ON THE INFLUENCE OF MULTIPLE LOADINGS TO PLASTIC PIPE ELEMENTS APPLICATIONS TO THE PIPE WHIP PROBLEM

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SUMMARY

The aim of this paper is a contribution to the study of the pipe whip accident considering various effects (pressurization, presence of bending in two normal planes, shears torque, normal force) which have been previously neglected by the author in precedent works on the same subject for simplicity reasons. A model for a strain hardening plastic pipe element has been proposed; the main hypotheses are briefly reviewed:

(a) plastic flow rule according to Prandtl-Reuss; Von Mises criterion to compute equivalent octaedral stress;

(b) a strain hardening rule is considered (generally Ramberg-Osgood type) the balance between applied external forces and internal stress resultant is considered, while the local equilibrium stress equations are waived; instead hypotheses on the stress distribution and/or on the strain distribution are introduced;

(c) two models have been proposed: in the so-called strain model, the section is supposed to remain plane and not to warp even under the influence of shear loads, in the so-called mixed model while the same hypothesis is made for axial strains (which corresponds to use a two terms Fourier expansion) a truncated Fourier expansion is assumed for the shear stress distribution;

(d) in this way a complete stiffness (and/or flexibility matrix) may be formulated correlating the generalized strains in each section of the pipe to the generalized stress resultants (moments, forces, presence);

(e) the influence of section deformation (ovalization due to radius changes in the plastic range (possibly loading to Brazier type instability) can be taken into consideration.

The two models (strain model and mixed model) are compared in terms of the results which may be obtained by the use of each of them: it results that in the cases investigated the differences are not large in terms of curvature changes (while it may be somewhat more important in terms of the shear strains). As earlier mentioned, the main aim of this development is an assessment of the relative importance of various parameters, among others one may quote:

(a) the influence of internal pressure;

(b) the influence of a slight torque due to asymmetries in the global geometry;

(c) the influence of bending moments on two normal planes;

(d) the influence of shear on the deformations of the pipe.

For all these cases tentative conclusions are drawn, in the form of graphs taking the differences with the basic simplified model as a main point. Finally, typical real load time histories for one pipe element are considered and the differences with simplified models in the displacements and stress-strain distribution are shown and discussed. All these analyses should serve as a practical guide to help in the decision where a refined model is required.

INTRODUCTION

The aim of this paper is a discussion and an appraisal of some effects, which could be important in the evaluation of the plastic behaviour of a pipe with particular reference to the so called "pipe whip accident" [1, 2]. In the past [3, 4] the author has presented a few analyses based on a 2D scheme by the use of the computer code FRUSTA.
However in those analyses the behaviour of the pipe was supposed such that deformations due to bending only were considered, neglecting strains and displacements due to shears, normal force etc. In this paper a multiaxial fully three dimensional plastic pipe element will be presented and discussed to clarify the influence of other phoenomena present during the pipe whip transitory. However the basic principales of the element have been descibed elsewhere [5] and they will be only briefly recalled here, rather the discussion will deal essentially with the application of the model to the pipe whip problem.

2. THE PIPE MODEL

Let us consider a pipe section and let r be the radius and t the thickness and let us assume that a generalized relationship exists between equivalent stress σ_p and strain ε_p of the type $\sigma_p = f(\varepsilon_p)$. Let us further indicate: $|F|$ generalized forces vector

$|\varepsilon|$ generalized strain vector

and let $[Kg]^{-1}$ be a generalized flexibility matrix correlating forces and strains:

$$(1) \quad |\Delta\varepsilon| = [Kg]^{-1} \cdot |\Delta F|$$

where an incremental approach has been considered and where Δ indicates increment in each load step. Pressure too is generally present in a pipe, however due to the fact that the pipe whip transitory is very quick and for added conservativism and for semplicity sake the pressure will be supposed to be a constant during the whole transitory; however the problem has been considered in ref. [5].
Omitting the details of the calculations the principal hypotheses will be briefly reviewed:
a) the local equilibrium equations among the stress components will be waived, rather the generalized equilibrium equations for the whole section will be retained.
Further the stress tensor will be simplified assuming that only the axial stress σ_z and hoop stress σ_θ exists together with the two components of the shear stress τ_{rz}, $\tau_{z\theta}$, (where z is the axial direction, and r and θ individuate the radial and tangential direction)
Then one can obviously formulate integral equations in terms of the internal resultants and stresses:

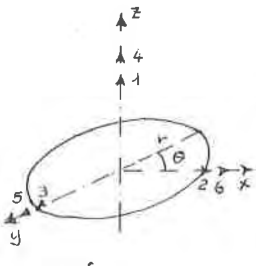

fig 1

$$\Delta F_1 = \int_0^{2\pi} rt\,\Delta\sigma_z\,d\theta \qquad \Delta F_5 = \int_0^{2\pi} r^2 t \cos\theta\,\Delta\sigma_z\,d\theta$$

$$\Delta F_6 = \int_0^{2\pi} r^2 t \sin\theta\,\Delta\sigma_z\,d\theta \qquad \Delta F_2 = \int_0^{2\pi} (\Delta\tau_{zr}\cos\theta + \Delta\tau_{z\theta}\sin\theta)\cdot r\,t\,d\theta$$

$$\Delta F_3 = \int_0^{2\pi} (\Delta\tau_{zr}\sin\theta + \Delta\tau_{z\theta}\cos\theta)\,rt\,d\theta$$

$$\Delta F_4 = \int_0^{2\pi} \Delta\tau_{z\theta}\,r^2\,t\,d\theta$$

b) as a substitute for the local equilibrium equations, hypotheses on the displacement and/or stress pattern can be made as suggested by Hodge [7]. It will be assumed a trun cated Fourier expansion for the axial strain, as follows:

(3a) $\varepsilon_z = A\cos\theta + B\sin\theta + C$

Besides if one assumes that no warping of the section takes place one has, letting γ_{zr} and $\gamma_{z\theta}$ be the appropriate shear strains:

(3b) $\gamma_{z\theta} = D\cos\theta + E\sin\theta + F$

(3c) $\gamma_{zr} = D\sin\theta + E\cos\theta$

Furthermore as the hoop strain ε_θ can be directly related to the pressure, one has the strain unknowns A, B, C, D, E, F to deal with and the six equilibrium equations (2a, f) to solve the problem.

However the hypotheses on the strain distribution can be not very significant as a presence of shear stress in the radial direction would result and besides the section would remain plane, while it is known to warp under the action of shear forces.

Alternatively one can make hypotheses on the stress pattern as follows:

(4a) $\tau_{z\theta} = \tau_1\cos\theta + \tau_2\sin\theta + \tau_3$

(4b) $\tau_{zr} = 0$

In this case the unknowns will be three stress parameters and three strain parameters, this is the reason why this model is known as the mixed model, while the other one will be the strain model.

c) Prandtl-Reuss equation will be used as the relationship connecting stress and the strain as follows:

(5a) $d\varepsilon_i = (\sigma_i - 0.5\sigma_j - 0.5\sigma_k)\,\dfrac{d\varepsilon_p}{\sigma_p} + \dfrac{1}{E}(d\sigma_i - \nu(d\sigma_j + d\sigma_k))$

(5b) $d\gamma_{ik} = \dfrac{3\tau_{ik}}{2}\,\dfrac{d\varepsilon_p}{\sigma_p} + \dfrac{d\tau_{ik}}{G2}$

where E, G are the elasticity modulus, ν is Poisson modulus σ_i, σ_j, σ_k, τ_{ik}, are the com ponent and ε_i, γ_{jk} are the components of the strain tensor [9] of the stress tensor and σ_p and ε_p are the equivalent stress and strain.

d) Von Mises equation will be used to correlate the equivalent stress to the components of the stress tensor:

$$(6)\quad \sigma_p^2 = \sigma_Z^2 + \sigma_\Theta^2 - \sigma_z \cdot \sigma_\Theta + 3\,(\tau_{z\Theta}^2 + \tau_{zr}^2)$$

c) A strain hardening rule will be assumed of the Ramberg - Osgood type

$$(7)\quad \sigma_p = A\,\varepsilon_p^m + E\,\varepsilon_p$$

If equations (1, 7) are used and the following parameters are introduced:

$$(8a)\quad H_\varepsilon = \partial\sigma_p / \partial\sigma_z$$
$$(8b)\quad H_{z\Theta} = \partial\sigma_p / \partial\tau_{z\Theta}$$
$$(8c)\quad H_{zr} = \partial\sigma_p / \partial\tau_{zr}$$

one has for the case of the strain model a relationship of the type

$$(9a)\quad \begin{vmatrix} d\sigma_z \\ d\tau_{z\theta} \\ d\tau_{zr} \end{vmatrix} = \left[\xi_s(\theta)\right] \cdot \begin{vmatrix} dA, & dB, & dC, & dD, & dE, & dF \end{vmatrix}$$

where the individual components of the 6 x 3 $\left[\xi_s(\Theta)\right]$ matrix are dependent on the angular position and on the local stress state. In the same way one will yield on equa tion for the strain model of the type:

$$\begin{vmatrix} d\sigma_z \\ d\tau_{z\Theta} \\ d\tau_{zr} \end{vmatrix} = \left[\xi_m(\Theta)\right] \cdot \begin{vmatrix} dA, & dB, & dC, & d\tau_1, & d\tau_3, & d\ _2 \end{vmatrix}$$

If equations (9a,b) are substituted into equations (2a,f) one will yield a relation - ship of the type:

$$(10)\quad \left|\Delta F_i\right| = \left[\eta_{ik}\right] \cdot \left|\Delta L_k^s\right|$$

where L_k^s are the strain parameters in equation 9a for the strain model.
For the mixed model the situation is similar, however one must define the strain para meters competent to the shear strain field, as warping and deformation of the section takes place; an equivalent average can be used in this case to define the relevant para meters of the section:

$$(11a)\quad \Delta\gamma_3 = \int_0^{2\pi} \frac{\Delta\gamma_{z\Theta}\, d\Theta}{2\pi}$$

$$(11b)\quad \Delta\gamma_1 = \int_0^{2\pi} \frac{\cos\Theta\,\Delta\gamma_{z\Theta}\, d\Theta}{2\pi}$$

$$(11c)\quad \Delta\gamma_2 = \int_0^{2\pi} \frac{\sin\Theta\,\Delta\gamma_{z\Theta}\, d\Theta}{2\pi}$$

In any case either directly by the use of equation (10) or by the use of equation (10) and (11) one may correlate the strain parameters and the force parameters as indicated in equa tion (1).

If equation (1) is integrated along the lenght of the pipe one will yield a relationship between the generalized displacement vector and the forces applied at the element terminals:

$$\left|\Delta W\right| = \left[K\right]^{-1} \cdot \left|\Delta F_i^{o}\right|$$

i.e. the element stiffness and/or element matrix can be formulated.

If equation (5a) is used for the case of the hoop strain one can obtain the values of the rate of change of the radius and the consequent changes in the section can be accordingly considered and accounted for in the formulation of the generalized equilibrium equations (2a,f). A Fortran program named PAPS T2G is available to produce the computation described in this paragraph, it is planned to introduce this element into a modified version of NONSAP [8] under development in SAIGE.

Four models are available:

a) the strain model

b) the strain model together with section ovalization due to the change of the radius of curvature in the pipe section

c) the mixed model

d) the mixed model together with section ovalization due to the change of radius of curva ture in the pipe section.

The use of models b) and d) would allow an evaluation of the behaviour of the pipe with respect to ovalization instability and its use will be reported later on, while this paper will be essentially based on the use of models a), c).

3. DISCUSSION

Three types of runs have been performed in order to study some particular effects, they are:

- runs P : to study the influence of the pressure on the lateral deflection characte ristics
- runs T : to study the particular behaviour of the pipe part overhanging from the re straint.
- runs M : to study the influence of the model on the deflection characteristics.

In all the cases, the geometry which has been considered is a cantilever beam, with various end loads; obviously PAPS T2G code can handle much more complicated situations,however a comparatively simple one has been selected in order to be able to perform comparisons with simpler models and for a better understanding of the phoenomenon.

3.1 Runs P

The results of runs P are shown in fig.2, where the details of the data are also given, all the cases were run by means of the mixed model. Together with a basic load situation (a large shear force and a small axial force, which are increased during the calculations) a constant pressure load is superimposed, three cases have been considered no pressure, a pressure of 60 Kg/cmq (producing a basic hoop stress of 15 Kg/mm^2), a pressure of 150 Kg/cmq (producing a basic hoop stress of 37.5 Kg/mm^2).

While the first two cases do not differ appreciably between themselves and with the results of the simplified analysis, (contribution of bending only as suggested in ref. [9] for beams and used in ref. [3,4]), the difference being largely due to the shear strain contribution, large differences found with the case of large pressure. However one may notice that in the last case the hoop stress is abnormally large for the common case of design, while the second case is rather large as well, then one might be able to suggest that the influence of pressure, for the common case of design is not vary large in assessing the behaviour of a pipe element under the effect of large lateral forces.

3.2 Runs T

During the pipe whip accident, the pipe part overhanging from the restraint is generally the most critical or one of the most critical ones [2], then there is a tendency to reduce the overhang length in order to minimize the bending moment in the section where the restraint is applied. However it was suggested during the discussion of section F5 of the third SMIRT Conference in London, August 1975 that shear could play an important role in assessing the behaviour of the pipe. As already mentioned in subparagraph 3.1 this tends to be true as the results of PAPS T2G code give larger displacements than the ones one would have obtained by means of the simplified model, this is due to both the presence of a shear strain deformation and to the increase of the σ_p stress as a consequence of the shear stress itself. The results of run T1 would confirm this tendency; besides two additional runs have been performed to study the influence of a slight torque produced by a small load eccentricity and of a slight bending moment on the other axis.

These cases were derived by a real case already run by means of FRUSTA code from which all the pertinent date have been taken. As one can see, in this case too the results of the simplified model give a reasonable reasonable somewhat understimated evaluation of the deflection.

All these cases have been run by means of the mixed model. In Fig. 4 the stress distribution at different axial locations in the pipe are given, as one can see the σ_p distribution tends to be rather uniform as suggested by Larson et al [10], obvious exception in the case when the shear stress is the dominant component for the evaluation of the equivalent stress σ_p.

3.3 Runs M

By means of these runs the influence of the model is studied, case T1 has been compared with the results one would have obtained if the strain model or the mixed model with consideration of ovalization in the section would have been used. As one can see the differences are, in this case, quite small, the mixed model tends to predict slightly larger displacements than the strain model, while the coincidence of the results of model 3 and model 4 is possibly due to the fact that, as the strains are comparatively small no large ovalization phoenomena take place. In fig. 6 the distributions of stresses in the section as predicted by the mixed and strain model are compared, no large difference is found in this case, with a slight tendency of strain model to produce a slight more uniform stress distribution, as one would have predicted.
In fig. 7 the curvature and shear strains are reported in these two cases, as one can see, while the difference is limited for the curvature, it is larger in the case of the shear strain, so accounting for the differences in the displacements.

4. CONCLUSIONS

A model for the appraisal of the deformation characteristics of a pipe under the effect of a multiple loading condition is presented. It allows an evaluation of the behaviour of the pipe under various loads combinations and an appraisal of the differences one would have obtained by means of the use of a simplified analysis.
Obviously it is quite difficult to draw general conclusions and the availability of the code to handle complex situations makes it perhaps irrelevant. However one may note that:

a) the pressure does not seem a decisive parameter, at least if it produces hoop stresses as usual in the pipe design
b) the shear stress may be responsible for some extra deformations and displacements of the pipe, particularely in the overhang portion, where a shear hinge might develop even if in rather extreme cases
c) the simplified model generally produces a reasonable, somewhat underestimated evaluation of the pipe deflections and strains
d) the use of a more refined element should be confined (for obvious cost reasons) to cases, where either geometry or load would produce an abnormal situation, (large pressure, large shears, large torque, a shear force changing its direction etc.) so that the use of both the simplified and complete element would be profitable
e) ovalization phoenomena eventually leading to Brazier type instability can be handled by this model.

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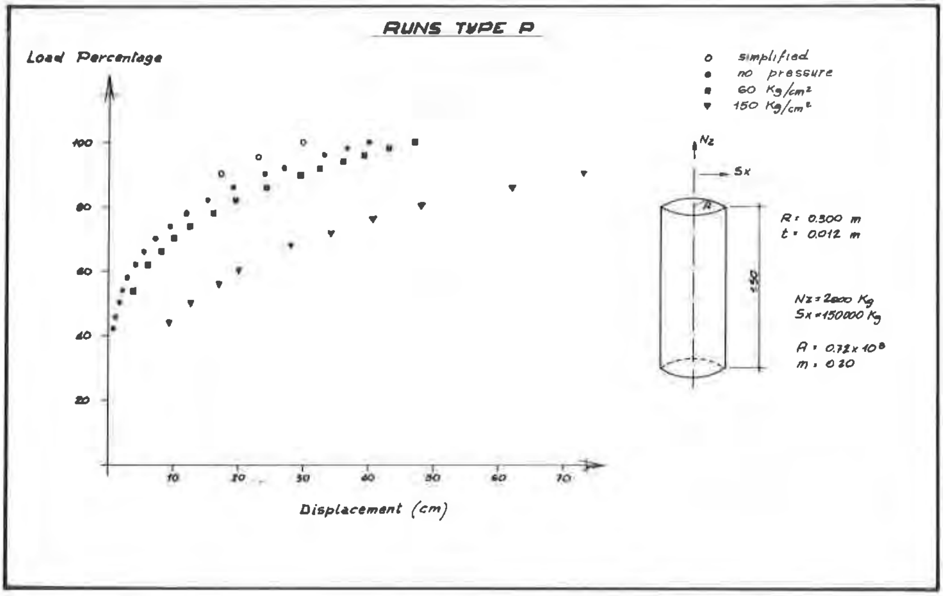

Fig.2 Runs type P influence of pressure on deflection

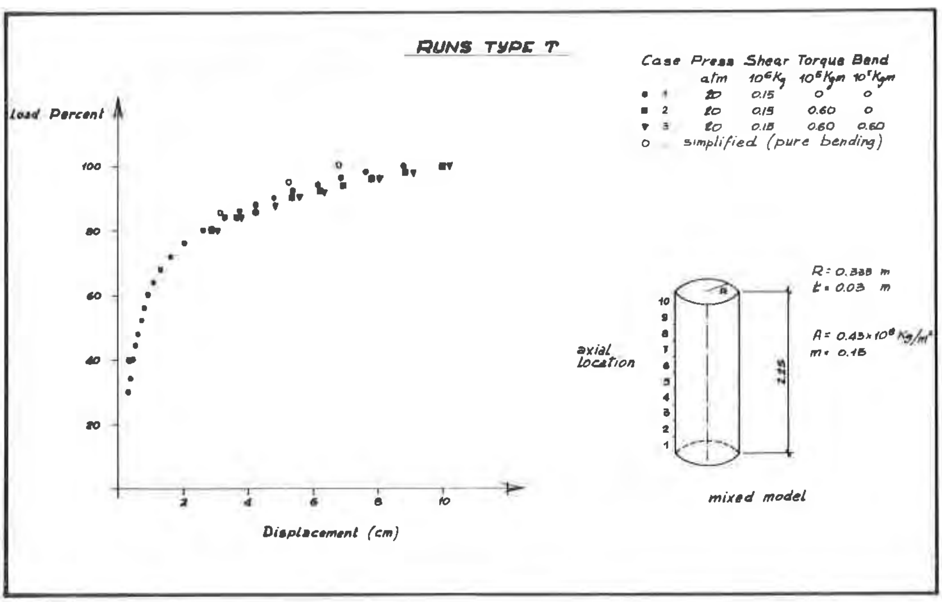

Fig.3 Runs type influence of secondary loads on deflection

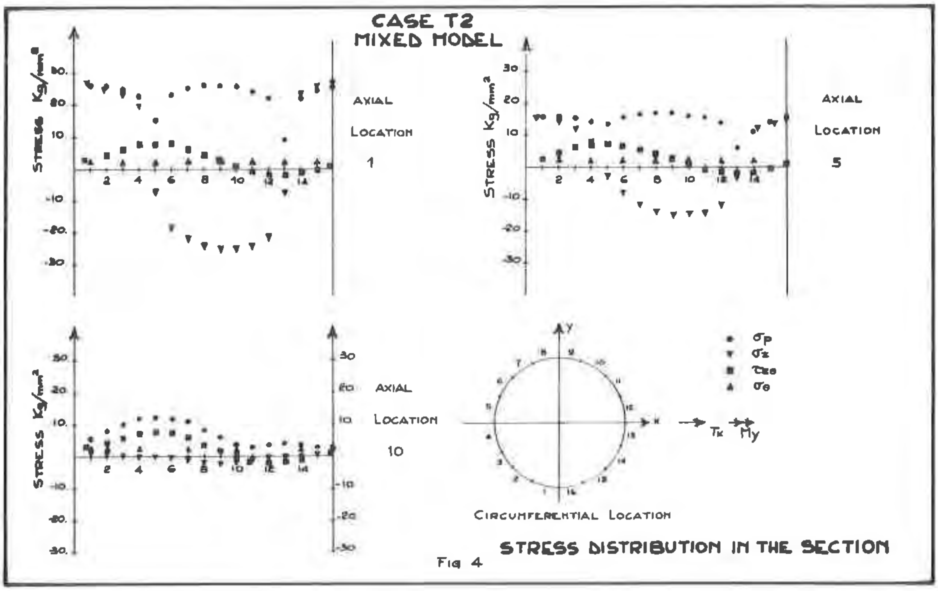

Fig.4 Runs type T stress distribution in the section

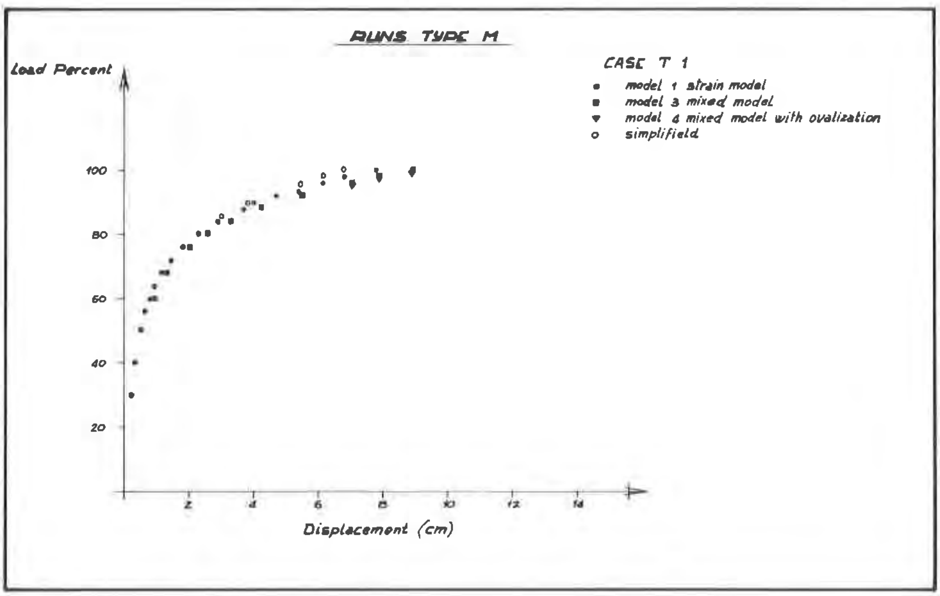

Fig.5 Runs type M influence of the model on the deflection

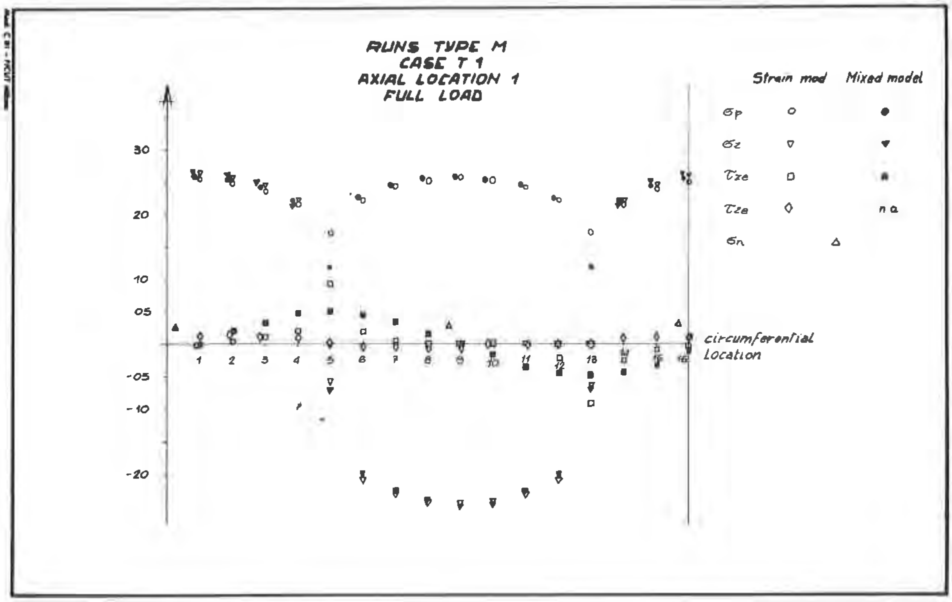

Fig.6 Runs type M stress distribution in the section

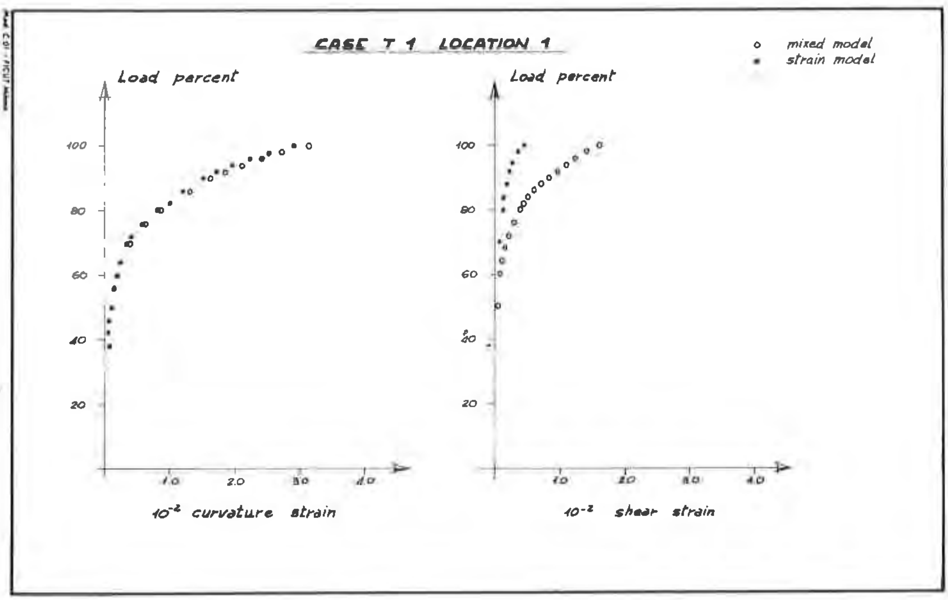

Fig.7 Runs type M influence of the model on strain in the section.