

Finite Element Validation of the Method Used to Estimate the Creep Crack Tip Charactering Parameters for Combined Primary and Secondary Stresses

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ABSTRACT

In this work, externally circumferentially cracked cylinders of elastic and elastic-plastic materials subjected to residual stress or combined primary and residual stresses under power law creep conditions are analysed using the finite element (FE) method. All the FE cases are reanalysed using the newly extended $J(t)$ and $C(t)$ estimation scheme of Ainsworth and Dean and the predicted $J(t)$ and $C(t)$ values are compared with the FE results. The results show that the $J(t)$ estimate is accurate for elastic-plastic material for both pure secondary stress and combined loading provided an appropriate value of the elastic follow up factor, Z , is used and the initial reference stress estimate is reasonably accurate. For elastic material, the short-term $J(t)$ for residual stresses only may be under estimated because of the strong elastic follow-up at the beginning of creep. The $C(t)$ estimate is conservative for residual stress only and may be slightly non-conservative for combined loading in short-term creep when using a factor of $\Phi = 1$ in the estimation formula. It is found that using a varying Φ factor in the $C(t)$ estimation can improve the accuracy of the equation for all the cases considered.

INTRODUCTION

Creep crack growth has to be considered in a residual life assessment using R5 [1] for defective components operating at elevated temperature. The rate of creep crack growth is correlated with an appropriate parameter, either $C(t)$, a function of time, t , or C^* (see e.g. [1,2]). Both $C(t)$ and C^* are creep crack tip parameters measuring the amplitude of the singularity of the creep crack tip fields. The former is appropriate for describing transient creep whereas the latter is applied to steady-state creep. $C(t)$ approaches C^* as t increases. In R5 Volume 4/5 [1], advice on the evaluation of $C(t)$ and C^* , based on the reference stress method, is given in Appendix A2 for primary load under elastic-plastic creep conditions and in Appendix A3 for combined loading and elastic creep behaviour. At the beginning of creep, the crack tip fields are actually controlled by J , an elastic-plastic fracture mechanics parameter. Under creep conditions, J in the near crack tip region may become path-dependent and the far-field J is time-dependent, even when the load applied to the defective component remains constant during creep. J is then no longer a crack tip parameter and, therefore, no advice on its evaluation is given in R5 [1]. However, J is closely related to $C(t)$ under creep conditions and it is implicitly evaluated in the procedure. Recent work by Ainsworth and Dean [3] has extended the R5 Appendix A3 method [1] for combined loading to elastic-plastic creep behaviour. The newly extended method reduces to the approach in the current Appendix A3 of R5 Volume 4/5 [1] for elastic materials and to the method in the current Appendix A2 of R5 Volume 4/5 for primary load only.

In this work, elastic and elastic-plastic finite element (FE) analyses are first performed for an externally full-circumferentially cracked cylinder subjected to residual stresses or combined primary and residual stresses under power law creep conditions. All cases analysed using the FE method are then reanalysed using the newly extended methods given in [3] and the predicted $J(t)$ and $C(t)$ values are then compared with the FE results.

BACKGROUND

In this section, the theory for $C(t)$ and $J(t)$ prediction as extended by Ainsworth and Dean [3], is briefly reviewed. The evaluation of the current reference stress is also discussed.

Evaluation of $J(t)$ and $C(t)$

The $J(t)$ and $C(t)$ estimation scheme developed by Ainsworth & Dean [3] is based on materials with power law strain hardening stress-strain relationship and power law creep behaviour, which may be expressed as follows in Eqs. (1) and (2), respectively.

$$\varepsilon = \frac{\sigma}{E} + B \sigma^m \quad (1)$$

$$\dot{\varepsilon}_c = D \sigma^n \quad (2)$$

where σ and \mathcal{E} are stress and total strain, respectively, B , D , m and n are material constants, E is Young's modulus and $\dot{\epsilon}_c$ is the creep strain rate.

For a general case of combined primary and secondary loading, J can then be expressed as a function of time, t , by [3]

$$\frac{J}{J_0} = \frac{\sigma_{ref}}{\sigma_{ref}^0} \left[1 + \frac{\dot{\epsilon}_c (\sigma_{ref}^p) t}{\epsilon_{ref}^0} - (Z-1) \frac{(\sigma_{ref} - \sigma_{ref}^0)}{E \epsilon_{ref}^0} \right] \quad (3)$$

where σ_{ref} and σ_{ref}^p are the total reference stress and the reference stress for primary load only, respectively, σ_{ref}^0 and ϵ_{ref}^0 are the total reference stress and strain at $t = 0$, respectively, $J = J_0$ at $t = 0$, $Z \geq 1$ is the elastic follow-up factor, and $\dot{\epsilon}_c (\sigma_{ref}^p)$ is the creep rate under the primary reference stress. Note that the current reference stress, σ_{ref} , is generally a function of time. For pure primary load, $\sigma_{ref} \equiv \sigma_{ref}^p$ and, using the reference stress approximations to J and C^* , Eq. (3) reduces to the J estimate for primary stress given in [4].

For combined primary and secondary loading, the following expression for $C(t)$ was obtained under an assumption of $m = n$ [3],

$$\frac{C(t)}{C_0^*} = \frac{\left(\frac{\sigma_{ref}}{\sigma_{ref}^0} \right)^{n+1} \left(\frac{\epsilon_{ref}}{\epsilon_{ref}^0} \right)^{n+1}}{\Phi \left[\left(\frac{\epsilon_{ref}}{\epsilon_{ref}^0} \right)^{n+1} - 1 \right] + \left[1 - \frac{\sigma_{ref}^0}{E \epsilon_{ref}^0} \right]} \quad (4)$$

where ϵ_{ref} is the total reference strain at time t and reference stress $\sigma_{ref}(t)$ and

$$C_0^* = \sigma_{ref}^0 \dot{\epsilon}_{ref}^0 R' \quad (5)$$

with

$$R' = \left(\frac{K^p}{\sigma_{ref}^p} \right)^2 \quad (6)$$

where K^p is the elastic stress intensity factor (SIF) for primary load. The factor Φ in Eq. (4) is a constant in the range

$$1 \leq \Phi \leq \frac{Z}{Z-1} \quad (8)$$

This factor will be further discussed below under "The Φ Factor". In Eq. (5), C_0^* is used as a normalisation rather than C^* . This change of normalisation can avoid problems when the primary stress is zero.

Relaxation of the Reference Stress for Combined Loading

In general, the reference stress for combined loading is a function of time and crack length, a , that is $\sigma_{ref}(a/w, t)$, where w is the width of the cracked body considered. When the crack growth is not significant, the basic reference stress rate expression in Section A3.4.3 of R5 Volume 4/5 [1] still applies, that is

$$\dot{\sigma}_{ref} = -\frac{E}{Z} \left(\dot{\epsilon}_c (\sigma_{ref}, \epsilon_c) - \dot{\epsilon}_c (\sigma_{ref}^p, \epsilon_c) \right) \quad (9)$$

where $\dot{\sigma}_{ref}$ is the reference stress rate and $\dot{\epsilon}_c(\sigma_{ref}, \epsilon_c)$ and $\dot{\epsilon}_c(\sigma_{ref}^p, \epsilon_c)$ are the creep strain rates for total and primary reference stresses, respectively, evaluated at the current creep strain, ϵ_c . The reference stress may then be obtained by solving Eq. (9) for given initial conditions, $\sigma_{ref} = \sigma_{ref}^0$ at $t = 0$.

The Φ Factor

For power law creep materials, the Φ factor may be expressed, according to [3], as

$$\Phi = \frac{Z}{\left(\frac{\sigma_{ref}^p}{\sigma_{ref}}\right)^n + (Z-1)} \quad (10)$$

From Eq. (10), the value of Φ is bounded above by $\Phi = Z/(Z-1)$ since $(\sigma_{ref}^p/\sigma_{ref})^n \geq 0$ and below by $\Phi = 1$ since $\sigma_{ref} \geq \sigma_{ref}^p$. It is obvious that, for the cases of secondary stress dominance, $\sigma_{ref} \gg \sigma_{ref}^p$, Φ is close to $Z/(Z-1)$ and assuming $\Phi = 1$ can lead to a very conservative prediction of $C(t)$ from Eq. (4) when the Z factor is close to unity.

Determination of the Initial Reference Stress and Reference Strain

Several methods may be used to evaluate the initial reference stress, σ_{ref}^0 , in practice, depending on the data available. In this paper, the initial reference stress is calculated directly using the FE method as follows. The reference stress for combined loading may be determined by performing cracked body elastic-plastic FE analysis when the J value for the combined loading, J_{com} , is available. A cracked body FE analysis under pure mechanical load may be performed and J evaluated. Adjusting the applied mechanical load, P , until $J = J_{com}$, the reference stress for combined loading is then obtained from

$$\sigma_{ref}^0 = \frac{P(J_{com})}{P_L} \sigma_y \quad (11)$$

where $P(J_{com})$ is the load corresponding to $J = J_{com}$ and P_L is the limit load of the cracked body. The initial reference strain, ϵ_{ref}^0 , can then be obtained from the stress strain relationship of the material.

FE ANALYSES

In order to validate the methodology described in the last section, selected cases are analysed using the FE method to obtain $J(t)$ and $C(t)$. The commercial finite element package ABAQUS [5] is used in the analyses.

The Cylinder Geometry and FE Model

The geometry considered in this work is a cylinder with an externally full-circumferential crack subjected to axisymmetric loads. The dimensions of the cylinder are given in Fig. 1. The FE mesh used is shown in Fig. 2. ABAQUS type CAX4 [5] elements are used to idealise the cylinder. Only a half of the cylinder is modelled because of symmetry, with the nodes on the uncracked ligament being constrained from displacing in the axial direction. A focused mesh arrangement is employed for evaluating J and $C(t)$, with 20 element rings around the crack tip as shown in Fig. 2.

Material Properties

The stress-strain curves of the materials used in the FE analyses may be described by the following equation

$$\epsilon = \begin{cases} \sigma/E & \text{for } \sigma \leq \sigma_0 \\ \sigma/E + \alpha(\sigma - \sigma_0)^{m_0} & \text{for } \sigma_0 < \sigma \leq \sigma_y \\ \sigma/E + B\sigma^m & \text{for } \sigma > \sigma_y \end{cases} \quad (12)$$

where α and m_0 are the material constants, σ_0 is the proportional limit and σ_y is the 0.2% proof stress. Note that Eq. (12) is basically a Ramberg-Osgood type stress-strain curve (Eq. (1)) for $\sigma > \sigma_y$. However, the curve for $\sigma < \sigma_y$ is modified

to make it more representative of a real material stress-strain relationship. The material parameters used are given in Table 1. In all cases, power-law creep (Eq. (2)) is used with $n = 5$ and $D = 1.0 \times 10^{-16}$ (MPa⁻⁵/h).

Table 1 Elastic-plastic material properties used in the FE analyses

E	2×10^5 (MPa)				
ν	0.3				
m	β ((MPa) ^{-m})	α ((MPa) ^{-m_0})	σ_0 (MPa)	m_0	$\sigma_y (= \sigma_{0.2})$ (MPa)
5	6.25×10^{-15}	2.68×10^{-8}	145	2.8	200

Applied Loads

The residual stress distribution was established by first imposing an initial radial temperature variation along the whole cylinder length, and then setting the whole model to a constant temperature. The resulting uncracked body, axial residual stress distribution, $(\sigma_{yy})_{res}$, along the crack plane is shown in Fig. 3. Note that, for the given temperature distribution, the resultant residual stress distribution depends on the material properties. It is seen in Fig. 3 that the residual stress at the crack tip position is about $0.92\sigma_y$ for the elastic-plastic material. The elastic residual stress profile is similar to that for elastic-plastic material, but the amplitude is up to $1.6\sigma_y$ at the crack tip position.

The mechanical load is applied by means of a remote axial force, P , at the free end of the cylinder. The mechanical load level is measured by L_r , the ratio of applied load, P , to limit load, P_L , as follows

$$L_r = \frac{P}{P_L}. \quad (13)$$

The limit load, P_L , is estimated according to R6 [6].

The SIF for the mechanical load was evaluated using the solution given in [7] and this resulted in a geometrical parameter, $R' = 3.89a$, where a is the crack depth.

Calculation of $J(t)$ and $C(t)$

$C(t)$ is calculated by the ABAQUS [5] in-built contour integration formulation on 20 contours around the crack tip. The radii of the 2nd and the 20th contours are about $1 \times 10^{-3}a$ and $0.9a$, respectively. Values of $C(t)$ presented in this paper are those obtained on the 2nd contour. Since $C(t)$ is path-dependent in the transient creep period, the correct value should be calculated at the crack tip. However, the $C(t)$ value obtained on the first contour near the crack tip is generally not reliable because of the inaccuracy of the FE stress, strain and displacement fields in the area very close to the crack tip. When steady-state conditions are achieved, $C(t)$ becomes path-independent and equal to C^* . The ABAQUS [5] in-built J computation does not evaluate J correctly when residual stresses exist [8,9]. Accordingly, the J -integral is calculated by an alternative ABAQUS post-processing program, developed by Lei et al. [8] on 20 contours around the crack tip. J values presented in this paper are the “far-field” values, i.e. the average of all values obtained on the 10th to 20th contours because J may be path-dependent inside the creep zone.

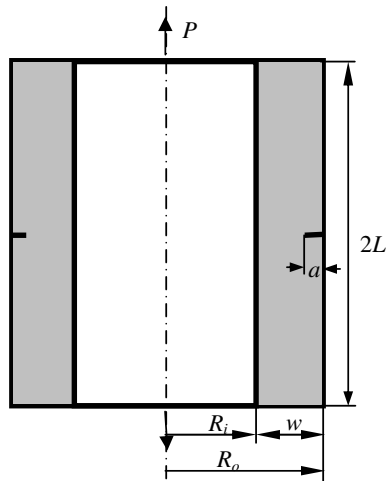


Fig. 1 Geometry and dimensions of the externally circumferentially cracked cylinder ($R_i = 225$ mm, $w = 50$ mm, $a/w = 0.2$, $L \approx 350$ mm)

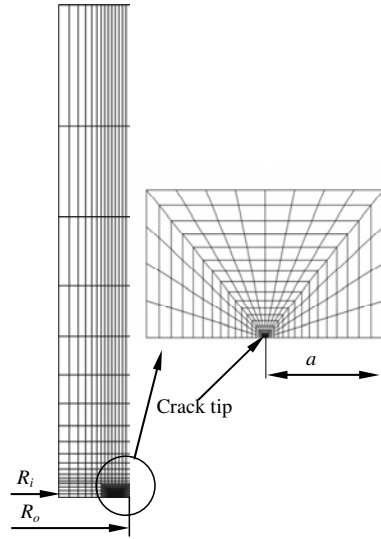
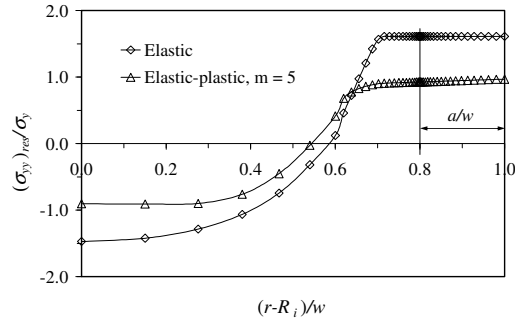


Fig. 2 Finite element mesh

Fig. 3 Uncracked body normalised axial residual stress distribution through the thickness of the cylinder at the crack plane (r is the radius variable)

ANALYSES AND RESULTS

Initial Reference Stress Estimation

The applied normalised mechanical load, L_r , and J values obtained from the FE analyses, J for the secondary stress, J^s , and for combined loading, J_{com} , are listed in Table 2. The initial reference stress has been estimated using the method described above are also given in Table 2, which are used in the prediction of $J(t)$ and $C(t)$ in this section.

Table 2 The initial reference stresses and the Z values

Case *	L_r	J^s (N/mm)	J_{com} (N/mm)	$\sigma_{ref}^0 / \sigma_y$	Z
Elastic, RS only	0	23.50	23.50	1.8295	2.1
Elastic, RS + AT	0.8	23.50	48.52	2.6307	4.2
m = 5, RS only	0	19.77	19.77	0.9955	1.9
m = 5, RS + AT	0.8	19.77	43.34	1.1165	4

* RS – Residual stress; AT – Axial tension.

Determining the Elastic Follow-up Factor, Z

The values of Z used in the calculations of this paper are selected such that the long-term FE J results are well predicted by Eq. (3). Z values obtained for all four cases considered in this report are also given in Table 2.

$J(t)$ and $C(t)$ Evaluation

The normalised $J(t)$ values obtained from the FE analyses for the four cases in Table 2 are plotted in Figs. 4(a) to (d) (labelled “FE”), against the normalised time, $t/(t_{red})_m$, where $(t_{red})_m$ is the redistribution time for a mechanical load corresponding to $J = J_0$ and the value of J is normalised by J_0 , the total J value at time $t = 0$. The normalised $J(t)$ values predicted using Eq. (3) for the four cases in Table 2 are also plotted in Figs. 4(a) – (d), labelled “Predicted”, for comparison.

Similarly, the normalised $C(t)$ values obtained from the FE analyses for the four cases are plotted in Figs. 5(a) to (d) (labelled “FE”), against the normalised time, $t/(t_{red})_m$, where $C(t)$ is normalised by $(C^*)_m$, the C^* value for a mechanical load corresponding to $J = J_0$. The normalised $C(t)$ values predicted using Eq. (4) for the four cases in Table 2 are also plotted in Figs. 5(a) – (d) for comparison. In Figs. 5(a) – (d), two predictions of $C(t)$ for each case are presented, one for $\Phi = 1$ (labelled “Predicted, $\Phi = 1$ ”) and another for Φ being evaluated using Eq. (10) (labelled “Predicted, $\Phi = \text{Eq. (10)}$ ”).

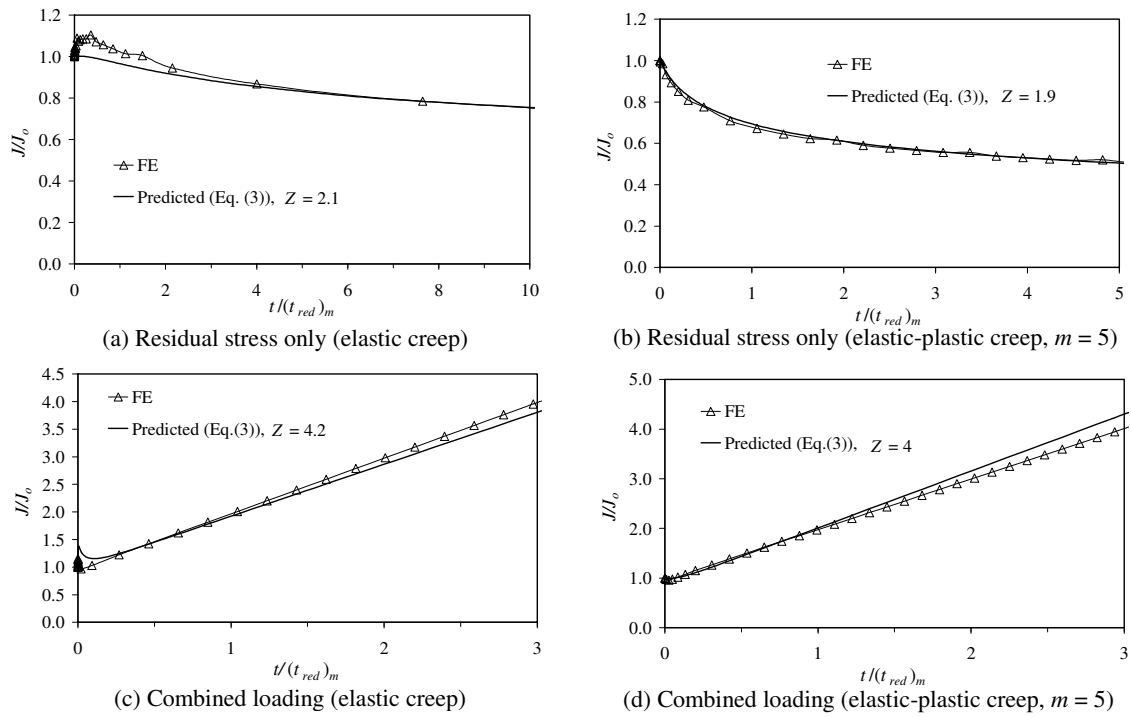


Fig. 4 Comparisons between predicted J and the FE results ($n = 5$)

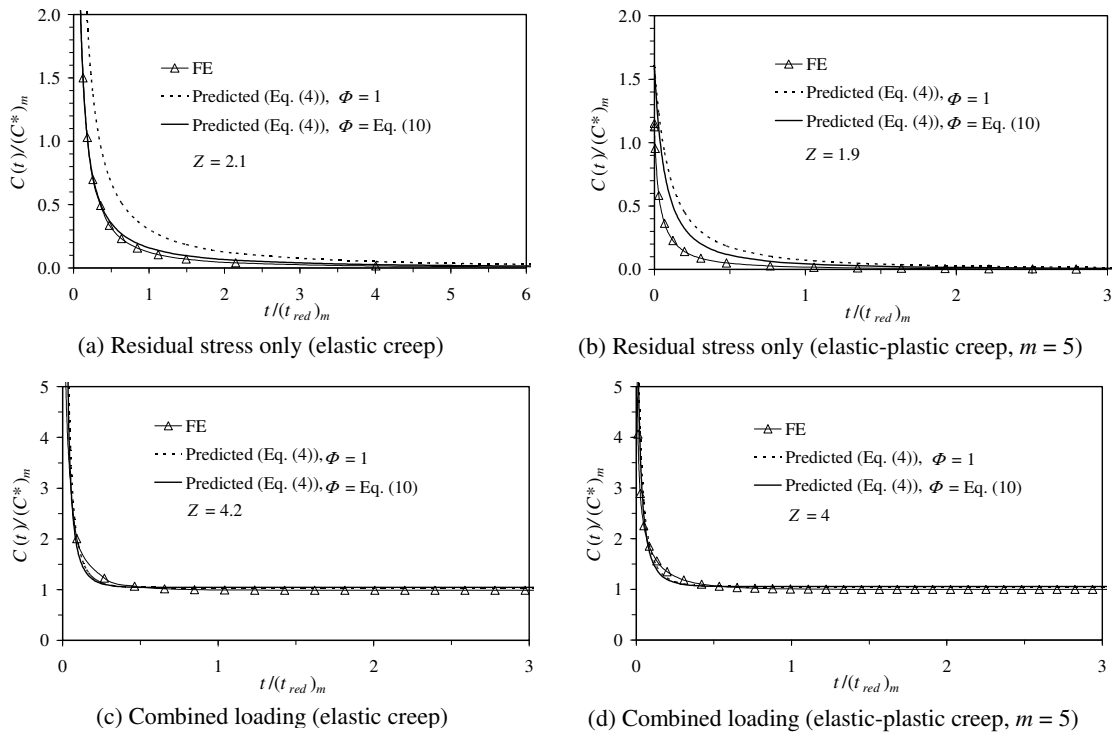


Fig. 5 Comparisons between predicted $C(t)$ and the FE results ($n = 5$)

DISCUSSION

J Prediction

From Figs. 4(a) to (d), the FE *J* results for long-term creep are accurately predicted by Eq. (3) when the *Z* values shown in Table 2 are used. This is expected because the *Z* values in Table 2 are calibrated using the FE *J* values for long-term creep. For short-term creep, the predictions using Eq. (3) are reasonably accurate for elastic-plastic creep (Figs. 4(b) and (d)) compared with the FE *J* data. However, the FE results for elastic creep under pure residual stress (Fig. 4(a)) are under-estimated. From Fig. 4(a), the normalised FE *J* shoots up at the beginning of creep and indicates a very strong elastic follow-up. Therefore, the *Z* factor for short-term creep may be much bigger than that shown in Table 2 for long-term creep. The degree of elastic follow-up reduces with increasing time. This indicates that the *Z* factor may vary during creep and should be considered as a function of time to obtain accurate *J* predictions. However, the *Z* factor is assumed to be a constant in Eqs. (9) and (3) and the possibility of varying with time has not been considered yet.

C(t) Prediction

The results for pure residual stress, Figs. 5(a) – (b), are first examined. From Figs. 5(a) and (b), the predictions of *C(t)* using Eq. (4) with $\Phi = 1$ are conservative and reasonably close to the FE results. When Φ is evaluated using Eq. (10), the predictions are still conservative, but are significantly more accurate. For the case of elastic creep, Fig. 5(a), the agreement between the FE results and the predictions is excellent when Φ is evaluated using Eq. (10).

The results for the cases of combined loading, Figs. 5(c) – (d), are next examined. Firstly, the predictions, both assuming $\Phi = 1$ and evaluating Φ from Eq. (10), are very similar to each other. This could be because the *Z* values for the cases of combined loading are large ($Z \geq 4$) and, in view of Eq. (10), the Φ values are, therefore, close to unity. For the two cases, the predictions using Eq. (4) are accurate but slightly non-conservative in the region $0.1 < t/(t_{red})_m < 0.5$ compared with the FE results.

Eq. (4) is strictly only valid for the cases with equal creep and plastic exponents, i.e. $m = n$, which was a basic assumption in its derivation. However, it has also been used to predict *C(t)* for elastic creep cases (Figs. 5(a) and (c)). From the above discussion, using this equation for the cases of elastic creep can still lead to reasonably accurate predictions of *C(t)* without increasing non-conservatism compared with the case $m = n$.

CONCLUSIONS

In this work, externally full-circumferentially cracked cylinders of elastic and elastic-plastic power law hardening (strain hardening exponent $m = 5$) materials subjected to residual stress or combined primary and residual stresses under power law creep (stress exponent $n = 5$) conditions have been analysed using the finite element (FE) method. All of the FE cases have then been reanalysed using the newly extended *J(t)* and *C(t)* estimates of Ainsworth and Dean [3]. The predicted *J(t)* and *C(t)* values have then been compared with the FE results and the conclusions drawn from this investigation are as follows.

1. The *J(t)* estimate proposed by Ainsworth and Dean is accurate for the elastic-plastic material for both pure residual stress and combined primary and residual stresses provided an appropriate value of the elastic follow up factor, *Z*, is used and the initial reference stress estimate is reasonably accurate. For elastic materials, the short-term *J(t)* for pure residual stresses may be under-estimated because of the high *Z* factor at the beginning of creep.
2. The *C(t)* estimate proposed by Ainsworth and Dean using $\Phi = 1$ is conservative for residual stress only and may be slightly non-conservative in the region of 0.1 ~ 0.5 of the redistribution time for combined loading.
3. Using a varying Φ factor, defined by Eq. (10), in the estimation of *C(t)* can improve the accuracy of the predictions for all the cases considered. However, the prediction is found to be slightly non-conservative in the region of 0.1 ~ 0.5 of the redistribution time for combined loading.
4. The work presented here has determined the value of elastic follow-up factor, *Z*, using the variation of *J* with time. Future work should consider developing an independent method for the determination of the *Z* for a cracked body.

ACKNOWLEDGMENT

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