferential order ν in fig. 3. The scatter of values extracted by different experimental procedures is considerably larger than in fig. 2, especially in case of small absolute ζ-values. Two different mechanisms are believed to be the cause of this phenomenon. One of them is due to the overwhelming numerical inferiority of the real part (damping term) $\zeta_n \omega_n$ in comparison with the imaginary part Ω_n in eq. (2). This disproportion may lead to numerical difficulties and contribute to scatter in case of extremely small ζ-values. The other relates to the strong variability of the contribution by structural damping due to variable prestressing of mutually rubbing surfaces (such as flanges). Depending on variations of the ambient temperature, it can deteriorate the reproducibility of the ζ-values especially when the respective measurements are performed on different days.

The intercomparison of two mode shapes identified by all four experimental procedures used is presented in fig. 4. Both plots represent a horizontal section through the shell mode of first axial and third circumferential order, measured 1000 mm above the bottom of the shell. The upper plot relates to the low-frequency (f = 70.66 Hz), the lower one to the high-frequency (f = 72.2 Hz) variant of the doublet. A relative good consistency of the corresponding global mode shapes can be observed, regardless of a strong scatter of individual points. Another interesting feature of fig. 4 is the presence of a distinct phase shift of approx. 23° between the two mode shapes represented. The causes of this phenomenon are two different symmetry axis of the square flanges of the test cylinder as well as the unsymmetry caused in manufacturing the test cylinder shell [14].

4. Concluding remarks

The comparative modal survey described in the proceeding section revealed a good consistency of the modal data extracted by all four experimental procedures used. Indisputable advantages of the relaxation technique in combination with the computer code EVA are the minimum test requirements regarding the duration and vibrational excitation as well as the high-frequency resolution necessary to identify the multiple modes occurring in typical shell structures investigated in nuclear safety. However, an efficient computer system is needed to perform the necessary calculations. To illustrate the computer requirements the following typical data obtained in the current surveys are presented here: number of accelerometer channels used R = 40, number of modes extracted N = 50; the corresponding sample size equals 5000 data/channel, direct access memory requirements are 4300 kbytes, CPU-time = 25 min; these data are characteristic of the computer system IBM 3033.

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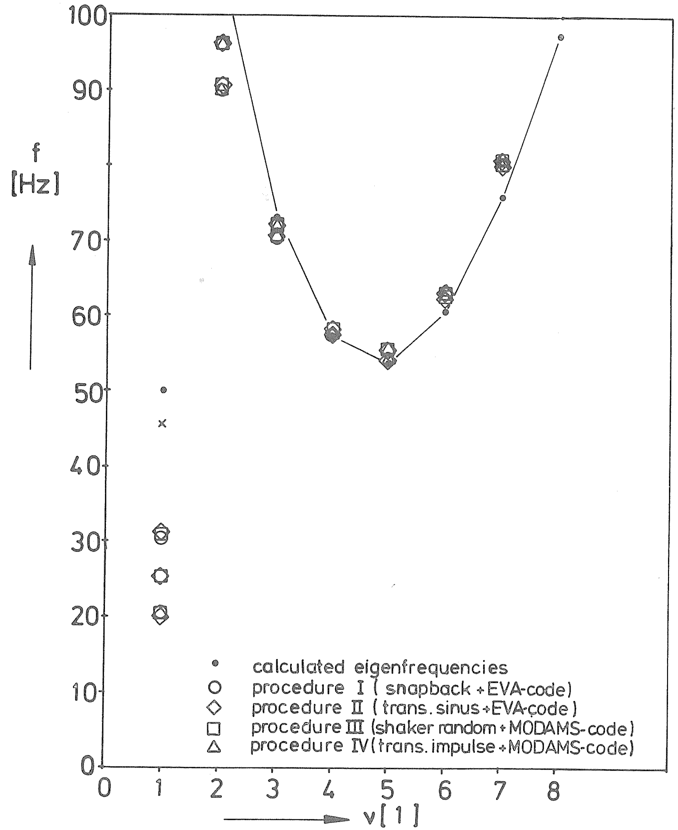

Fig. 1: Test cylinder

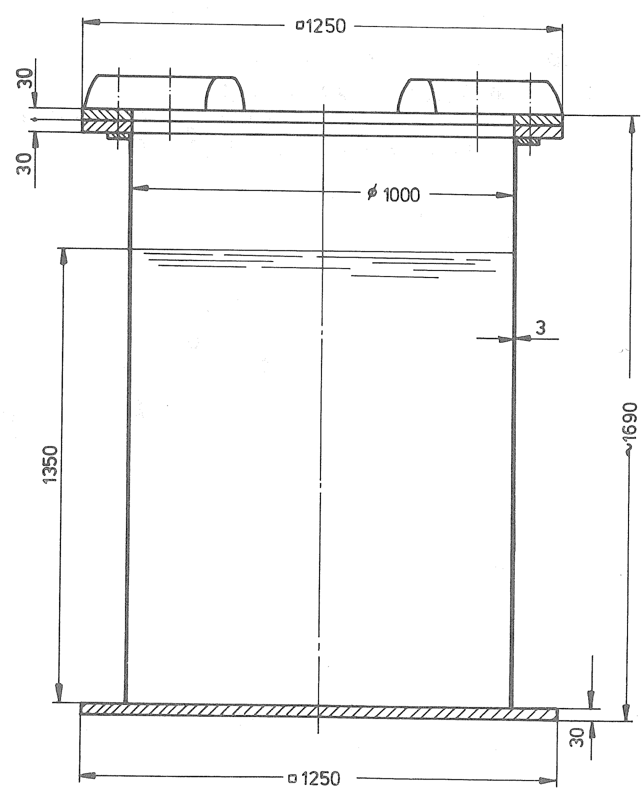

Fig. 2: Calculated and extracted natural frequency values of the test cylinder

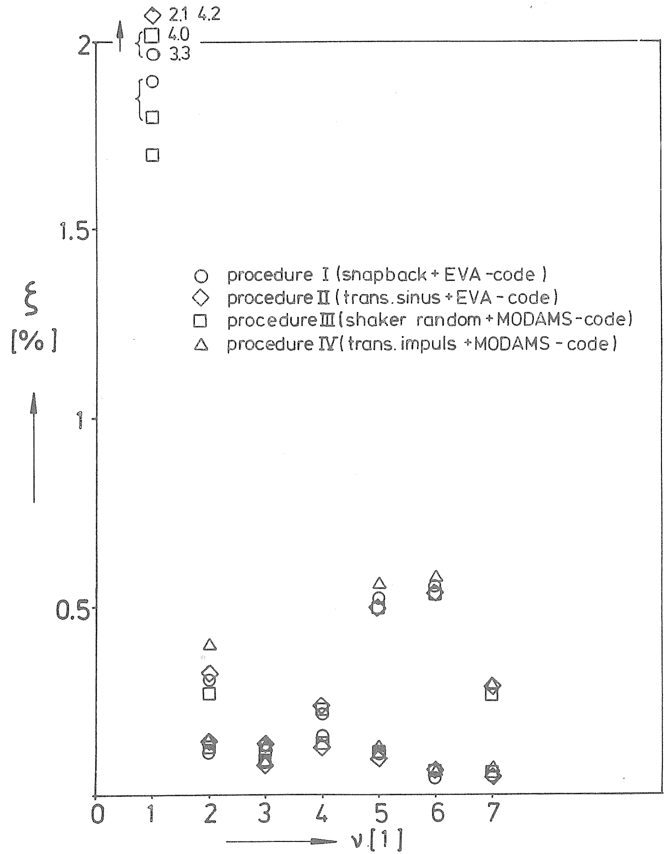

Fig. 3: Critical damping ratios extracted

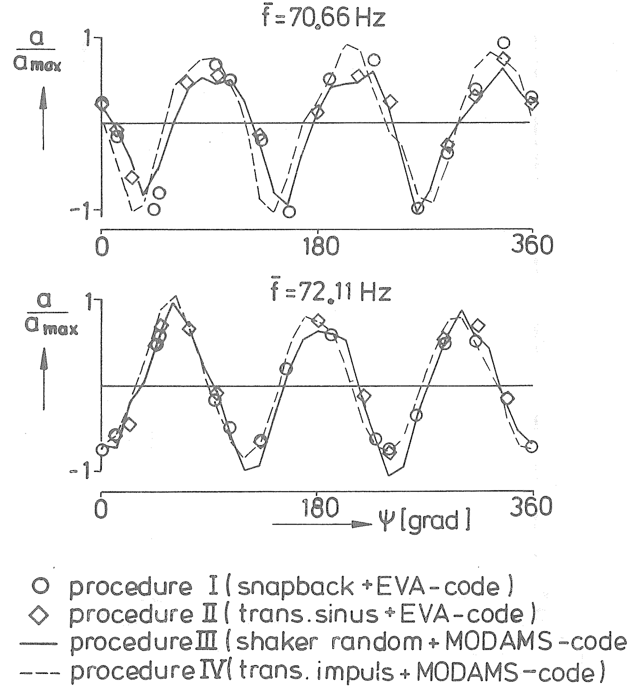

Fig. 4: Comparison of two typical mode shapes identified by four experimental procedures