TRANSIENT THERMAL STRESSES IN A HOLLOW CIRCULAR CYLINDER UNDER CONTINUOUS POINT HEAT SOURCE

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SUMMARY

Recently the transient thermoelastic problems of a solid finite circular disk due to an instantaneous point heat source have been solved by Y. Takeuti, *Bulletin of JSME*, 10-39 (1967), 423, and T.R. Hsu, *Trans. ASME*, E, 36-1 (1969), 113, independently. The case of an instantaneous point heat source on a hollow circular disk has been solved by Y. Takeuti and N. Noda, *Nucl. Engng. Design* (to be published). On the other hand, recently transient thermal stresses on a solid finite disk due to a continuous point heat source were evaluated by T.R. Hsu, *Trans. ASME*, B, 92-2 (1970), 357.

The case of a hollow circular disk, however, introduces more difficulty into the mathematical analysis owing to the existence of the dislocation problem in a multiply-connected region.

This paper contains exact solutions for the transient temperature distribution and the associated quasi-static thermal stresses which arise in a hollow circular disk of finite radius subjected to a continuous point heat source acting on its arbitrary point. Analysis is developed by the Laplace transform. The solutions are given in the form of double infinite series, and graphical representations of the solutions in dimensionless terms are included. The solutions are applicable to problems which occur in welding engineering and nuclear technology.

1. Statement of the Problem

The problem of a cylinder subjected to a heat source (or sink) can occur in nuclear technology. The thermoelastic problems of an instantaneous or a continuous point heat source in circular disk have been of interest to many authors [1]-[4]. The present analysis is based on a hollow circular disk under a continuous point heat source at the arbitrary point in the interior part of the disk, but this result can be applied to the problem of a long hollow circular cylinder subjected to a line heat source. The analysis is approached on the basis of classical linear quasi-static uncoupled thermoelasticity, and all physical properties are regarded as independent of temperature. This paper determine the transient temperature and the hoop, radial, shear stress distributions in a hollow circular disk with a continuous point heat source of constant strength.

2. Soluiton of the Temperature Problem

The disk is thermally insulated on both its flat faces and is in contact with a surrounding fluid. The heat-conduction equation of transient state of disk takes the form:

$$\Delta\tau = (1/\kappa)\,\partial\tau/\partial t \tag{1}$$

inwhich the initial condition is

$$\tau(r,\theta,0) = 0 \tag{2}$$

where
τ : temperature rise
t : time
κ : thermal diffusivity
Δ : Laplacian $= \partial^2/\partial r^2 + r^{-1}\partial/\partial r + r^{-2}\partial^2/\partial\theta^2$

Since the convex surfaces of the disk are in contact with the surrounding fluid, the boundary conditions of eq.(1) have the form as

$$[\partial\tau/\partial r - h_a\tau]_{r=a} = 0, \quad [\partial\tau/\partial r + h_b\tau]_{r=b} = 0 \tag{3}$$

where h_a, h_b are inner and outer surface heat-transfer coefficients respectively. Further, for the hollow circular disk in the presence of a continuous point heat source of strength Q on the arbitrary point $(r_1, 0)$ of the disk, as shown in Fig.1, the general solution of the heat conduction equation can be expressed by [5]:

$$\tau(r,\theta,t) = \tau_g + \tau_1 \tag{4}$$

where τ_g satisfies both eq.(1) and the condition of

$$\lim_{R\to 0} R\frac{\partial\tau}{\partial R} = -\frac{Q}{\kappa}$$

R = distance between the source and observation points.

Above expression satisfies the finite heat energy at the source point. Integrating the Green's function of an instantaneous point heat source for the range (0,t), the derivation of a solution τ_g for this problem can be obtained in the form

$$\tau_g = \int_0^t \frac{Q}{4\pi\kappa t} e^{-\frac{R^2}{4\kappa t}}\,dt \tag{5}$$

where $$R^2 = r^2 + r_1^2 - 2rr_1 \cos\theta$$

We may consider the Laplace transform on G_t:

$$G_t^*(r,\theta,p) = \frac{Q}{2\pi\kappa}\cdot\frac{1}{P}K_o(qR) \tag{6}$$

where $q^2 = P/\kappa$

P = parameter to Laplace transform

K_o = the modified Bessel function of order 0

By the Neuman's addition theorem, eq.(6) is equal to

$$\left.\begin{aligned} G_t^*(r,\theta,P) &= \frac{Q}{2\pi\kappa P}\sum_{n=o}^{\infty} E_n I_n(qr)K_n(qr_1)\cos n\theta \qquad r<r_1 \\ &= \frac{Q}{2\pi\kappa P}\sum_{n=o}^{\infty} E_n I_n(qr_1)K_n(qr)\cos n\theta \qquad r>r_1 \end{aligned}\right\} \tag{7}$$

where I_n, K_n = the modified Bessel functions of order n

E_n = constants, $E_n = 1$ for $n = 0$; $E_n = 2$ for $n>0$

The governing equation for τ_1 is

$$\Delta\tau_1 = \frac{1}{\kappa}\cdot\frac{\partial\tau_1}{\partial t} \tag{8}$$

$$\tau_1(r,\theta,0) = 0 \qquad \text{at} \quad t = 0 \tag{9}$$

Now, the Laplace transform of eq.(8) can be shown as

$$\Delta\tau_1^* = q^2\tau_1^* \tag{10}$$

Considering the symmetry for $\theta=0$, the solution of this equation is

$$\tau_1^* = \sum_{n=o}^{\infty}\{A_nI_n(qr)+B_nK_n(qr)\}\cos n\theta \tag{11}$$

where A_n and B_n are unknown coefficients to be determined from the boundary conditions. Substituting eqs.(7) and (11) into eq.(3), we can determine the coefficients A_n and B_n. Thus, we have

$$\begin{aligned} \tau^* =& \frac{Q}{2\pi\kappa P}\cdot\frac{1}{D_o'}\Big[q\{I_o(qr)K_1(qa)+I_1(qa)K_o(qr)\}+h_a\{I_o(qr)K_o(qa)-I_o(qa)K_o(qr)\}\Big] \\ &\times\Big[q\{I_o(qr_1)K_1(qb)+I_1(qb)K_o(qr_1)\}-h_b\{I_o(qr_1)K_o(qb)-I_o(qb)K_o(qr_1)\}\Big] \\ &+\frac{Q}{\pi\kappa P}\sum_{n=1}^{\infty}\frac{1}{D_n'}\Big[q\{I_n(qr)K_{n-1}(qa)+I_{n-1}(qa)K_n(qr)\}+(h_a+\tfrac{n}{a})\{I_n(qr)K_n(qa) \\ &-I_n(qa)K_n(qr)\}\Big]\Big[-q\{I_n(qr_1)K_{n-1}(qb)+I_{n-1}(qb)K_n(qr_1)\}+(h_b-\tfrac{n}{b})\{I_n(qr_1)K_n(qb) \\ &-I_n(qb)K_n(qr_1)\}\Big]\cos n\theta \qquad \text{for} \quad r<r_1 \end{aligned} \tag{12}$$

$$\begin{aligned} \tau^* =& \frac{Q}{2\pi\kappa P}\cdot\frac{1}{D_o'}\Big[q\{I_o(qr_1)K_1(qa)+I_1(qa)K_o(qr_1)\}+h_a\{I_o(qr_1)K_o(qa)-I_o(qa)K_o(qr_1)\}\Big] \\ &\times\Big[q\{I_o(qr)K_1(qb)+I_1(qb)K_o(qr)\}-h_b\{I_o(qr)K_o(qb)-I_o(qb)K_o(qr)\}\Big]+\frac{Q}{\pi\kappa P}\sum_{n=1}^{\infty}\frac{1}{D_n'} \end{aligned}$$

$$\times\Big[q\{I_n(qr_1)K_{n-1}(qa)+I_{n-1}(qa)K_n(qr_1)\}+(h_a+\tfrac{n}{a})\{I_n(qr_1)K_n(qa)-I_n(qa)K_n(qr_1)\}\Big]$$
$$\times\Big[-q\{I_n(qr)K_{n-1}(qb)+I_{n-1}(qb)K_n(qr)\}+(h_b-\tfrac{n}{b})\{I_n(qr)K_n(qb)-I_n(qb)K_n(qr)\}\Big]$$
$$\times\cos n\theta \qquad \text{for} \quad r>r_1 \tag{13}$$

where

$$D_o'=q^2\{I_1(qb)K_1(qa)-I_1(qa)K_1(qb)\}+qh_a\{I_o(qa)K_1(qb)+I_1(qb)K_o(qa)\}$$
$$+qh_b\{I_1(qa)K_o(qb)+I_o(qb)K_1(qa)\}+h_a h_b\{I_o(qb)K_o(qa)-I_o(qa)K_o(qb)\}$$

$$D_n'=(h_a+\tfrac{n}{a})(h_b-\tfrac{n}{b})\{I_n(qa)K_n(qb)-I_n(qb)K_n(qa)\}-q(h_a+\tfrac{n}{a})\{I_n(qa)K_{n-1}(qb)$$
$$+I_{n-1}(qb)K_n(qa)\}-q(h_b-\tfrac{n}{b})\{I_{n-1}(qa)K_n(qb)+I_n(qb)K_{n-1}(qa)\}$$
$$+q^2\{I_{n-1}(qa)K_{n-1}(qb)-I_{n-1}(qb)K_{n-1}(qa)\}$$

From the inversion of the Laplace transform of eqs.(12) and (13), we obtain

$$\tau=\sum_{n=o}^{\infty}F_n(r)\cos n\theta+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty}e^{-\kappa\beta_{nm}^2 t}F_{nm}(r)\cos n\theta \tag{14}$$

where

$$F_o(r)=\frac{Q}{2\pi\kappa}\cdot\frac{1}{D_o}\Big(\frac{1}{bh_b}+\ln\frac{b}{r_1}\Big)\Big(\frac{1}{ah_a}+\ln\frac{r}{a}\Big) \qquad r<r_1$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \tag{15}$$
$$=\frac{Q}{2\pi\kappa}\cdot\frac{1}{D_o}\Big(\frac{1}{ah_a}+\ln\frac{r_1}{a}\Big)\Big(\frac{1}{bh_b}+\ln\frac{b}{r}\Big) \qquad r>r_1$$

$$F_n(r)=\frac{Q}{2\pi\kappa}\cdot\frac{1}{D_n}\Big\{\Big(1-\frac{n}{bh_b}\Big)\Big(\frac{r_1}{b}\Big)^n-\Big(1+\frac{n}{bh_b}\Big)\Big(\frac{b}{r_1}\Big)^n\Big\}\Big\{\Big(1+\frac{n}{ah_a}\Big)\Big(\frac{r}{a}\Big)^n-\Big(1-\frac{n}{ah_a}\Big)\Big(\frac{a}{r}\Big)^n\Big\} \qquad r<r_1 \tag{16}$$

$$=\frac{Q}{2\pi\kappa}\cdot\frac{1}{D_n}\Big\{\Big(1+\frac{n}{ah_a}\Big)\Big(\frac{r_1}{a}\Big)^n-\Big(1-\frac{n}{ah_a}\Big)\Big(\frac{a}{r_1}\Big)^n\Big\}\Big\{\Big(1-\frac{n}{bh_b}\Big)\Big(\frac{r}{b}\Big)^n-\Big(1+\frac{n}{bh_b}\Big)\Big(\frac{b}{r}\Big)^n\Big\} \qquad r>r_1 \tag{17}$$

$$D_o=\frac{1}{ah_a}+\frac{1}{bh_b}+\ln\frac{b}{a}$$
$$\qquad\qquad\qquad\qquad \tag{18}$$
$$D_n=n\Big\{\Big(1-\frac{n}{bh_b}\Big)\Big(1-\frac{n}{ah_a}\Big)\Big(\frac{a}{b}\Big)^n-\Big(1+\frac{n}{ah_a}\Big)\Big(1+\frac{n}{bh_b}\Big)\Big(\frac{b}{a}\Big)^n\Big\}$$

$$F_{om}(r)=-\frac{Q}{2\kappa}\frac{\left[\{J_1(\beta_{om}b)Y_o(\beta_{om}r_1)-J_o(\beta_{om}r_1)Y_1(\beta_{om}b)\}/bh_b+\{J_o(\beta_{om}r_1)Y_o(\beta_{om}b)-J_o(\beta_{om}b)Y_o(\beta_{om}r_1)\}/b\beta_{om}\right]\left[\{J_1(\beta_{om}a)Y_o(\beta_{om}r)-J_o(\beta_{om}r)Y_1(\beta_{om}a)\}/ah_a+\{J_o(\beta_{om}a)Y_o(\beta_{om}r)-J_o(\beta_{om}r)Y_o(\beta_{om}a)\}/a\beta_{om}\right]}{\left(\frac{\beta_{om}}{h_a\cdot bh_b}+\frac{1}{a\beta_{om}}\right)\{J_o(\beta_{om}a)Y_1(\beta_{om}b)-J_1(\beta_{om}b)Y_o(\beta_{om}a)\}+\left(\frac{1}{ah_a}-\frac{1}{bh_b}\right)\times\{J_1(\beta_{om}a)Y_1(\beta_{om}b)-J_1(\beta_{om}b)Y_1(\beta_{om}a)\}+\left(\frac{1}{ah_b}-\frac{1}{bh_a}\right)\{J_o(\beta_{om}a)Y_o(\beta_{om}b)-J_o(\beta_{om}b)Y_o(\beta_{om}a)\}+\left(\frac{\beta_{om}}{ah_a\cdot h_b}+\frac{1}{b\beta_{om}}\right)\{J_1(\beta_{om}a)Y_o(\beta_{om}b)-J_o(\beta_{om}b)Y_1(\beta_{om}a)\}} \tag{19}$$

$$F_{nm}(r)=-\frac{Q}{\kappa}\frac{\left[\left(1+\frac{n}{ah_a}\right)\{J_n(\beta_{nm}a)Y_n(\beta_{nm}r)-J_n(\beta_{nm}r)Y_n(\beta_{nm}a)\}-\frac{\beta_{nm}}{h_a}\{J_{n-1}(\beta_{nm}a)Y_n(\beta_{nm}r)-J_n(\beta_{nm}r)Y_{n-1}(\beta_{nm}a)\}\right]\times\left[\left(1-\frac{n}{bh_b}\right)\{J_n(\beta_{nm}b)Y_n(\beta_{nm}r_1)-J_n(\beta_{nm}r_1)Y_n(\beta_{nm}b)\}+\frac{\beta_{nm}}{h_b}\{J_{n-1}(\beta_{nm}b)Y_n(\beta_{nm}r_1)-J_n(\beta_{nm}r_1)Y_{n-1}(\beta_{nm}b)\}\right]}{\left\{a\beta_{nm}\left(1+\frac{n}{ah_a}\right)\left(1-\frac{n}{bh_b}\right)+\frac{b\beta_{nm}^3}{h_ah_b}\right\}\{J_{n-1}(\beta_{nm}a)Y_n(\beta_{nm}b)-J_n(\beta_{nm}b)Y_{n-1}(\beta_{nm}a)\}+\left\{2n\left(1+\frac{n}{ah_a}\right)\left(1-\frac{n}{bh_b}\right)-\frac{a\beta_{nm}^2}{h_a}\left(1-\frac{n}{bh_b}\right)+\frac{b\beta_{nm}^2}{h_b}\left(1+\frac{n}{ah_a}\right)\right\}\{J_n(\beta_{nm}b)Y_n(\beta_{nm}a)-J_n(\beta_{nm}a)Y_n(\beta_{nm}b)\}+ab\beta_{nm}^2\left(\frac{1}{bh_b}-\frac{1}{ah_a}\right)\{J_{n-1}(\beta_{nm}a)Y_{n-1}(\beta_{nm}b)-J_{n-1}(\beta_{nm}b)Y_{n-1}(\beta_{nm}a)\}+\left\{b\beta_{nm}\left(1+\frac{n}{ah_a}\right)\left(1-\frac{n}{bh_b}\right)+\frac{a\beta_{nm}^3}{h_ah_b}\right\}\{J_n(\beta_{nm}a)Y_{n-1}(\beta_{nm}b)-J_{n-1}(\beta_{nm}b)Y_n(\beta_{nm}a)\}} \tag{20}$$

and β_{om} for each value m is the root of the equation

$$\left\{\frac{\beta}{h_a}J_1(\beta a)+J_o(\beta a)\right\}\left\{\frac{\beta}{h_b}Y_1(\beta b)-Y_o(\beta b)\right\}$$

$$-\left\{\frac{\beta}{h_b}J_1(\beta b)-J_o(\beta b)\right\}\left\{\frac{\beta}{h_a}Y_1(\beta a)+Y_o(\beta a)\right\}=0 \tag{21}$$

and β_{nm} for each value of m is the root of the equation

$$\left\{\left(1+\frac{n}{ah_a}\right)J_n(\beta a)-\frac{\beta}{h_a}J_{n-1}(\beta a)\right\}\left\{\left(1-\frac{n}{bh_b}\right)Y_n(\beta b)+\frac{\beta}{h_b}Y_{n-1}(\beta b)\right\}-\left\{\left(1-\frac{n}{bh_b}\right)J_n(\beta b)\right.$$

$$\left.+\frac{\beta}{h_b}J_{n-1}(\beta b)\right\}\left\{\left(1+\frac{n}{ah_a}\right)Y_n(\beta a)-\frac{\beta}{h_a}Y_{n-1}(\beta a)\right\}=0 \tag{22}$$

and J_n and Y_n are the Bessel functions of order n. For the convenience of the foregoing treatment, we may express the steady terms in (14) by the following form:

$$\tau_o=\frac{Q}{2\pi\kappa}\{a_o+b_o\ \ln r+\sum_{n=1}^{\infty}(a_n r^n+b_n r^{-n})\cos n\theta-\ln R\} \tag{23}$$

where

$$a_o=\left(\ln b+\frac{1}{bh_b}\right)\left(\ln\frac{r_1}{a}+\frac{1}{ah_a}\right)/D_o$$

$$b_o=\left(\ln\frac{b}{r_1}+\frac{1}{bh_b}\right)/D_o$$

$$a_n=\left(1-\frac{n}{bh_b}\right)\left\{\left(1+\frac{n}{ah_a}\right)\left(\frac{r_1}{a}\right)^n-\left(1-\frac{n}{ah_a}\right)\left(\frac{a}{r_1}\right)^n\right\}\frac{1}{b^nD_n}$$

$$b_n=\left(1-\frac{n}{ah_a}\right)\left\{\left(1+\frac{n}{bh_b}\right)\left(\frac{b}{r_1}\right)^n-\left(1-\frac{n}{bh_b}\right)\left(\frac{r_1}{b}\right)^n\right\}\frac{a^n}{D_n}$$

Then, the exact solution of temperature distribution for this problem is obtained entirely.

3. Transient thermoelastic problem

The fundamental equation in quasi-static two-dimensional thermoelasticity is

$$\Delta\Delta\chi=-k\Delta\tau \tag{24}$$

where

- χ : stress function
- k : material constant, i.e., $k=\alpha_t E$ for plane stress problems and $k=\alpha_t E/(1-\nu)$ for plane strain problems
- α_t : coefficient of thermal expansion
- E : Young's modulus
- ν : Poisson's ratio

The thermal stresses can be expressed as;

$$\sigma_{ij} = (\delta_{ij}\Delta - \partial_i\partial_j)\chi \qquad i,j = x,y \quad \text{(when } i=j \text{ not to be summed)} \tag{25}$$

Where δ_{ij} is Kronecker's delta.
The boundary conditions are given by

$$\left.\begin{aligned} \chi &= c_1x+c_2y+c_3 \\ \partial_r\chi &= c_1 \cos\,(n,x)+c_2 \cos\,(n,y) \end{aligned}\right\} \quad \text{at} \quad r=a \tag{26}$$

$$\chi = \partial_r\chi = 0 \qquad \text{at} \quad r=b \tag{27}$$

The Michells conditions are

$$\left.\begin{aligned} &\int_o^{2\pi} \partial_r(\Delta\chi+k\tau)rd\theta=0 \qquad \int_o^{2\pi} (r\cos\theta\partial_r-\sin\theta\partial_\theta)(\Delta\chi+k\tau)rd\theta=0 \\ &\int_o^{2\pi} (r\cos\theta\partial_r+\sin\theta\partial_\theta)(\Delta\chi+k\tau)rd\theta=0 \end{aligned}\right\} \tag{28}$$

In the present problem, it is convenient to split the solution of eq.(24) into the bi-harmonic function and the particular solution; these will denoted by χ_c and χ_p, respectively. A particular solution, χ_p of

$$\Delta\chi = -k\tau \tag{29}$$

is also a particular solution of eq.(24). Thus the required solution of eq.(24) may be written,

$$\chi = \chi_p + \chi_c \tag{30}$$

Considering the form of eqs.(14) and (23), χ_p is expressed to this problem:

$$\chi_p=\frac{kQ}{8\pi\kappa}R^2 \ln R+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty} e^{-\kappa\beta_{nm}^2 t}\frac{k}{\beta_{nm}^2}F_{nm}(r)\cos n\theta \tag{31}$$

The problem is symmetrical about the x-axis. The general solution of bi-harmonic equation χ_c in plane polar coordinates can be represented as

$$\begin{aligned} \chi_c&=A_o+B_or^2+C_o\ln r+D_or^2\ln r+(A_1r+B_1r^{-1}+C_1r^3+D_1r\ln r)\cos\theta \\ &+\sum_{n=2}^{\infty}(A_nr^n+B_nr^{-n}+C_nr^{n+2}+D_nr^{-n+2})\cos n\theta \end{aligned} \tag{32}$$

Substituting eqs.(31), (32) and (14) into eqs.(28), we obtain

$$D_1 = -\frac{kQb_1}{4\pi\kappa} \quad , \qquad D_o = -\frac{kQb_o}{8\pi\kappa} \tag{33}$$

The unknown coefficients are now determined by satisfying the boundary conditions of eqs.(26) and (27). Comparing the coefficients of equivalent terms in these equations of boundary conditions, we can determined the remaining unknown coefficients. Therefore the final form of the stress function may be determined after substituting these values of the constants. The thermal stress components are

$$\sigma_{rr}=\frac{1}{r^2}\cdot\frac{\partial^2\chi}{\partial\theta^2}+\frac{1}{r}\frac{\partial\chi}{\partial r}\quad,\quad\sigma_{\theta\theta}=\frac{\partial^2\chi}{\partial r^2}\quad,\quad\sigma_{r\theta}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\chi}{\partial\theta}\right)\tag{34}$$

The components of the thermal stresses for this problem can thus be derived by the differentiation of the function χ in eq. (30) in accordance with eqs. (34).

For simplicity and generality, the temperature and stresses distribution in the cylinder or disk obtained from the above analysis will be expressed in dimensionless quantities. Thus by defining the dimensionless quantities as follows:

$$\left.\begin{array}{l}\rho = r/a,\ \rho_o = b/a,\ \rho_i = r_1/a,\ t_d = \kappa t/a^2\\ H_a = ah_a,\ H_b = ah_b,\ \alpha_{nm} = a\beta_{nm}\end{array}\right\}\tag{35}$$

and the quantities

$$T=\pi\kappa\tau/Q\ ,\ \widehat{\theta\theta}=\pi\kappa\sigma_{\theta\theta}/kQ\ ,\ \widehat{\rho\rho}=\pi\kappa\sigma_{rr}/kQ\ ,\ \widehat{\rho\theta}=\pi\kappa\sigma_{r\theta}/kQ\tag{36}$$

The final solutions in dimensionless terms for temperature, and stresses in the cylinder are given as follows:

$$T(\rho,\theta,t_d)=\frac{1}{2}\{\bar{a}_o+\bar{b}_o\ \ln\rho+\sum_{n=1}^{\infty}(\bar{a}_n\rho^n+\bar{b}_n\rho^{-n})\cos n\theta-\ln\bar{R}\}$$
$$+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty}e^{-k\beta_{nm}^2t_d}\,\bar{F}_{nm}(\rho)\cos n\theta\tag{37}$$

where

$$\bar{a}_o=(\ln\rho_o+1/\rho_oH_b)(\ln\rho_i+1/H_a)/\bar{D}_o$$

$$\bar{b}_o=(\ln\rho_o/\rho_i+1/\rho_oH_b)/\bar{D}_o$$

$$\bar{a}_n=\left(1-\frac{n}{\rho_oH_b}\right)\left\{\left(1+\frac{n}{H_a}\right)\rho_i{}^n-\left(1-\frac{n}{H_a}\right)\frac{1}{\rho_i{}^n}\right\}\frac{1}{\rho_o{}^n\bar{D}_n}$$

$$\bar{b}_n=\left(1-\frac{n}{H_a}\right)\left\{\left(1+\frac{n}{\rho_oH_b}\right)\left(\frac{\rho_o}{\rho_i}\right)^n-\left(1-\frac{n}{\rho_oH_b}\right)\left(\frac{\rho_i}{\rho_o}\right)^n\right\}\frac{1}{\bar{D}_n}$$

$$\bar{D}_o=\frac{1}{H_a}+\frac{1}{\rho_oH_b}+\ln\rho_o$$

$$\bar{D}_n=n\left\{\left(1-\frac{n}{\rho_oH_b}\right)\left(1-\frac{n}{H_a}\right)\frac{1}{\rho_o{}^n}-\left(1+\frac{n}{\rho_oH_b}\right)\left(1+\frac{n}{H_a}\right)\rho_o{}^n\right\}$$

$$\bar{R}^2=\rho^2+\rho_i{}^2-2\rho\rho_i\cos\theta$$

$$\bar{F}_{om}(\rho)=-\frac{\pi}{2}\,\frac{\left[\{J_1(\alpha_{om}\rho_o)Y_o(\alpha_{om}\rho_i)-J_o(\alpha_{om}\rho_i)Y_1(\alpha_{om}\rho_o)\}/\rho_oH_b+\{J_o(\alpha_{om}\rho_i)Y_o(\alpha_{om}\rho_o)\right.}{\left(\frac{\alpha_{om}}{H_a\rho_oH_b}+\frac{1}{\alpha_{om}}\right)\{J_o(\alpha_{om})Y_1(\alpha_{om}\rho_o)-J_1(\alpha_{om}\rho_o)Y_o(\alpha_{om})\}}$$

$$\frac{-J_o(\alpha_{om}\rho_o)Y_o(\alpha_{om}\rho_i)\}/\rho_o\alpha_{om}\Big]\Big[\{J_1(\alpha_{om})Y_o(\alpha_{om}\rho)-J_o(\alpha_{om}\rho)Y_1(\alpha_{om})\}/H_a}{+(\frac{1}{H_a}-\frac{1}{\rho_oH_b})\{J_1(\alpha_{om})Y_1(\alpha_{om}\rho_o)-J_1(\alpha_{om}\rho_o)Y_1(\alpha_{om})\}+(\frac{1}{H_b}-\frac{1}{\rho_oH_a})\{J_o(\alpha_{om})}$$

$$\frac{+\{J_o(\alpha_{om})Y_o(\alpha_{om}\rho)-J_o(\alpha_{om}\rho)}{\times Y_o(\alpha_{om}\rho_o)-J_o(\alpha_{om}\rho_o)Y_o(\alpha_{om})\}+(\frac{\alpha_{om}}{H_aH_b}+\frac{1}{\rho_o\alpha_{om}})\{J_1(\alpha_{om})Y_o(\alpha_{om}\rho_o)}$$

$$\frac{\times Y_o(\alpha_{om})\}/\alpha_{om}\Big]}{-J_o(\alpha_{om}\rho_o)Y_1(\alpha_{om})\}} \tag{38}$$

$$\bar{F}_{nm}(\rho)=\frac{-\pi\Big[(1+\frac{n}{H_a})\{J_n(\alpha_{nm})Y_n(\alpha_{nm}\rho)-J_n(\alpha_{nm}\rho)Y_n(\alpha_{nm})\}-\frac{\alpha_{nm}}{H_a}\{J_{n-1}(\alpha_{nm})Y_n(\alpha_{nm}\rho)}{\{\alpha_{om}(1+\frac{n}{H_a})(1-\frac{n}{\rho_oH_b})+\frac{\rho_o\alpha_{nm}^3}{H_aH_b}\}\{J_{n-1}(\alpha_{nm})Y_n(\alpha_{nm}\rho_o)-J_n(\alpha_{nm}\rho_o)Y_{n-1}(\alpha_{nm})\}}$$

$$\frac{-J_n(\alpha_{nm}\rho)Y_{n-1}(\alpha_{nm})\}\Big]\Big[(1-\frac{n}{\rho_oH_b})\{J_n(\alpha_{nm}\rho_o)Y_n(\alpha_{nm}\rho_i)-J_n(\alpha_{nm}\rho_i)Y_n(\alpha_{nm}\rho_o)\}}{\times\{\ 2n(1+\frac{n}{H_a})(1-\frac{n}{\rho_oH_b})-\frac{\alpha_{nm}^2}{H_a}(1-\frac{n}{\rho_oH_b})+\frac{\rho_o\alpha_{nm}^2}{H_b}(1+\frac{n}{H_a})\}\{J_n(\alpha_{nm}\rho_o)Y_n(\alpha_{nm})}$$

$$\frac{+\frac{\alpha_{nm}}{H_b}\{J_{n-1}(\alpha_{nm}\rho_o)Y_n(\alpha_{nm}\rho_i)}{-J_n(\alpha_{nm})Y_n(\alpha_{nm}\rho_o)\}+\rho_o\alpha_{nm}^2(\frac{1}{\rho_oH_b}-\frac{1}{H_a})\{J_{n-1}(\alpha_{nm})Y_{n-1}(\alpha_{nm}\rho_o)-J_{n-1}(\alpha_{nm}\rho_o)}$$

$$\frac{-J_n(\alpha_{nm}\rho_i)Y_{n-1}(\alpha_{nm}\rho_o)\}\Big]}{Y_{n-1}(\alpha_{nm})\}+\{\rho_o\alpha_{nm}(1+\frac{n}{H_a})(1-\frac{n}{\rho_oH_b})+\frac{\alpha_{nm}^3}{H_aH_b}\}\{J_n(\alpha_{nm})Y_{n-1}(\alpha_{nm}\rho_o)}$$

$$\frac{}{-J_{n-1}(\alpha_{nm}\rho_o)Y_n(\alpha_{nm})\}} \tag{39}$$

$$\begin{aligned}\widehat{\rho\rho}=&2\bar{B}_o+\bar{C}_o\rho^{-2}-\bar{b}_o(2\ \ln\ \rho+1)/8+(-2\bar{B}_1\rho^{-3}+2\bar{C}_1\rho-b_1\rho^{-1}/4)\cos\ \theta-\sum_{n=2}^{\infty}\{\bar{A}_n(n-1)n\rho^{n-2}\\&+\bar{B}_n(n+1)n\rho^{-n-2}+\bar{C}_n(n+1)(n-2)\rho^n+\bar{D}_n(n-1)(n+2)\rho^{-n}\}\cos\ n\theta+(2\ \ln\ \bar{R}\ +1\\&+2\rho_i^2\sin^2\ \theta/\bar{R}^2)/8+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty}\ e^{-\alpha_{nm}^2t}\frac{d}{\alpha_{nm}^2}\frac{1}{}\{\frac{1}{\rho}\frac{d\bar{F}_{nm}(\rho)}{d\rho}-\frac{n^2}{\rho^2}\bar{F}_{nm}(\rho)\}\cos\ n\theta\end{aligned} \tag{40}$$

$$\widehat{\theta\theta}=2\bar{B}_o-\bar{C}_o\rho^{-2}-\bar{b}_o(2\ln\rho+3)/8+\{2\bar{B}_1\rho^{-3}+6\bar{C}_1\rho-b_1\rho^{-1}/4\}\cos\theta+\sum_{n=2}^{\infty}\{\bar{A}_n n(n-1)\rho^{n-2}$$
$$+\bar{B}_n n(n+1)\rho^{-n-2}+\bar{C}_n(n+1)(n+2)\rho^{n}+\bar{D}_n(n-2)(n-1)\rho^{-n}\}\cos n\theta+\{(2\ln\bar{R}+1)$$
$$+2(\rho-\rho_i\cos\theta)^2/\bar{R}^2\}/8+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_{nm}^2 t}d\frac{1}{\alpha_{nm}^2}\frac{d^2\bar{F}_{nm}(\rho)}{d\rho^2}\cos n\theta \tag{41}$$

$$\widehat{\rho\theta}=(-2\bar{B}_1\rho^{-3}+2\bar{C}_1\rho-b_1\rho^{-1}/4)\sin\theta+\sum_{n=2}^{\infty}\{\bar{A}_n n(n-1)\rho^{n-2}-\bar{B}_n n(n+1)\rho^{-n-2}+\bar{C}_n n(n+1)\rho^{n}$$
$$-\bar{D}_n n(n-1)\rho^{-n}\}\sin n\theta-2(\rho-\rho_i\cos\theta)\rho_i\sin\theta/8\bar{R}^2+\sum_{n=o}^{\infty}\sum_{m=1}^{\infty}e^{-\alpha_{nm}^2 t}d\frac{n}{\alpha_{nm}^2\rho}$$
$$\times\{\frac{d\bar{F}_{nm}(\rho)}{d\rho}-\frac{1}{\rho}\bar{F}_{nm}(\rho)\}\sin n\theta \tag{42}$$

where

$(\bar{A}_n,\bar{B}_n,\bar{C}_n,\bar{D}_n) = (A_n,B_n,C_n,D_n)\pi\kappa/kQ$

The foregoing solution will be illustrated numerically by solving a hollow circular cylinder under a continuous point heat source specified by the following values:

$\rho_o = 3, \qquad \rho_i = 2$

Fig.2 shows the non-dimensional temperature variation for dimensionless time. The variations of hoop stresses are shown in Figs. 3-4 for various times. Figs.5 and 6 illustrate the variations in $\widehat{\rho\rho}$ and $\widehat{\rho\theta}$ respectively along the x-axis.

Acknowledgments

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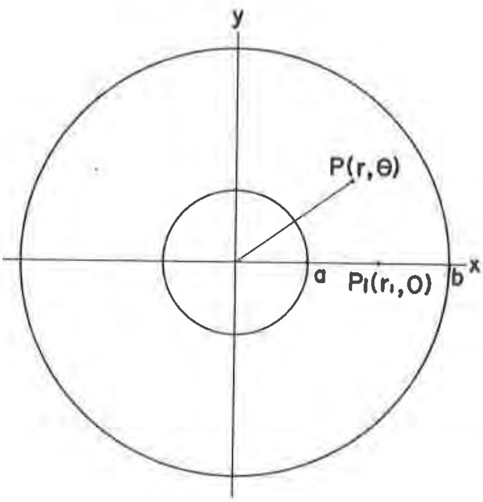

Fig.1 Continuous point heat source on the arbitrary point $(r_1,0)$ on the hollow circular disk.

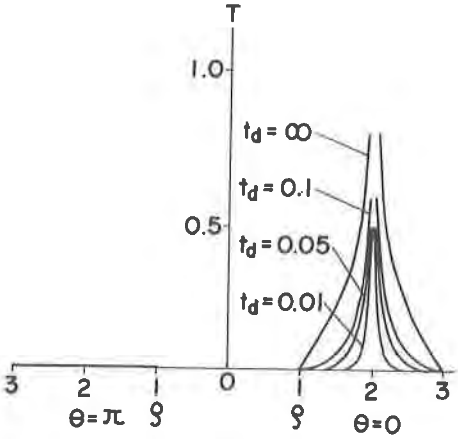

Fig.2 Temperature distribution for various times.

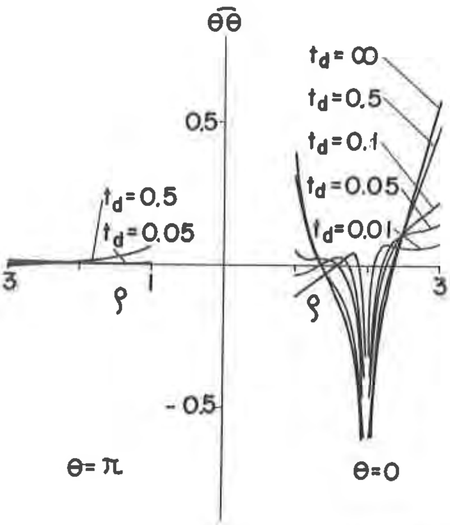

Fig.3 Variation in $\widehat{\theta\theta}$ along the $\theta = 0$ and $\theta = \pi$

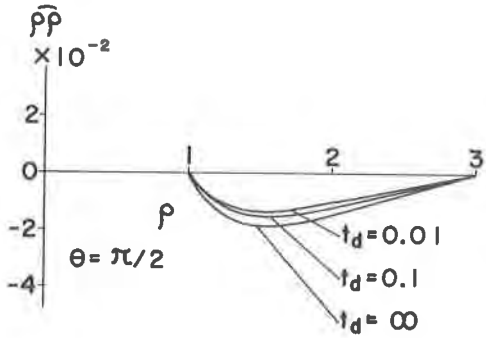

Fig 4 Variation in $\widehat{\rho\rho}$ along the $\theta = \pi/2$

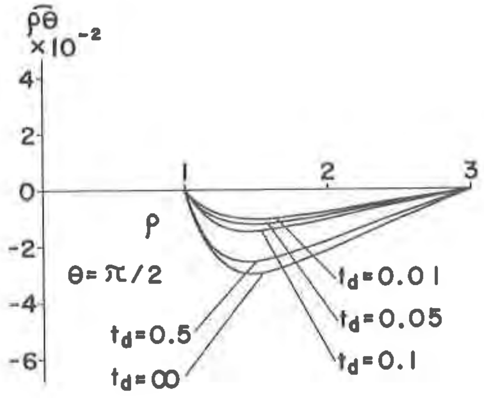

Fig.5 Variation in $\widehat{\rho\theta}$ along the $\theta = \pi/2$

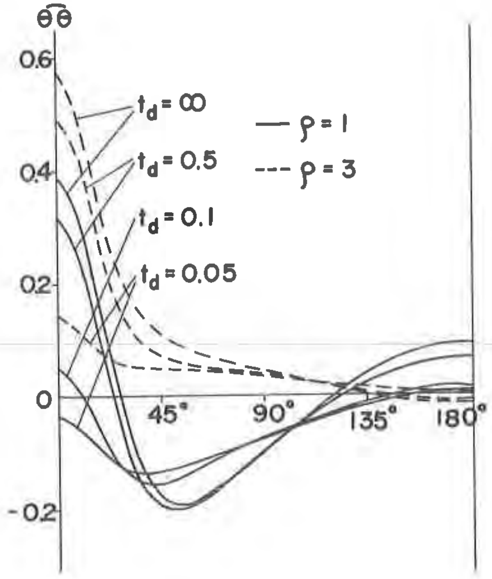

Fig.6 Hoop stress distribution round the edge of a hole.