GENERATION OF EMPIRICAL ENVELOPE FUNCTIONS AND SYNTHETIC TIME HISTORIES FOR STRONG GROUND MOTION

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ABSTRACT

Simulation of earthquake ground motion is necessary for estimating seismic hazard for various nuclear reactor systems. The objective of present work is to develop a procedure for computation of empirical acceleration envelopes and to simulate the ground motion from the parameters of a given earthquake. A nonlinear optimization algorithm is used to obtain the parameters of the envelope function from the rise time and duration of a given earthquake. A quadratic correlation has been developed for rise time and duration from the strong motion data base of Himalayan earthquakes. Simulation of acceleration envelopes will be carried out for various earthquakes and compared with the recorded accelerograms. The synthetic time histories are obtained by inverting Fourier Amplitude Spectrum (FAS) with random phase angles. FAS of earthquake ground motion is evaluated using a semi analytical model. This is a point source model which evaluates FAS in terms of magnitude and distance. The synthetic time histories are generated by multiplying the random time histories with envelope function. In the present work, envelope functions are generated for various Himalayan earthquakes. Synthetic time histories are generated for two earthquakes and compared with recorded accelerograms.

INTRODUCTION

Earthquake is an important consideration in the design of Nuclear Power Plants and other critical facilities. The design basis earthquake ground motion is specified in terms of peak ground acceleration (PGA), response spectral shape and acceleration time history [1]. Usually a spectrum compatible time-history of ground acceleration is specified such that the time-history generated response spectrum is compatible with a specified design response spectrum. The time-history of ground acceleration is required for time-history analysis of structures on ground and for generation of response spectra at various floor levels.

For seismic-hazard analysis of nuclear power plants, the ground motions from earthquakes of moment magnitude more than five, at distances enveloping 300 km from the source are considered [2]. In the present work, an analytical model given in the literature [2] is used to obtain FAS of the ground acceleration. This model allows the evaluation of Fourier Amplitude Spectrum (FAS) of earthquake ground motion in terms of magnitude, distance and is based on seismological considerations.

ESTIMATION OF RISE TIME AND DURATION FOR A GIVEN EARTHQUAKE

Rise time and duration are expressed as quadratic functions of magnitude and distance of a given earthquake. The correlation is given as

$$t_R = a_o + a_1 M + a_2 R + a_3 MR + a_4 M^2 + a_5 R^2 \quad (1)$$

$$t_D = b_o + b_1 M + b_2 R + b_3 MR + b_4 M^2 + b_5 R^2 \quad (2)$$

Where, a_0, a_1, a_2, a_3, a_4 & a_5 and b_0, b_1, b_2, b_3, b_4 & b_5 are constants which are evaluated from the strong motion data base of various historic earthquakes. Table 1 gives the rise time and duration for various earthquakes. The acceleration records are obtained from Indian Institute of Technology, Roorkee Strong Motion Database [4]. The rise time and duration for each earthquake are obtained from the recorded acceleration time histories. The rise time is taken as the time interval from beginning to the time corresponds to peak acceleration. The duration of earthquake is taken as the time interval between 5% of peak acceleration from both sides of peak value. These limits are considered to cover the significant portion of the recorded ground motion.

Table 1: Rise time, duration and parameters of envelope function for various earthquakes

S. No.	Name of earthquake & date	Station name	M	R (km)	Rise Time (s)	Duration (s)	α	β
1	Chamoli (14.12.2005)	CHM_20051214_070941	5.2	54.3	3.95	22.345	0.1585	0.1618
2	Chamoli (14.12.2005)	PAU_20051214_070943	5.2	97.28	7.735	26.39	0.141	0.146
3	Chamoli (14.12.2005)	UTT_20051214_071012	5.2	83.81	8.545	30.005	0.1472	0.1474
4	Uttarkashi (22.07.2007)	Roo_20070722_230251	5	151.1	14.745	55.415	0.0753	0.0892
5	Uttarakhand (25.02.2009)	CHA_20090225_040427	3.7	20.99	4.035	22.685	0.3032	0.8238
6	Uttarkashi,Uttarakhand (18.03.2009)	BAR_20090318_112351	3.3	44.69	2.36	24.485	0.1166	0.1646
7	Chamoli, Uttarakhand (15.05.2009)	CHA_20090515_184252	4.1	9.971	2.695	21	0.5221	0.6302
8	Uttarkashi,Uttarakhand (21.09.2009)	Bageshwar	4.7	135.1	14.835	64.995	0.0475	0.0667
9	Uttarkashi,Uttarakhand (21.09.2009)	Barkot	4.7	43.57	12.665	40.32	0.1013	0.1391
10	Uttarkashi,Uttarakhand (21.09.2009)	Chamoli	4.7	58.2	11.165	36.305	0.0929	0.1072
11	Uttarkashi,Uttarakhand (21.09.2009)	Garsain	4.7	105.4	14.22	53.415	0.0671	0.0672
12	Uttarkashi,Uttarakhand (21.09.2009)	Ghanshyali	4.7	66.99	11.155	62.775	0.0871	0.0955
13	Uttarkashi,Uttarakhand (21.09.2009)	Lansdown	4.7	124.6	14.65	46.15	0.0459	0.0763
14	Uttarkashi,Uttarakhand (21.09.2009)	RudraPrayag	4.7	69.08	9.685	37.565	0.0872	0.0916
15	Bageshwar (03.10.2009)	Bageshwar	4.3	22.59	4.11	55.14	0.1608	0.2202
16	Bageshwar (03.10.2009)	Champawat	4.3	76.41	14.87	63.85	0.0505	0.0854
17	Distt. Bageshwar Uttarakhand (01.05.2010)	Bageshwar	4.6	32.73	4.24	21.93	0.125	0.1514
18	Distt. Bageshwar Uttarakhand (01.05.2010)	Roorkee	3.5	76.35	23.075	76.145	0.0507	0.0592

The data of Table 1 is used to fit rise time and duration as a quadratic function of magnitude and distance of earthquake. A multi linear regression technique is used to evaluate the constants of the quadratic functions. Table 2 gives the constants of quadratic functions for rise time and duration. Fig. 1 and 2 show the bar charts of the estimated and recorded values for rise time and duration respectively.

Table 2: Constants of quadratic functions for rise time (t_R) and duration (t_D)

Constant	t_R (s)	Constant	t_D (s)
a_0	-86.6719	b_0	-377.316
a_1	33.8348	b_1	169.4168
a_2	0.9497	b_2	2.6927
a_3	-0.1759	b_3	-0.5283
a_4	-3.1277	b_4	-17.6506
a_5	-0.0001	b_5	0.0005

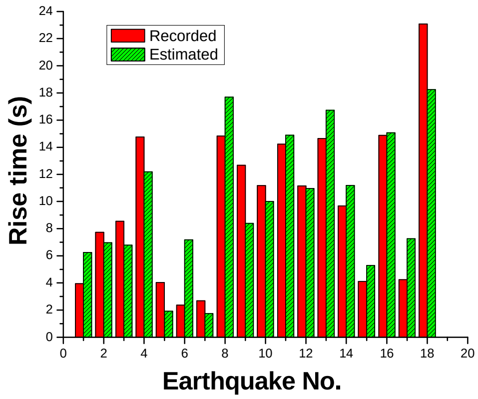

Fig. 1: Comparison of simulated and recorded rise time of various earthquakes

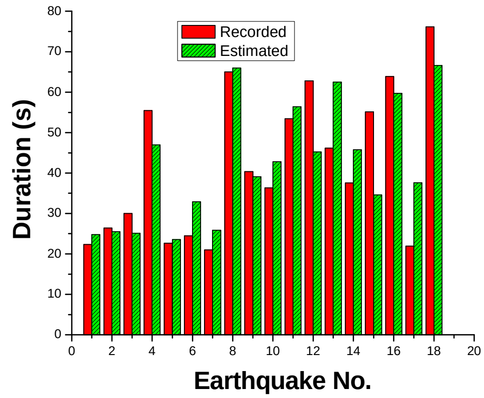

Fig. 2: Comparison of simulated and recorded duration of various earthquakes

GENERATION OF ENVELOPE FUNCTIONS

The envelope function is a function which envelopes the acceleration wave form. It is an important parameter in the generation of synthetic time histories using stochastic simulation technique. In this technique, a random time history is generated by inverting the Fourier Amplitude Spectrum (FAS) with random phase angles. Then, the time history is multiplied by an enveloping function to obtain the synthetic time history. In this study, the following envelope function E(t) is used [3].

$$E(t) = \exp(-\alpha t) - \exp(-\beta t) \tag{3}$$

Where, α and β are the parameters which are chosen as functions of Magnitude (M) and Hypocentral distance (R) of a given earthquake. By equating the first derivative of E(t) to zero, the rise time t_R can be obtained as,

$$\frac{1}{\beta-\alpha}\ln\frac{\beta}{\alpha} - t_R = 0 \tag{4}$$

The duration of earthquake is taken as the time interval between 5% of peak acceleration. The times corresponding to 5% peak acceleration are δt and (tD+ δt). By substituting in Eq. (2), we get

$$0.05\left[e^{-\alpha t_R} - e^{-\beta t_R}\right] - e^{-\alpha\delta t} + e^{-\beta\delta t} = 0 \tag{5}$$

$$0.05\left[e^{-\alpha t_R} - e^{-\beta t_R}\right] - e^{-\alpha(t_D+\delta t)} + e^{-\beta(t_D+\delta t)} = 0 \tag{6}$$

The optimum values of α and β are obtained by solving Eqs. (3, 4 and 5) in the least square sense using a nonlinear optimization algorithm. This optimization procedure requires the estimation of rise time and duration for a given earthquake in terms of magnitude and distance. Table 1 gives the parameters (α and β) for various earthquake records. Fig. 3 to 6 show the comparison of envelope functions with recorded accelerograms for various earthquakes listed in Table 1.

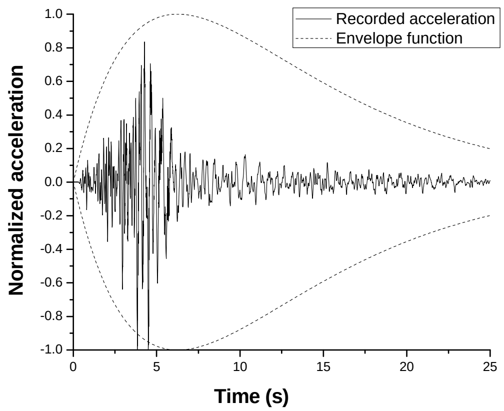

Fig. 3: Comparison of envelope function with recorded accelerogram for 1st record

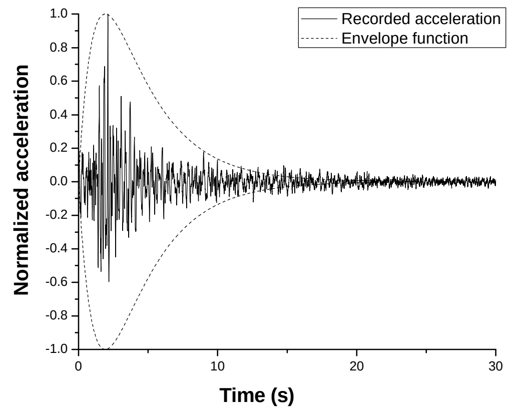

Fig. 4: Comparison of envelope function with recorded accelerogram for 5th record

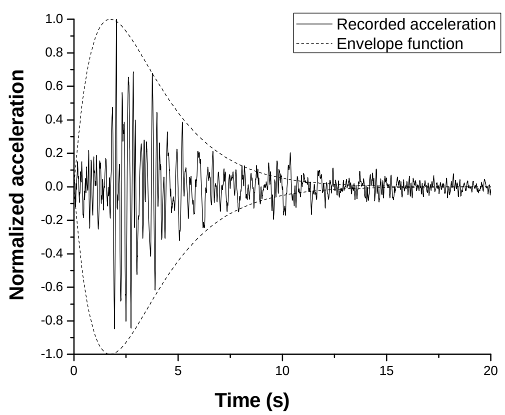

Fig. 5: Comparison of envelope function with recorded accelerogram for 7th record

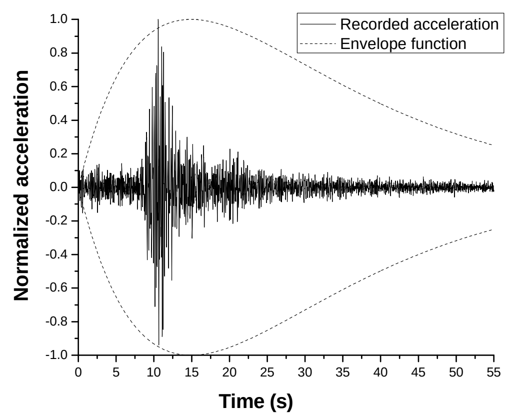

Fig. 6: Comparison of envelope function with recorded accelerogram for 11th record

SIMULATION OF EARTHQUAKE GROUND MOTION

The various steps involved in the generation of earthquake ground motion [3] are as follows. First, a random time history is generated by inverting the FAS with random phase angles distributed uniformly in (0,2π). Then, the time history is multiplied by an enveloping function to obtain the synthetic time history. FAS of the ground acceleration is computed using two different seismological models which are described below.
The model given by Atkinson [2] for the Fourier Amplitude Spectrum (FAS) of ground acceleration is,

$$A(f) = CM_o(2\pi f)^2 \exp(-\pi f \kappa_o)\exp(-\pi f R/Q\beta)/R\left[1+(f/f_o)^2\right] \quad (7)$$

where, M0 is seismic moment and f0 is corner frequency, which is given by

$$f_o = 4.9x10^6 \beta(\Delta\sigma / M_o)^{1/3} \quad (8)$$

where, $\Delta\sigma$ is stress parameter in bars, M0 is in dyne centimeters, and β is shear-wave velocity in kilometers per second. The constant C is given by,

$$C = R_{\theta\phi} FV / (4\pi\rho\beta^3) \quad (9)$$

where, $R_{\theta\phi}$ is radiation pattern (average value of 0.55 for shear waves), F is free surface amplification (2.0), V is partition onto two horizontal components (0.71), ρ is the density, and R is hypocentral distance. Various constants in the Eq. (1) are taken from the paper by Atkinsion [2]. The term $\exp(-\pi f \kappa_o)$ is a high-cut filter to account for near-surface attenuation effects, which describe the commonly observed rapid spectral decay at high frequencies, $\exp(-\pi f R / Q\beta)/R$ is the attenuation term. The quality factor, *Q(f)* is an inverse measure of anelastic attenuation. Through this equation, the spectrum is diminished with distance to account for empirically defined attenuation behaviour.

In the present work, a case study is carried out using the strong motion data recorded for the Chamoli, Uttarakhand (S.No. 7 in Table 1) and Uttarkashi, Uttarakhand (S.No. 11 in Table 1) earthquakes. These two earthquakes are considered for the present study to understand the effect of envelope function in synthetic time history generation. In the first case, the envelope function closely envelopes the recorded accelerogram (Fig. 4) where as wide envelope for the second one (Fig. 6).

The Chamoli earthquake struck the Uttarkhand on 15th May, 2009. The earthquake and aftershocks produced a rich set of strong ground motion recordings. The acceleration records are obtained from Indian Institute of Technology, Roorkee Strong Motion Database [4]. The acceleration records are also available for the Uttarkashi earthquake of 21st September, 2009. The parameters of the events are listed in Table 3.

Table 3: Parameters for Chamoli and Uttarkashi earthquakes

Parameter	Chamoli earthquake (2009)	Uttarkashi Earthquake (2009)
Magnitude	4.1	4.7
Hypocentral distance (R), *km*	9.971	105.4
Hypocenter Latitude	30.5^{o} N	30.9^{o} N
Hypocenter Longitude	79.3^{o} E	79.1^{o} E
Station name	CHA	GAR
Station Latitude	30.412^{o} N	30.051^{o} N
Station Longitude	79.32^{o} E	79.588^{o} E
Shear wave velocity (V_{s30}), *km/s*	3.25	3.25

Figs. 7 and 8 show the acceleration record and normalized Fourier Amplitude Spectra (FAS) for Chamoli earthquake in horizontal direction. The peak ground acceleration of the record is 4.45 cm/s^2. Figs. 9 and 10 show the acceleration record and normalized Fourier Amplitude Spectra (FAS) for Uttarkashi earthquake in horizontal direction. The peak ground acceleration of the record is 14 cm/s^2.

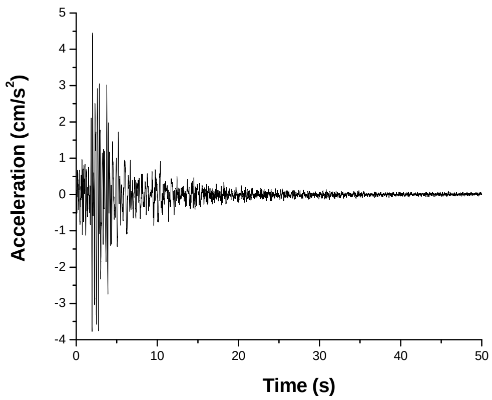

Fig. 7 Acceleration time history from station (Chamoli earthquake, 2009)

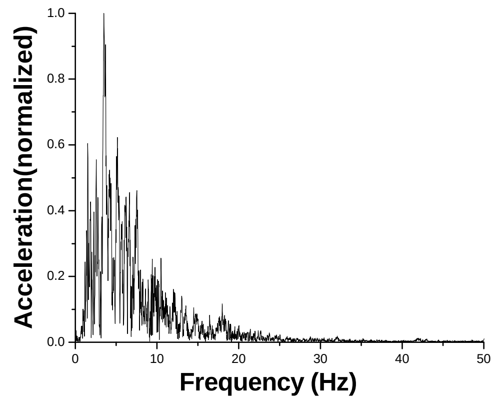

Fig. 8 Normalized FAS of the ground motion (Chamoli earthquake, 2009)

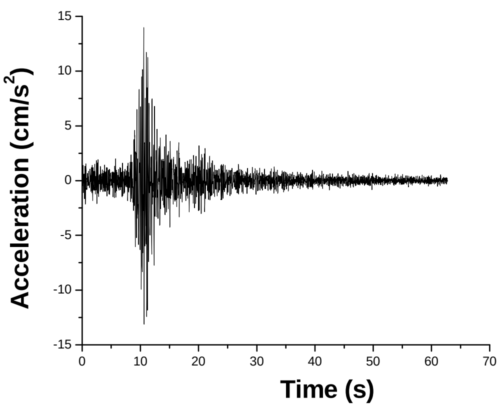

Fig. 9 Acceleration time history from station (Uttarkashi earthquake, 2009)

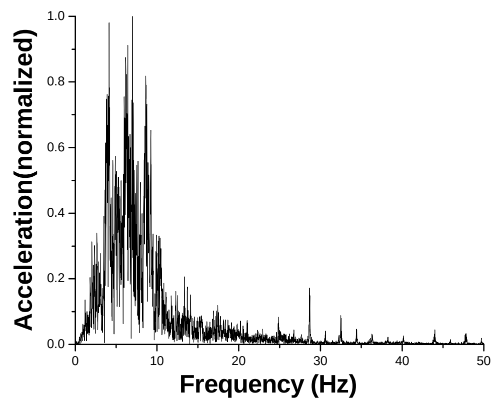

Fig. 10 Normalized FAS of the ground motion (Uttarkashi earthquake, 2009)

Stochastic simulation of Ground Motion

Fourier Amplitude Spectrum (FAS) for these earthquakes are computed using Eq. (6). A random time history is generated by inverting the FAS with random phase angles which are distributed uniformly in $(0,2\pi)$. The optimum values of the envelope function parameters are obtained by solving Eqs. (3, 4 and 5) in the least square sense using a nonlinear optimization algorithm. The envelope function parameters α and β for Chamoli earthquake are 0.5221 and 0.6302 respectively. α and β for Uttarkashi earthquake are 0.0671 and 0.0672 respectively. The random time history is multiplied by the enveloping function to obtain the synthetic time history. The resulting synthetic time histories are shown in Figs. 11 and 13 for Chamoli and Uttarkashi earthquakes respectively. The normalized FAS of the synthetic time histories are shown in Figs. 12 and 14. The frequency content of the simulated time history is closely matching with that of accelerogram for Chamoli earthquake. For the Uttarkashi earthquake, the frequency content of synthetic time history is wider as compared to that of accelerogram. The closer envelope for Chamoli earthquake has produced closer simulation while broader simulation for Uttarkashi can be attributed to wider envelope. Thus envelope function plays a vital role for ground motion simulation.

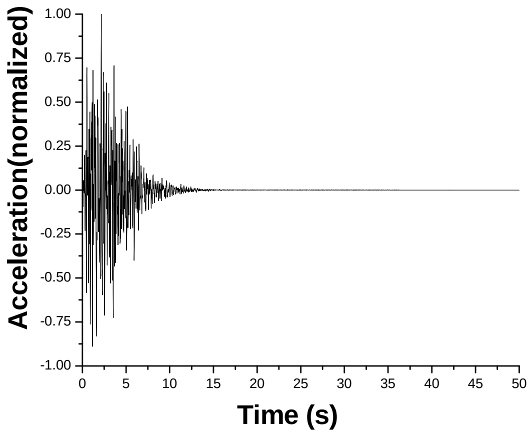

Fig. 11 Synthetic acceleration time history (Chamoli earthquake, 2009)

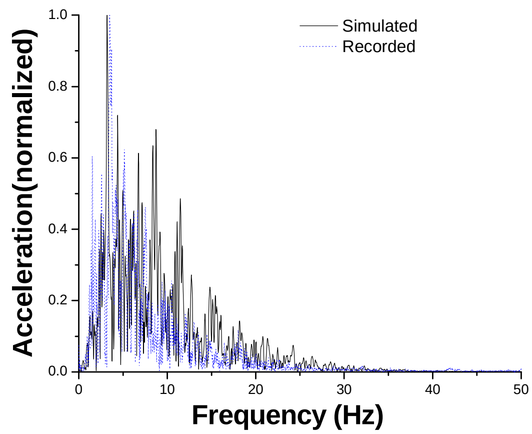

Fig. 12 Normalized FAS of the Synthetic acceleration time history (Chamoli earthquake, 2009)

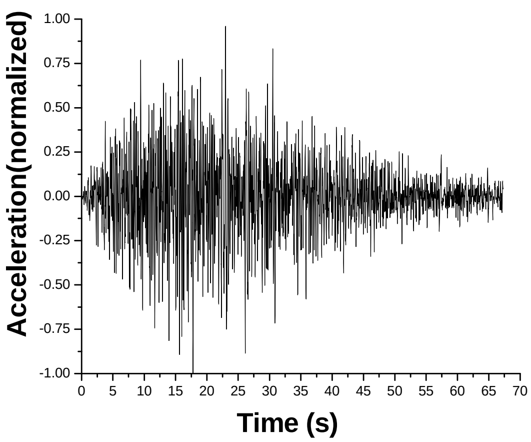

Fig. 13 Synthetic acceleration time history (Uttarkashi earthquake, 2009)

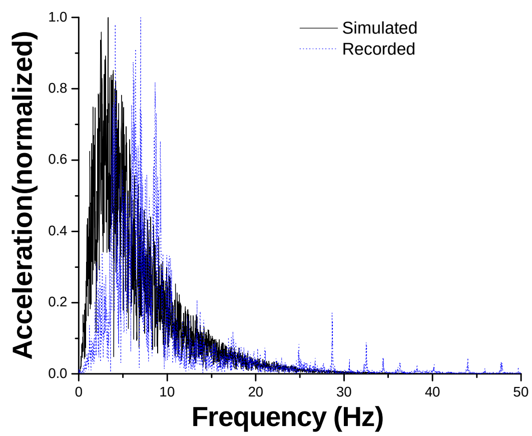

Fig. 14 Normalized FAS of the Synthetic acceleration time history (Uttarkashi earthquake, 2009)

Conclusions

In the present work, envelope functions are generated for various Himalayan earthquakes and compared with corresponding accelerograms. A nonlinear optimization algorithm is used to obtain the parameters of the envelope function from the rise time and duration of earthquake. The rise time and duration of a given earthquake are evaluated using a quadratic function of magnitude and distance. The constants of the quadratic function are obtained from the strong motion data base of Himalayan earthquakes. Semi analytical approach is used to simulate the ground motion generated due to earthquake. The random time histories are generated by inverting Fourier Amplitude Spectrum (FAS) with random phase angles distributed uniformly. FAS of earthquake ground motion is evaluated using a point source model. The synthetic time histories are generated by multiplying the random time histories with the generated envelope function. Artificial time histories are generated for Chamoli and Uttarkashi earthquakes and their FAS are compared with those of acceleration records. From the comparison, it is concluded that envelope function plays a vital role for ground motion simulation.

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