CONDITIONAL COLLAPSE PROBABILITY UNDER GIVEN LOADS OF PLASTIC PLATES WITH RANDOM STRENGTH

A. BARATTA

Istituto di Scienza delle Costruzioni, Facoltà di Ingegneria,
Università di Napoli, Naples, Italy

SUMMARY

In the probabilistic context of structural safety, it is commonly accepted that, in order to judge the adequacy of a structure to sustain an assigned set of loads, the strength of the structure be described as a random variable, and that the significant parameter is the probability of failure.

The paper deals with the problem to calculate as small as possible upper bounds on the collapse probability of a perfectly plastic isotropic supported plate subject to uniform loading.

It is assumed that the strength of the plate (expressed by the "yield moment") in every point is a random variable, and that, possibly, different strengths are realized in different points.

A discrete model of the plate is referred to, in way that a number of "check points", in which the respect of the yield condition must be checked, are individuated, while the strength parameters (the yield moments) in these points are assumed to be a set of statistical variables whose joint probability distribution is assigned.

A first question rises by observing that it is not realistic to assume statistical independence of these random variables: if it was so, in fact, the collapse probability would approach unity by simply increasing the number of check points. It is necessary, therefore, to introduce the possibility that some degree of statistical correlation runs among the strengths in different points. Moreover, the correlative link between two points probably decreases with increasing their distance. To overcome most of the practical and theoretical difficulties connected with the introduction of the correlation, an approach is proposed leaving the usual concept of the "coefficient of linear correlation", and introducing an "extinction length" as the minimum distance between two material points whose strengths can be considered statistically independent, and assuming that the correlation varies according to an arbitrary function of the reciprocal distance of the points (but with the purpose to fit with future experimental data), in a way that the corresponding random variables can vary from the absolute identity to the perfect statistical independence.

Once the probabilistic settlement of the data has thus been given, the collapse probability is calculated by the static approach, as formulated in SMiRT-2 Paper M 7/8, by investigating a class of parabolic moment fields of the type

$$m_x = A(1 - x^2); \qquad m_y = B(1 - y^2/\beta^2); \qquad m_{xy} = Cxy \tag{1}$$

where A, B are arbitrary constants, and C is defined by the equilibrium condition. The number of couples A, B to be introduced in the expression of the collapse probability is limited to 10 for computational reasons, but these ten couples are chosen by investigating a much larger number of couples, according to the criterion to minimize the calculated collapse probability.

After discussing the computational detail, the influence of the ratio of the extinction length to the geometrical size of the plate on the collapse probability is investigated.

1. - Introduction

In some recent papers [1, 2, 3, 4], it has been proved that an upper bound on the probability of plastic collapse of any elastic-perfectly plastic structure with random strength variations can be calculated for any set of applied loads, by the Static Approach of Probabilistic Limit Analysis.

This approach is a direct consequence of the extension of the classical Static (or Equilibrium) Theorem of deterministic L. A. to the case that the local strengths of the structure are suitably described by a set of random variables, like it may be in problems of structural safety and decision. Essentially the Theorem states that the actual conditional probability of plastic collapse under a given set of applied loads $\underline{W}$, say $P_f(\underline{W})$, is not larger than the probability $P_\psi(\underline{W})$ that, assigned a set $(b_1 \dots b_n)$ of any n stress fields equilibrating the applied loads $\underline{W}$, none of them is statically admissible[(1)], i. e.

$$P_\psi(\underline{W}) = \text{Prob}\left\{ \nexists\, b_i \in (b_1, \dots, b_n) \text{ stat. adm.} \right\} \geq P_f(\underline{W}) \tag{1}$$

and $P_\psi(\underline{W})$ is referred to as a static approximation of $P_f(\underline{W})$.

It is therefore obvious that any static approximation is a safe bound on the collapse probability $P_f(\underline{W})$.

In the paper, static approximations on the plastic collapse probability of a simply-supported isotropic elastic-perfectly-plastic plate with random strength variations are calculated under uniformly distributed loads, and some basic ideas to keep account of eventual strength-correlations among the various points of the plate are proposed.

2. - Basic statements

Consider the rectangular plate of Fig. 1, of constant thickness "s", simply supported along the four edges and subjected to a uniform loading of intensity W.
Assume that the plate is made by perfectly-plastic material, and that the yield locus in any point of the plate obeys the Mises' law

$$\varphi = \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2 = \sigma_o^2 \tag{2}$$

If the simple theory of plastic bending of thin plates holds [5, 6, 7], eq. (2) can be better expressed in terms of generalized stress components (bending moments M_X, M_Y and twisting moment M_{XY})

$$\varphi = M_X^2 - M_X M_Y + M_Y^2 + 3\,M_{XY}^2 = M_o^2 \tag{3}$$

M_o being the yield moment in pure bending of the plate.
If a moment field $b = [M_X(X,Y), M_Y(X,Y), M_{XY}(X,Y)]$ is in equilibrium with the applied load, the following well known differential equation must be satisfied

$$\frac{\partial^2 M_X}{\partial X^2} + 2\frac{\partial^2 M_{XY}}{\partial X \partial Y} + \frac{\partial^2 M_Y}{\partial Y^2} = -W \tag{4}$$

with the boundary conditions

$$\begin{aligned} M_X &= 0 \quad \text{for} \quad X = \pm A \\ M_Y &= 0 \quad \text{for} \quad Y = \pm B \end{aligned} \tag{5}$$

(1) - A stress field "b" is said to be statically admissible if it does not violate the material limit yield condition in any point of the structure.

With the positions [5, 6]

$$m_x = \frac{M_X}{M_0} ; \; m_y = \frac{M_Y}{M_0} ; \; m_{xy} = \frac{M_{XY}}{M_0} \tag{6}$$

$$x = \frac{X}{A} ; \; y = \frac{Y}{A} ; \quad \beta = \frac{B}{A} ; \; w = \frac{W A^2}{M_0} \tag{7}$$

the yield condition, eq. (3), can be written in terms of non-dimensional variables

$$\varphi = m_x^2 - m_x m_y + m_y^2 + 3 m_{xy}^2 = 1 \tag{8}$$

and the equilibrium eqs. (4) - (5)

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = - w \tag{9}$$

$$m_x = 0 \quad \text{for} \quad x = \pm 1 ; \quad m_y = 0 \quad \text{for} \quad y = \pm \beta \tag{10}$$

Let "B" be the set of all generalized stress fields equilibrating the applied uniform load w. Clearly, B is the general solution of eq. (9) and boundary conditions, eqs. (10). Unfortunately, this solution is not available in closed form. Leaving out the interesting possibility to get finite-element solutions of eqs. (9)-(10), only some restrincted class of equilibrated stress fields can be individuated, and let it be the subset B_1 of B defined by assuming the following expressions for bending and twisting moments

$$m_x = C_x(1-x^2) ; \; m_y = C_y(1-y^2/\beta^2) ; \; m_{xy} = C_{xy} \, xy \tag{11}$$

C_x and C_y being arbitrary constants, while C_{xy} is related to C_x, C_y and to the load intensity w by the eq. (9) which yields

$$C_{xy} = C_x + C_y / \beta^2 - w/2 \tag{12}$$

Thus, any choose of C_x and C_y yields an equilibrated stress field of B_1. In particular, n couples (C_{xi}, C_{yi}) of values of C_x, C_y can be considered, obtaining correspondingly n moment fields $(b_1, \dots, b_n)$. equilibrating w.

Assume now that the strength of the plate cannot be deterministically individuated, but, because of various sources of uncertainty (as workmanship, working processes of materials, etc.), can only be described as a random variable, and that the realization of this random variable can also be different in different points.

The admissibility of the stresses in the generic point Q of the plate can then be expressed as follows

$$\varphi[b(Q)] = m_x^2 - m_x m_y + m_y^2 + 3 m_{xy}^2 \leq \tilde{\varphi}_0 (Q) \tag{13}$$

$\tilde{\varphi}_0(Q)$ being the realization of the random function φ_0 in the point Q.
The admissibility of the field b over the whole surface U of the plate requires then that

$$\varphi[b(Q)] \leq \tilde{\varphi}_0 (Q) \qquad \forall Q \in U \tag{14}$$

whence it apears clear that $\tilde{\varphi}_0$ (Q) expresses the stochastic variations of the strength throughout the plate.

According to the Static Theorem of Probabilistic L. A., recalled in the Introduction, a safe bound on the plastic collapse probability $P_f(w)$ under an assigned intensity of the uniform load w, can be got by calculating the probability P_ψ (w) that the set $(b_1, \dots, b_n)$ does not contain any admissible stress field, i. e. that none of the considered couples (C_{xi}, C_{yi}) yields, by eqs. (11) a moment field satisfying the ineq. (14).

Obviously, if R_ψ (w) denotes the probability that, on the contrary, at least one of the quoted couples (C_{xi}, C_{yi}) does yield an admissible field, P_ψ (w) is the complement to unity of this latter probability, i. e.

$$P_\psi (w) = 1 - R_\psi (w) \tag{15}$$

Therefore, it is immaterial to calculate P_ψ (w) or R_ψ (w).

3. - Probabilistic Limit Analysis of a square plate

3. 1. - Consider the square plate (β = 1) of Fig. 2, and assume that it is sufficient, for technical purposes, to investigate only the check-points Q_j $(j=1, \dots, m)$ individuated by the mesh shown in Fig. 2, in order to check the admissibility of any moment field b_i.

For any couples (C_{xi}, C_{yi}), denote by φ_{ij} the value that the yield function (leftward member of eq. (13))assumes in the point $Q_j(x_j, y_j)$ in connection with the moments

$$m_x = C_{xi}(1-x^2) \; ; \; m_y = C_{yi}(1-y^2) \; ; \; m_{xy} = (C_{xi}+C_{yi} - \frac{w}{2}) \, xy \tag{16}$$

Thus

$$\varphi_{ij}=C_{xi}^2(1-x_j^2)^2-C_{xi}C_{yi}(1-x_j^2)(1-y_j^2)+C_{yi}^2(1-y_j^2)^2+3(C_{xi}+C_{yi} - \frac{w}{2})^2x_j^2y_j^2 \tag{17}$$

The admissibility of the i-th moment field b_i is therefore expressed by

$$\varphi_{ij} \leq \tilde{\varphi}_{oj} \qquad \forall j \in (1, \dots, m) \tag{18}$$

($\tilde{\varphi}_{01}, \dots, \tilde{\varphi}_{0m}$) being a set of random variables expressing the limit strength of the plate in the check-points.

The probability that the i-th stress field is admissible is given by

$$R_i(w) = \text{Prob} \left\{ \varphi_{ij} \leq \tilde{\varphi}_{oj} \quad \forall j \in (1, \dots, m) \right\} \tag{19}$$

and the probability that at least one of the investigated couples (C_{xi}, C_{yi}) yields an admissible field by

$$R_\psi (w) = \text{Prob} \left\{ \exists \, i \in (1, \dots, n) : \; \varphi_{ij} \leq \tilde{\varphi}_{oj} \quad \forall j \in (1, \dots, m) \right\} \tag{20}$$

R_ψ (w) can be furtherly explicitated by putting, for any number $k \leq n$

$$R_{i_1 \dots i_k} = \text{Prob} \left\{ (\max_{\ell=(1 \dots k)} \varphi_{i_\ell j}) \leq \tilde{\varphi}_{oj} \qquad \forall j \in (1, \dots, m) \right\} \tag{21}$$

$(i_1 \dots i_k)$ being any combination of class k of the first n natural numbers. In fact, $R_{i_1 \dots i_k}$ is the probability that the moment fields $b_{i_1 \dots i_k}$ are all admissible, and by well known formulas of Theory of Probabilities

$$R_\psi (w) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{(i_1 \dots i_k)} R_{i_1 \dots i_k} \tag{22}$$

3. 2. - Assume now that the strengths in the check-points are a set of statistically indepen-

dent variables $\tilde{\varphi}_{oj}$, obeying the Normal Probability law, with equal mean values $\bar{\varphi}_{oj}$ and standard deviations $S_{\varphi j}$

$$\bar{\varphi}_{oj} = \bar{\varphi}_o = 1 ; \quad S_{\varphi j} = S_{\varphi} = 0.1 ; \quad \forall j \in (1, \ldots, m) \tag{23}$$

Because of the assumed independence, eq. (21) can be written

$$R_{i_1 \ldots i_k} = \prod_{j=1}^{m} \text{Prob} \left\{ \tilde{\varphi}_{oj} \geq \max_{\ell=(1\ldots k)} \varphi_{i_\ell j} \right\} \tag{24}$$

An important point to get the best (i.e. the smallest static approximation of $P_f(w)$ is the choose of the n couples (C_{xi}, C_{yi}) that are to be introduced into the expression, eq. (20), of R_ψ (w).

In order to investigate this question, consider the expected plate, i.e. the plate with deterministic strength equal to the expected value, eq. (23), of $\tilde{\varphi}_o$. It is easy to verify [5, 6] that in this case the best lower bound of the collapse load over the subset B_1 is always obtained for $C_x = C_y$, and is approximately equal to 5.155. Consider then the plane C_x, C_y (Fig. 3), and the line $C_x = C_y$. Numerical results show that the maximum individual reliability, eq. (19), of a stress field is got on the line $C_x = C_y$. It is therefore possible to individuate the point on this line yielding this maximum (Fig. 3a). The associated field b_1 is assumed as the first element of the set to be investigated.

As a matter of fact, and according to the general considerations formulated in [4], it is reasonable to expect that the largest reliability R_ψ (w) can be got by investigating a suitable neighborhood of b_1.

Limiting to 9 the maximum number n of stress fields that are to be checked for admissibility (1), the best static approximation under this restriction can be found by the following procedure and looking, for instance, only at square neighborhoods of b_1(Fig. 3b).

Consider a square of side Δ, with center in the point $b_1(C_{x1}, C_{y1})$, cover this square by a mesh of, say, 100 nodes, and let b_j be the j-th node (Fig. 4). Start from the point b_1, and calculate, for any b_j the probability that, simultaneously, b_j is admissible and b_1 is not. Denote it by

$$R (b_j \cap \bar{b}_1) \tag{25}$$

and put (2)

$$R_1 = R (b_1) \tag{26}$$

$$R_2 = \max_j R (b_j \cap \bar{b}_1) = R (b_{\ell_1} \cap \bar{b}_1) \tag{27}$$

Next, consider the probabilities

$$R (b_j \cap \bar{b}_1 \cap \bar{b}_{\ell_1}) \tag{28}$$

and put

$$R_3 = \max_j R(b_j \cap \bar{b}_1 \cap \bar{b}_{\ell_1}) = R(b_{\ell_2} \cap \bar{b}_1 \cap \bar{b}_{\ell_1})$$

(1) - Remember that, as can be inferred by eq. (22), the number of operations required to calculate R_ψ (w) grows like 2^n.

(2) - Obviously : $R (b_1 \cap \bar{b}_1) = 0$

and so on up to get 9 points, and a relibility

$$R_{\psi}(w) = \sum_{i=1}^{9} R_i \tag{30}$$

This result can be looked at as the best possible approximation, under the condition that n is not larger than 9, of the probability that the assigned square contains an admissible field, and must be regarded as a function of the square size

$$R_{\psi}(w, \Delta) = 1 - P_{\psi}(w, \Delta) \tag{31}$$

This function ought to have a maximum for $\Delta = \bar{\Delta}$. The best bound $P_{\psi}(w)$ can then be defined by

$$P_{\psi}(w) = 1 - R_{\psi}(w, \bar{\Delta}) \tag{32}$$

As an example, consider the case that the collapse probability must be safely bounded under the load intensity w=1. 3 . The starting point b_1, i. e. the equilibrated stress field of B_1 yielding the maximum individual reliability, is given by $C_{x1} = C_{y1} = 0.655$.

Next, consider firstly a square neigborhood of b_1 of side $\Delta_o = 0.3275 = C_{x1}/2$ (Fig. 4a). The points yielding the maximum of $R_{\psi}(w, \Delta_o)$ are marked by • in the figure , and show some tendency to be attracted by the center of the square. When the side is reduced, put $\Delta_1 = \Delta_o/2$, a new square and a new mesh are obtained, and some of the points yielding $R_{\psi}(w, \Delta_1)$, marked by ★ in the figure, result to be placed on the boundary of the square, but the calculated reliability (~ 1-O. 64 E-04) is not very different from the previous one. When the side is again reduced, put $\Delta_2 = \Delta_1/2$ (Fig. 4b), the stress fields ○ leading to $R_{\psi}(w, \Delta_2)$ are definitily centrifugated, and $R_{\psi}(w, \Delta_2)$ is reduced of about one order of magnitude with respect to $R_{\psi}(w, \Delta_1)$. The plot of $P_{\psi}(w, \Delta)$ versus $\log_2(\Delta_o/\Delta)$, shows that the minimum of $P_{\psi}(w, \Delta)$ is attained app. ly for $\Delta = \Delta_o/2$ (Fig. 5), i. e. as soon as centrifugal tendencies are exhibited by the optimal points.

4. - Basic ideas for accounting strength-correlation

Consider the case that the chosen check-points exhibit random strengths which cannot be considered statistically independent, but that it is necessary to keep into account some degree of correlation among the strengths in the different points.

Rather than approaching the question by introducing the well known coefficients of linear correlation, a simplified, though not rigorous in principle, approach can be proposed as follows.

The basic step is the calculation of the probability $R_{i_1, \dots i_k}$, eq. (21), which, if the strengths in the points Q_j are not statistically independent, cannot be simply expressed by the product at the leftward side of eq. (24). In order to express it as a product, put

$$\varphi_j = \max_{\ell=1,\dots,k} \varphi_{i_\ell j} \tag{33}$$

then, by well known formulas,

$$R_{i_1 \dots i_k} = \text{Prob}\{\varphi_1 \le \tilde{\varphi}_{o1}\} \cdot \text{Prob}\{\varphi_2 \le \tilde{\varphi}_{o2} | \varphi_1 \le \tilde{\varphi}_{o1}\} \dots \cdot \text{Prob}\{\varphi_m \le \tilde{\varphi}_{om} | \varphi_1 \le \tilde{\varphi}_{o1}, \dots, \varphi_{m-1} \le \tilde{\varphi}_{o,m-1}\} \tag{34}$$

In this expression, the quantity

$$\text{Prob}\left\{\varphi_j \le \tilde{\varphi}_{oj} / \varphi_1 \le \tilde{\varphi}_{o1}, \ldots, \varphi_{j-1} \le \tilde{\varphi}_{o,j-1}\right\} = R_{j\,|\,j-1} \qquad (35)$$

is the conditional probability that none of the considered fields ($b_1, \ldots, b_n$) violates the yield condition in the point Q_j, assumed that the same happens in the points $Q_1, \ldots, Q_{j-1}$ which precede Q_j in the chosen layout of the check-points.

Put, for any j and for any i

$$R_j = \text{Prob}\left\{\varphi_j \le \tilde{\varphi}_{oj}\right\} \; ; \qquad d_{ij} = \text{distance}\,(Q_i, Q_j) \qquad (36)$$

and consider the ratio

$$R_j / R_{j\,|\,j-1} = f_j \; ; \; f_1 = 1 \qquad (37)$$

This ratio will be, in general, a function of $\varphi_1, \ldots, \varphi_j$, of the statistical properties of strength, and of the degree of correlation among the strengths in different points. It is here postulated that this latter influence can be expressed in terms of the reciprocal distances d_{ij} of the check-points

$$R_j / R_{j\;\;j-1} = f_j\,(d_{1j}, \ldots, d_{j-1,\,j}) \qquad (38)$$

According to eq. (38), the probability $R_{i_1 \ldots i_k}$ can be expressed by

$$R_{i_1 \ldots i_k} = \left(\prod_{j=1}^{m} R_j\right) \left[\prod_{j=1}^{m} f_j(d_{1j}, \ldots, d_{j,\,j-1})\right] \qquad (39)$$

It is easy to show that eq. (38) cannot be, by itself, a satisfactory estimate of the ratio $R_j / R_{j\;\;j-1}$. In fact, put for instance m=3, and consider the probability $R_{1.\,2.\,3}$ that $\tilde{\varphi}_{oj} \geq \varphi_j \;\; \forall j = (1, 2, 3)$

$$R_{123} = R_1\, R_2\, R_3 \cdot f_2\,(d_{12}) \cdot f_3\,(d_{13}, d_{23}) \qquad (40)$$

If the layout of the points $Q_1\, Q_2\, Q_3$ is changed, take for instance $Q_2\, Q_3\, Q_1$ the same probability would be given by

$$R_{231} = R_1\, R_2\, R_3\, f_2\,(d_{23}) \cdot f_3\,(d_{21}, d_{31}) \qquad (41)$$

which is, in general, different from eq. (40). The postulate (38) can, therefore be accepted only if the layout of the check-points is determined; in this sense the dependence of the ratio on $R_{j-1}, \ldots, R_1$ can be expressed by ordering the points $Q_1, \ldots, Q_m$ in the direction of increasing R_j.

In this agreement, the probability $R_{i_1 \ldots i_k}$ is uniquely defined, and the postulate (38) is formally consistent.

In order to express function f_j, consider firstly the case j=2

$$R_2 / R_{2/1} = f_2(d_{21}) \qquad (42)$$

Assume the existance of an extinction length "d_o", i.e. of a distance beyond which the strengths of two points of the plate can be considered statistically independent, and put

$$\varrho_{ij} = \min\,(1,\; d_{ij}/d_o) \qquad (43)$$

Thus

$$R_2 / R_{2/1} = f_2\,(\varrho_{12}) \qquad (44)$$

If $\varrho_{12} = 0$, $Q_1 = Q_2$, and $f_2(0) = R_2 = R_1$. If $\varrho_{12} = 1$, $\tilde{\varphi}_{O1}$ and $\tilde{\varphi}_{O2}$ are statistically independent, and $f_2(1) = 1$. Function f_2 can be interpolated on the interval (0, 1) between these two values.

A possible expression for f may be of the type (Fig. 6)

$$f_2(\varrho_{12}) = 1 + (R_2 - 1)\,(1 - \varrho_{12})^{\alpha} \tag{45}$$

α being any positive real number. This expression of f_2 can be generalized to the case $j > 2$. Put in fact

$$\varrho_j = \prod_{\ell=1}^{j-1} \varrho_{\ell j} \tag{46}$$

$$f_j(\varrho_j) = 1+(R_j-1)(1-\varrho_j) \tag{47}$$

Then

$$R_j/R_{j\,|\,j-1} = 1 + (R_j-1)(1-\varrho_j)$$

This position is consistent with the basic requirements. In fact, if $\varrho_{\ell j}=0$ for at least one value of ℓ, $R_{j\,|\,j-1} = 1$, and $f_j(0) = R_j$, while if $\varrho_{\ell j}=1$ for any ℓ $R_{j\,|\,j-1}=R_j$, and $f_j(1) = 1$ (Fig. 6).

Finally, in Fig. 7 are quoted numerical results of the static approximation P_ψ (w) versus the extinction length d_o still for w=1. 3

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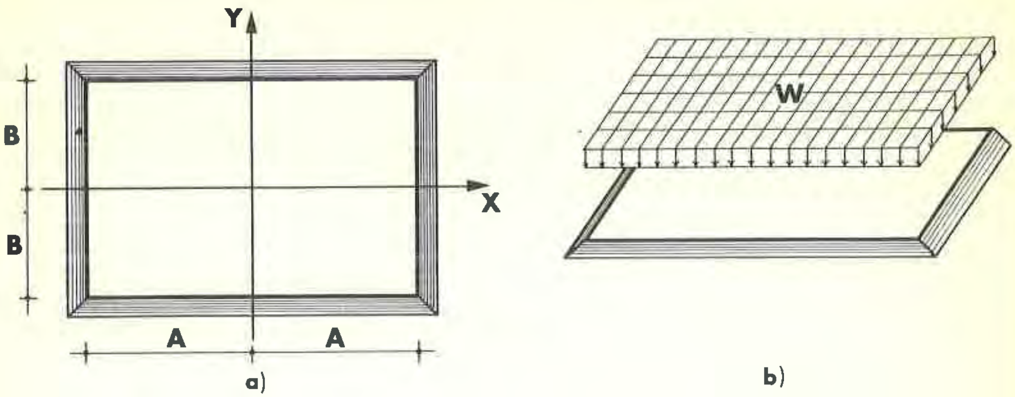

Fig. 1 - Simply supported rectangular plate under uniform loading

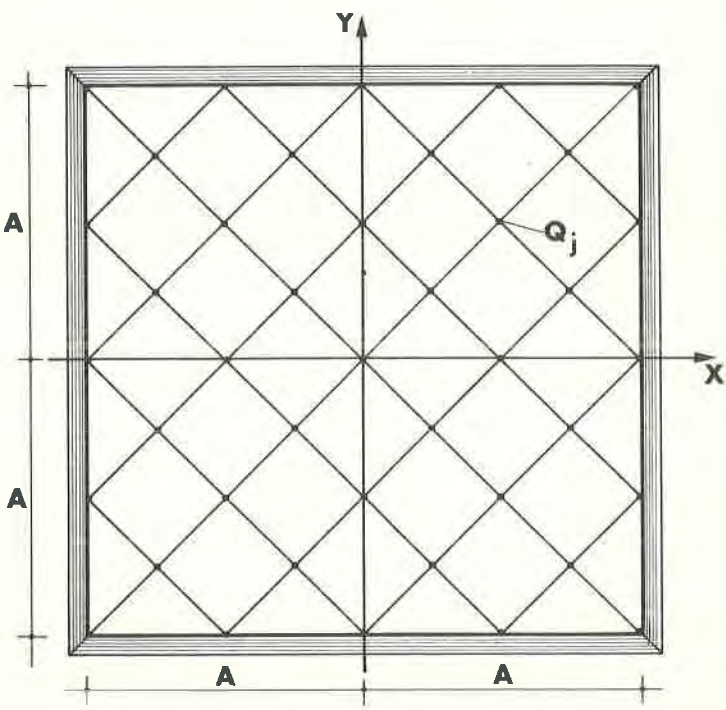

Fig. 2 - The investigated plate

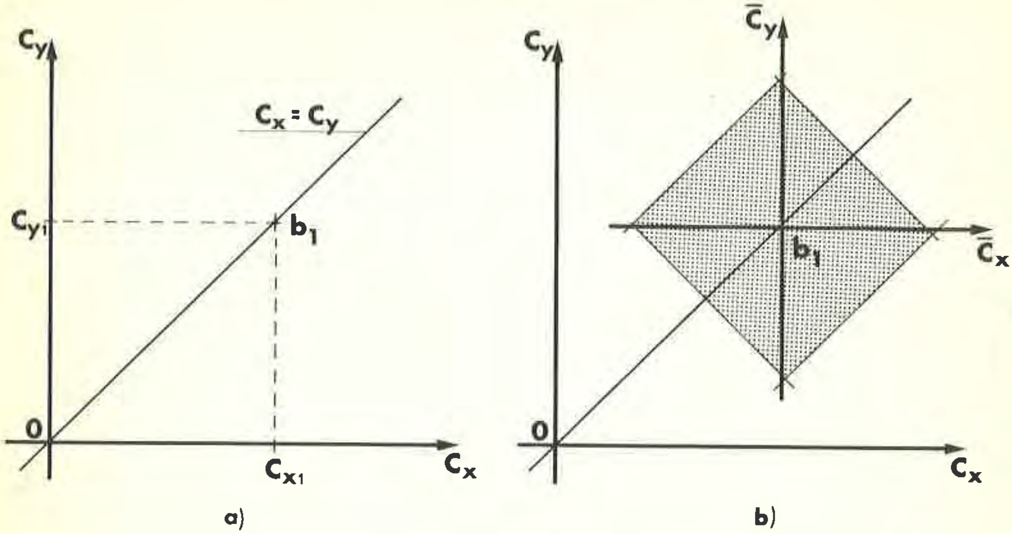

Fig. 3 - The plane of the equilibrated stress fields
$\bar{C}_x = C_x - C_{x1}$; $\bar{C}_y = C_y - C_{y1}$

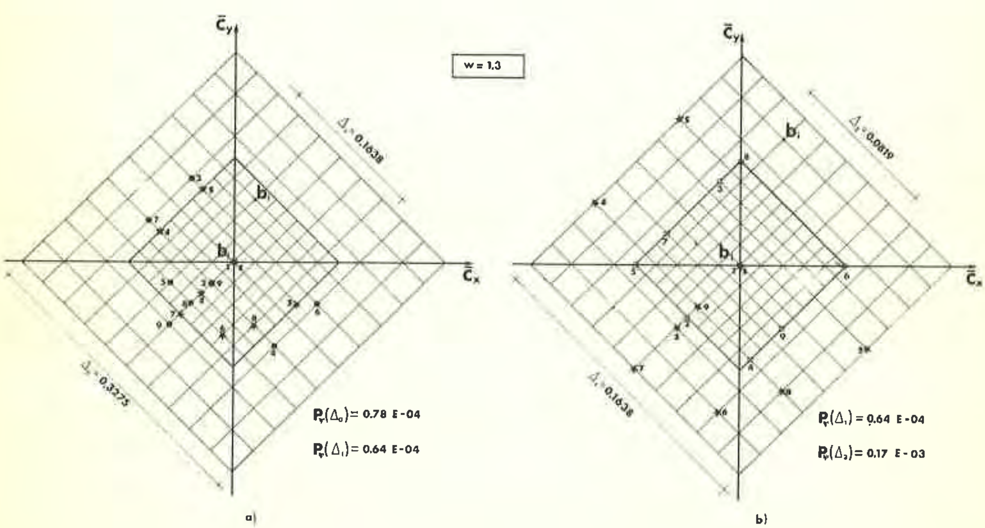

Fig. 4 - Optimal points for various values of Δ

● : $\Delta = \Delta_o$; ★: $\Delta = \Delta_1$; O : $\Delta = \Delta_2$

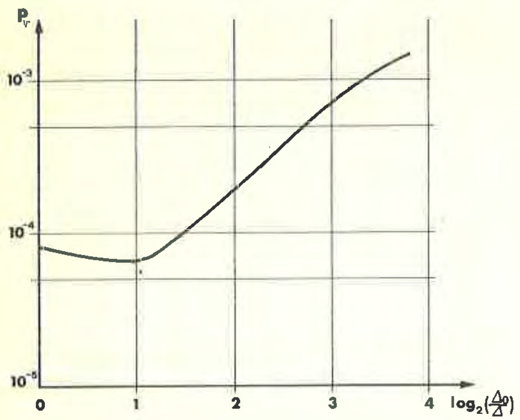

Fig. 5 - The function P_{ψ} (w, Δ)

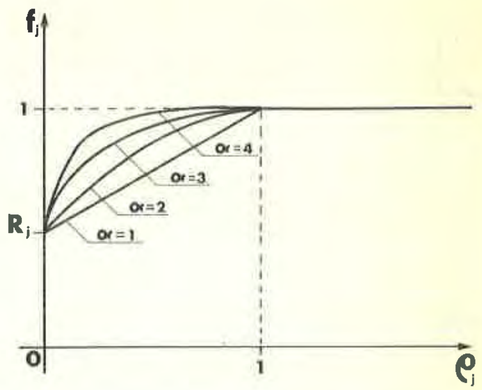

Fig. 6 - Possible approximations of the function f_j

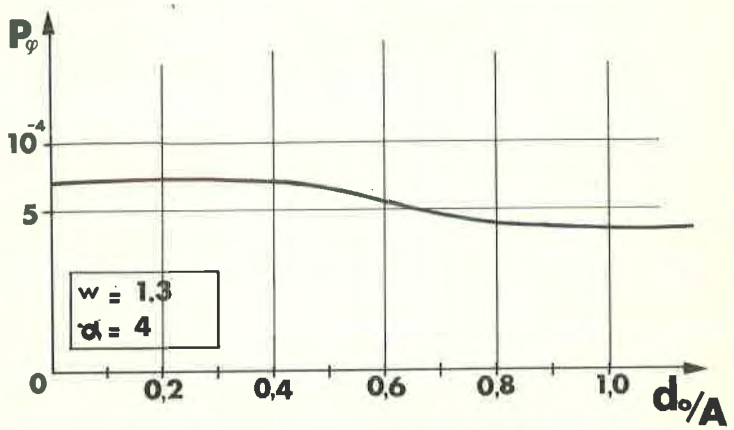

Fig. 7 - Static approximation of the collapse probability versus the extinction length d_o