Reliability analysis of large scaled structures by optimization technique

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1 INTRODUCTION

The reliability analysis of the highly redundant structure is very important for the probabilistic safety assessment of the ultimate limit state of large-scaled structures such as an offshore platform or a transmission tower. In order to evaluate the plastic collapse probability of the whole structure based on the kinematic approach of plastic analysis, it is needed to take into account for all possible collapse modes of the structure and their mutual effects of correlation. While many studies (Moses and Kinser 1976, Yonezawa et al. 1978, Shiraishi et al. 1980) have been devoted to the reliability analysis of redundant structures, there are a few studies on the highly redundant systems having a large number of possible collapse modes.

For example, Ditlevsen and Bjerager (1985) applied the lower bound theorem of plastic analysis to evaluate the upper bound of collapse probability. Murotsu et al. (1983) proposed the automatic generation method to find the most dominant collapse mode by applying the elastic-plastic analysis technique. However, there are quite a few studies on the reliability analysis using optimization technique. Only Ang and Ma (1985) suggested a method finding the most probable collapse mode by using mathematical programming based on the basic collapse modes. In these methods mentioned above tedious calculations are still required to find the collapse probability of the structure and, as such, it will be difficult to them to apply to the optimal reliability-based design problem of the large structures.

To this end, this paper presents a reliability analysis based on the optimization technique using PNET (Probabilistic Network Evaluation Technique) method for the highly redundant structures having a large number of collapse modes. That is, this approach makes the best use of the merit of the optimization technique in which the idea of PNET method is used. The analytical process involves the minimization of safety index of the representative mode, subjected to satisfaction of the mechanism condition and the the positive external work. The procedure entails the sequential performance of a series of the NLP (Nonlinear Programming) problems, where the correlation condition as the idea of PNET method pertaining to the representative mode is taken as an additional constraint to the next analysis. Upon succeeding iterations, the final analysis is achieved when a collapse probability at the subsequent mode is extremely less than the value at the 1st mode. The approximate collapse probability of the structure is defined as the sum of the collapse probabilities of the representative modes classified by the extent of correlation.

Then, in order to confirm the validity of the proposed method, the conventional Monte Carlo simulation is also revised by using the collapse load analysis. Finally, two fairly large structures were analyzed to illustrate the scope and application of the approach.

2 RELIABILITY ANALYSIS

2.1 Outline of PNET Method (Ang and Ma 1979)

The conventional PNET method has been developed for the analysis of activity networks by Ang et al. (1975) and this method is applied to the reliability analysis of framed structures (Ang and Ma 1979,1981). In the study (Ang and Ma 1979) it is based on the premise that the collapse modes that are highly correlated may be assumed to be perfectly correlated, and the other collapse modes that are lowly correlated may be assumed to be statistically independent. Then, all collapse modes can be divided into some groups, and modes within each group can be "represented" by the modes which has the largest collapse probability in the group. The collapse probability will be approximated by the following equation, when all possible collapse modes are known.

$$P_f = \sum_{n=1}^{Nr} P_{fn} \tag{1}$$

where n and Nr are the index and the total number of representative modes, respectively.

The calculation procedure by the conventional PNET method is performed as follows:

(1) Find all possible collapse modes, and arrange them according to their collapse probabilities.

(2) Calculate the coefficient of correlation (ρ_{nj}) between the mode with the largest collapse probability (the n-th representative mode) and the other mode j.

(3) Classify as the n-th group modes in which ρ_{nj} is greater than the specified demarcating correlation ρ_0 and evaluate P_{fn} (n=1,2,...,Nr) of the n-th representative modes which has the largest collapse probability in the n-th group.

(4) Repeat the steps (2), (3) for the rest modes.

(5) Finally, estimate Pf by Eq.(1).

The collapse probability calculated by PNET method is an approximate value, but its simple calculation procedure will be able to extend to the design problem of the large scaled structures. However, in order to apply to the reliability analysis and design of the large scaled structures, it is difficult to find all possible collapse modes and their mutual correlations. Therefore, the conventional PNET method will be revised and extended by making the best use of the optimization technique.

2.2 Formulation by Optimization Technique

2.2.1 The 1st Representative Mode

The 1st representative mode is a mode which has the largest collapse probability among all possible collapse modes. Authors (1986) have proposed the plastic analysis for finding the mode with the smallest safety index, i.e. the largest collapse probability.

The analysis is formulated by using the kinematic approach of plastic analysis as follows:

(1) All possible collapse modes can be devided into the elementary modes and combined modes. It should be noted that combined modes can be expressed as a linear combination of elementary modes (Cohn et al. 1972) because of the assumption of rigid-plastic behavior. That is,

$$\theta_j = \sum_{k=1}^{K} C_{kj}\, t_k \quad (j=1,2,..,J) \tag{2}$$

where θ_j is the value of relative rotation of section j in the combined mode; C_{kj} is the value of relative rotation of section j in the elementary mode k; t_k is the factor defining the combination of an elementary mode k; k and K are the index and the total number of elementary modes.

(2) Collapse probability P_{fi} of a combined mode i can be expressed as the function of the safety index β_i as follows:

$$P_{fi} = 1 - \Phi(\beta_i) = 1 - \int_{-\infty}^{\beta_i} \exp(-t^2/2)\, dt \tag{3}$$

where P_{fi} and β_i are the collapse probability and the safety index of a combined mode i, respectively. The safety index β_i can be expressed as:

$$\beta_i = \bar{Z}_i \,/\, \sigma_{Zi} = (\bar{R}_i - \bar{S}_i) \,/\, (\sigma_{Ri}^2 + \sigma_{Si}^2) \tag{4}$$

where Z_i, R_i and S_i are the safety margin, the internal work and external work of the combined mode i, respectively; $\bar{X}$ and σ_X are the mean value and the standard deviation of X, respectively; Therefore, $\bar{R}_i$, $\bar{S}_i$, σ_{Ri} , and σ_{Si} can be written as follows:

$$\bar{R}_i = \sum_{h=1}^{H} \bar{M}_{ph} \left(\sum_{j \in h} |\theta_j| \right) \tag{5a}$$

$$\bar{S}_i = \sum_{k=1}^{K} \bar{e}_k\, t_k \tag{5b}$$

$$\sigma_{Ri} = \sum_{h=1}^{H} \bar{M}_{ph}\, V_{mh} \left(\sum_{j \in h} |\theta_j| \right) \tag{5c}$$

$$\sigma_{Si} = \sum_{k=1}^{K} \bar{e}_k\, V_{ek}\, t_k \tag{5d}$$

where $\bar{M}_{ph}$ is the mean value of plastic moment capacity of linked member h; $\bar{e}_k$ is the mean value of virtual external work of elementary mode k; V_{mh} and V_{ek} are the coefficients of variation of M_{ph} and e_k; h is the index of linked members whose strengthes are perfectly correlated.

By using Eqs.(3), (4) and (5) the collapse probability P_{fi} can be expressed as the function of unknown t_k and θ_j through the safety index β_i. As both R_i and S_i are assumed as normal distribution, the collapse mode with the maximum collapse probability is equal to the mode with the minimum safety index. Consequently, the analysis for the 1st representative mode with the maximum collapse probability can be formulated as the following NLP problem that entails the minimization of the objective function β_1 of Eq.(4) while satisfying the conditions of collapse mechanism of Eq.(2) and of the positive external work of Eq(5b).

Given: $\bar{M}_{ph},\ \bar{e}_k,\ C_{kj},\ V_{mh},\ V_{ek}$

Find: $\theta_j\ (j=1,2,..,J),\ t_{k1}\ (k=1,2,..,K)$

Object: $\beta_1 = (\bar{R}_1 - \bar{S}_1)\ /\ (\sigma_{R1}^2 + \sigma_{S1}^2)^{\frac{1}{2}} \longmapsto \min$ (6a)

Subject to: $$\theta_j = \sum_{k=1}^{K} C_{kj}\, t_{k1} \quad (j=1,2,..,J) \qquad (6b)$$

$$\sum_{k=1}^{K} \bar{e}_k\, t_{k1} > 0 \qquad (6c)$$

In order to enhance the computational efficiency, the NLP problem of Eq.(6) can be transformed into the following NLP problem, in which the number of unknown variables is decreased by substituting Eq.(6b) into Eq.(6a), and the nonlinearlity of the objective function of Eq.(6a) is weakened by introducing β_1 as an unknown variable.

Given: $\bar{M}_{ph},\ \bar{e}_k,\ C_{kj},\ V_{mh},\ V_{ek}$

Find: $\beta_1,\ t_{k1}\ (k=1,2,..,K)$

Object: $\beta_1 \longrightarrow \min$ (7a)

Subject to:

$$\sum_{h=1}^{H} \bar{M}_{ph} \Big(\sum_{j\in h} \Big| \sum_{k=1}^{K} C_{kj}\, t_{k1} \Big| \Big) - \sum_{k=1}^{K} \bar{e}_k\, t_{k1} - \beta_1 \cdot \Big[\sum_{h=1}^{H} \Big\{ \bar{M}_{ph}^2 V_{mh}^2 \Big(\sum_{j\in h} \Big| \sum_{k=1}^{K} C_{kj}\, t_{k1} \Big| \Big)^2 \Big\} + \sum_{k=1}^{K} (\bar{e}_k^2 V_{ek}^2 t_{k1}^2) \Big]^{\frac{1}{2}} = 0 \qquad (7b)$$

$$\sum_{k=1}^{K} \bar{e}_k\, t_{k1} > 0 \qquad (7c)$$

where t_{k1} is the factor defining the combination of the 1st representative mode by the elementary mode k.

The optimization problem of Eq.(7) involves the determination of safety index β_1 of the 1st representative mode and the factor t_{k1}. The SLP (Sequential Linear Programming) algorithm specially devised is used to solve Eq.(7) and, as such, the collapse probability of the 1st representative mode P_{f1} is calculated from Eq.(3).

2.2.2 The 2nd Representative Mode

The 2nd representative mode can be realized as the mode that has the maximum collapse probability of all modes whose coefficients of correlation with the 1st representative mode are less than the specified demarcating correlation ρ_0. The coefficient of correlation $\rho_{(1,n)}$ with the 1st representative mode and the other mode n is expressed as,

$$\rho_{(1,n)} = Cov(Z_1, Z_n)\ /\ \sigma_{Z1}\, \sigma_{Zn} \qquad (8)$$

where $Cov(Z_1,Z_n)$ is the covariance between safety margin of the 1st representative mode Z_1 and that of other collapse mode Z_n; σ_{Z1} and σ_{Zn} are the standard deviation of Z_1 and Z_n, respectively. $Cov(Z_1,Z_n)$, σ_{Z1} and σ_{Zn} can be written as follows:

$$Cov(Z_1,Z_n) = \sum_{h=1}^{H} \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{k1} \right| \right) \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{kn} \right| \right) \bar{M}_{ph}^2 V_{mh}^2 + \sum_{k=1}^{K} t_{k1} t_{kn} \bar{e}_k^2 V_{ek}^2 \tag{9a}$$

$$\sigma_{Z1} = \left[\sum_{h=1}^{H} \bar{M}_{ph}^2 V_{mh}^2 \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{k1} \right| \right)^2 + \sum_{k=1}^{K} \bar{e}_k^2 V_{ek}^2 t_{k1}^2 \right]^{\frac{1}{2}} \tag{9b}$$

$$\sigma_{Zn} = \left[\sum_{h=1}^{H} \bar{M}_{ph}^2 V_{mh}^2 \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{kn} \right| \right)^2 + \sum_{k=1}^{K} \bar{e}_k^2 V_{ek}^2 t_{kn}^2 \right]^{\frac{1}{2}} \tag{9c}$$

where t_{k1} and t_{kn} are the factors defining the combination of the known 1st mode and the other mode n by the elementary mode k, respectively.

It should be noted that $\rho_{(1,n)}$ can be expressed as the function of unknown t_{kn} and given t_{k1}, as evidenced in Eqs.(8) and (9).

Therefore, the 2nd representative mode can be obtained by adding the constraint of correlation to Eq.(7) as follows:

Given: $\bar{M}_{ph}$, $\bar{e}_k$, C_{kj}, V_{mh}, V_{ek}, ρ_o, t_{k1} (k=1,2,..,K)

Find: β_2, t_{k2} (k=1,2,..,K)

Object: $\beta_2 \longrightarrow \min$ (10a)

Subject to:

$$\sum_{h=1}^{H} \bar{M}_{ph} \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{k2} \right| \right) - \sum_{k=1}^{K} \bar{e}_k t_{k2} - \beta_2 \cdot \left[\sum_{h=1}^{H} \left\{ \bar{M}_{ph}^2 V_{mh}^2 \left(\sum_{j\in h} \left| \sum_{k=1}^{K} C_{kj} t_{k2} \right| \right)^2 \right\} + \sum_{k=1}^{K} \left(\bar{e}_k^2 V_{ek}^2 t_{k2}^2 \right) \right]^{\frac{1}{2}} = 0 \tag{10b}$$

$$\sum_{k=1}^{K} \bar{e}_k t_{k2} > 0 \tag{10c}$$

$$\rho_{(1,2)} = Cov(Z_1, Z_2) / \sigma_{Z1} \sigma_{Z2} < \rho_o \tag{10d}$$

where t_{k2} is the factor defining the combination of the 2nd representative mode by the elementary mode k. Solving Eq.(10) by SLP algorithm, the collapse probability P_{f2} of the 2nd representative mode can be found by Eq.(3) through the safety index β_2.

2.2.3 The N-th Representative Mode

When the 1st, 2nd, ... , (N-1)-th representative modes are known, the N-th representative mode with the minimum safety index β_N can be obtained by adding the constraints of correlation of the 1st, 2nd, ... , (N-1)-th representative modes with other modes to Eq.(7). That is,

Given: $\bar{M}_{ph}$, $\bar{e}_k$, C_{kj}, V_{mh}, V_{ek}, ρ_o, t_{kn} (k=1,2,..,K, n=1,2,..,N-1)

Find: β_N, t_{kN} (k=1,2,..,K)

Object: $\beta_N \longrightarrow \min$ (11a)

Subject to:

$$\sum_{h=1}^{H} \bar{M}_{ph} \left(\sum_{j \in h} \left| \sum_{k=1}^{K} C_{kj}\, t_{kN} \right| \right) - \sum_{k=1}^{K} \bar{e}_k\, t_{kN} - \beta_N \cdot \left[\sum_{h=1}^{H} \left\{ \bar{M}_{ph}^2 V_{mh}^2 \left(\sum_{j \in h} \left| \sum_{k=1}^{K} C_{kj}\, t_{kN} \right| \right)^2 \right\} + \sum_{k=1}^{K} \left(\bar{e}_k^2 V_{ek}^2 t_{kN}^2 \right) \right]^{\frac{1}{2}} = 0 \qquad (11b)$$

$$\sum_{k=1}^{K} \bar{e}_k\, t_{kN} > 0 \qquad (11c)$$

$$\rho_{(n,N)} = Cov(Z_n, Z_N) \,/\, \sigma_{Zn}\, \sigma_{ZN} < \rho_o \quad (n=1,2,..,N-1) \qquad (11d)$$

where t_{kN} is the factor defining the combination of the N-th representative mode by the elementary mode k.

It is noted that the calculation of Eqs.(7),(10),(11) is very efficient, because only elementary modes are required as the input data. Therefore, all representative modes can be found by using the K elementary modes.

2.2.4 Evaluation of Collapse Probability

The representative modes can be found by solving Eqs.(7),(10) and (11) in succession and their collapse probabilities P_{fn} are calculated by Eq.(3). The collapse probability of the new representative mode is obviously less than one of the previous mode and, therefore, the analysis will be terminated when the ratio of the N-th representative mode to the 1st mode becomes the following value.

$$P_{fn} \,/\, P_{f1} < \varepsilon \qquad (12)$$

where ε is the small quantity, for example, $\varepsilon = 10^{-3}$.

Finally, the collapse probability of the structure is evaluated by Eq.(1) as follows:

$$P_f = \sum_{n=1}^{N} P_{fn} \qquad (13)$$

where N is the total number of representative modes found by Eqs.(7), (10), (11), and N is less than the total number Nr when all possible collapse modes are known.

2.3 Calculation Procedure

The calculation procedure of the proposed method is performed by using Eqs.(7), (10),(11) and the flow chart is shown in Fig.1.

On the other hand, as for

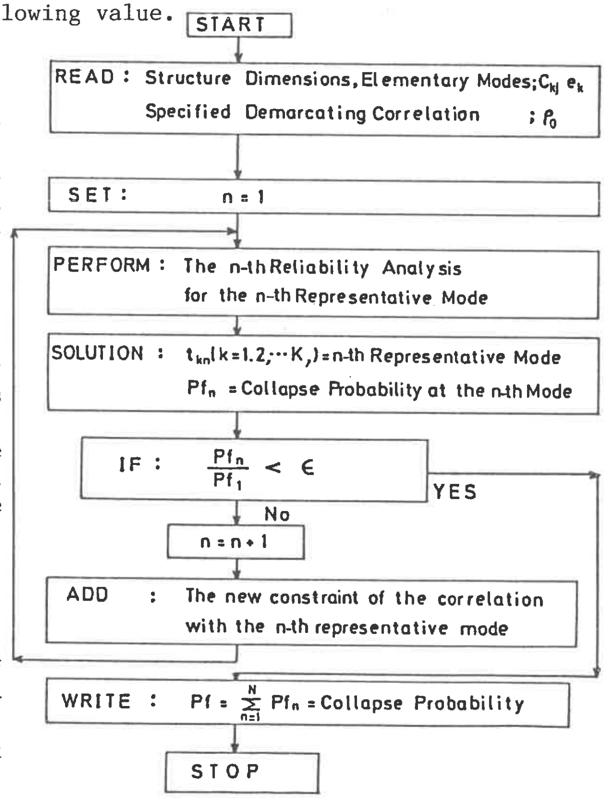


Fig. 1 Flow Chart of Caluculation Procedure

the value of the demarcating correlation ρ_o, it is used as 0.6 by Ang et al. (1979), but it should be noted that in general the value of ρ_o (ρ_o<1) should be taken as larger one (e.g., ρ_o=0.9-0.95), if the collapse probability P_f is smaller (e.g., $P_f<10^{-3}$) from the results of the 1st representative mode by Eqs.(7).

3 MONTE CARLO SIMULATION

3.1 Conventional Monte Carlo Simulation

The reliability analysis procedure by conventional Monte Carlo simulation is performed as follows:
(1) Random variables i.e., strength of section and external loads which are assumed to be probabilistic values are initially sampled according to the normal distribution function.
(2) Then, it is judged whether the structure is in the collapse state or not for each set of sample.
(3) Repeat steps (1),(2) until enough number of trial will be performed.
(4) Evaluate the collapse probability as the proportion of the structural collapse number to all trial number.
It should be noted that the judge of step (2) needs generally the value of safety margin of all possible collapse modes, because it is judged whether the safety margin of all modes is positive or not. However, it is very difficult to find all possible collapse modes for the highly redundant structures with the great number of collapse modes.

3.2 Revised Monte Carlo Simulation Using Collapse Load Analysis

This paper also proposes the revised method of Monte Carlo simulation using collapse load analysis by linear programming in order to find the exact collapse probability of the large-scaled structures. The procedure is performed as follows (See Fig.2);
(1) First, it is assumed that a set of probabilistic sample values in the s-th trial is the plastic moment $M_{ph}(s)$ and the external load $e_k(s)$.
(2) Then, the collapse load analysis for a set of sample whose values are deterministic is performed by the following LP problem.

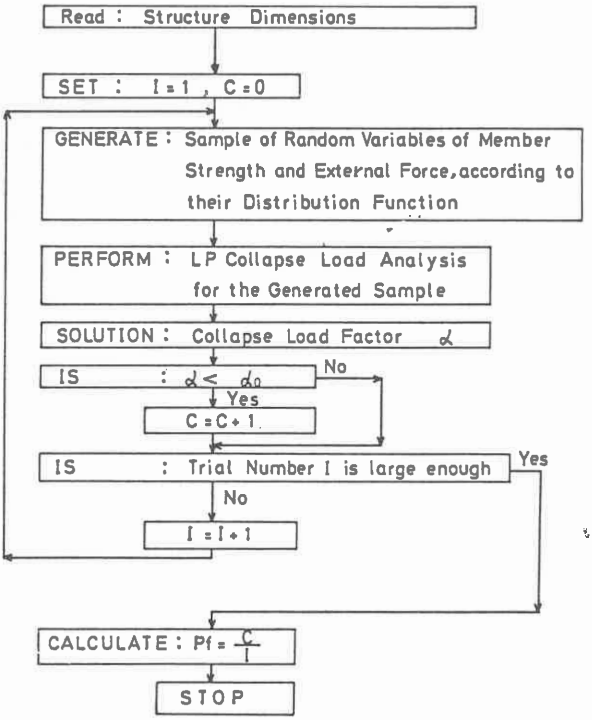


Fig. 2 Revised Monte Carlo Simulation

Given: $M_{ph}(s)$, $e_k(s)$, C_{kj}

Find: θ_j (j=1,2,..,J), t_k (k=1,2,..,K)

$$\text{Object:} \quad \alpha = \sum_{h=1}^{H} M_{ph}(s) \left(\sum_{j \in h} |\theta_j| \right) \longrightarrow \min \tag{14a}$$

$$\text{Subject to:} \quad \theta_j = \sum_{k=1}^{K} C_{kj}\, t_k \quad (j=1,2,..,J) \tag{14b}$$

$$\sum_{k=1}^{K} e_k(s)\, t_k = 1 \tag{14c}$$

where α is the collapse load factor for a set of sample s and it plays a role of judgement whether the structure is in the collapse state or not. That is, if the value of α is larger than the specified ultimate load factor α_o the structure is strong in the load carrying capacity. However, if the value of α is less than α_o ($\alpha < \alpha_o$), the structure is weak in the whole strength and, therefore, it will form the collapse mechanism.

(3) By repeating the performance of steps (1) and (2), the collapse probability of a structure can be evaluated by the proportion of the total collapse number (C) to the total trial number (I), i.e., P_f=C/I.

This approach has the merits that only elementary modes are required as the input data and it can be also applied to the highly redundant structures, although it consumes more CPU time.

4 EXAMPLES

4.1 Example 1: 2-Story 1-Span Frame

The 2-story 1-span frame as shown in Fig.3 is to be analyzed to illustrate the validity of this approach. This frame has 8 elementary modes (K=8) shown in Fig.4, where C_{kj} and e_k are also identified. The demarcating correlation is herein assumed as ρ_o=0.9.

Initially, performing the analysis to find the 1st representative mode by Eq.(7), the safety index β_1=3.49 and the 1st representative mode are found to be shown in Fig.5(a). Therefore, the collapse probability of the 1st representative mode is calculated by Eq.(3) as $P_{f1}=0.239*10^{-3}$.

Then, the analysis to find the 2nd representative mode is performed by adding the constraint of correlation with the 1st representative mode to Eq.(7), as formulated by Eq.(10). The results are obtained as β_2= 3.76 and the 2nd representative mode shown in Fig.5(b).

Successively, performing the analyses for the 3rd, 4th and 5th representative modes by using Eq.(11), the results are found to be shown in Fig.5. It is noted that $P_{f5}=0.91*10^{-7}$ is extremely less than $P_{f1}=0.239*10^{-3}$ and, as such, the analysis is terminated.

Finally, the collapse probability of the structure $P_f=0.403*10^{-3}$ is evaluated by Eq.(13).

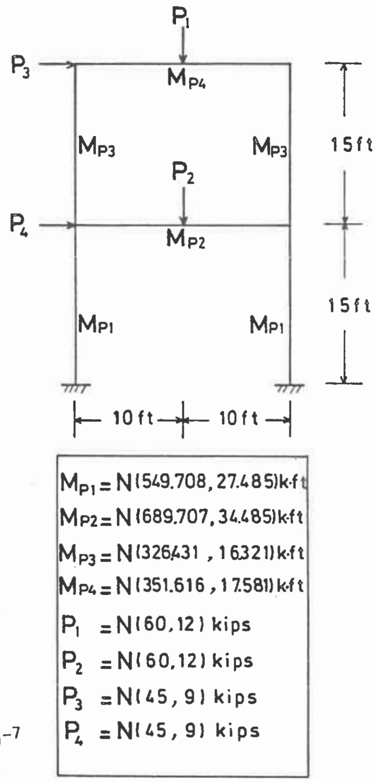


Fig. 3 Example 1: 2-Story 1-Span Frame

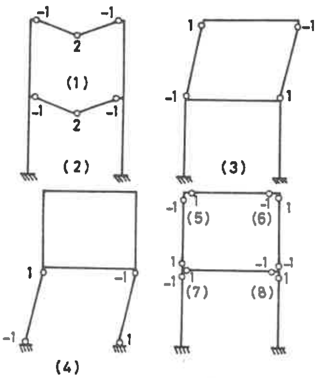


Fig. 4 Example 1: Elementary Modes

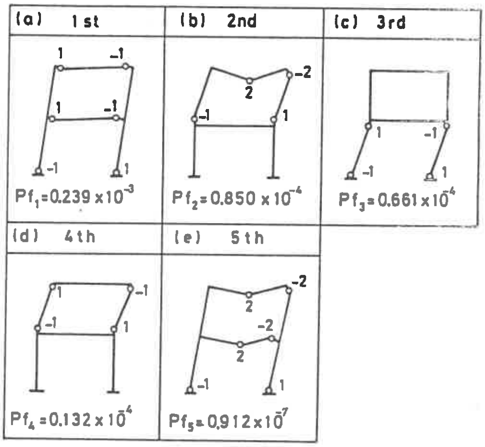


Fig. 5 Example 1: Representative Modes

This result is very close to the one obtained by Yonezawa's method (1978) as shown in Table 1, which considers 19 dominant collapse modes and their mutual correlations.

Furthermore, the conventional Monte Carlo simulation (100 000 trial) which considered 60 all possible collapse modes and the revised Monte Carlo simulation (100 000 trial) by collapse load analysis have been performed by too-much time consuming. That is, it takes 1.7min. and 3.2min. in CPU time, respectively. The latter method takes about 2 times of the former method in CPU times, but the input data of the latter method only requires elementary modes. It should be also noted from Table 1 that both Monte Carlo simulation results are completely coincided with each other and the proposed method which takes CPU time=45sec. is very near to these results by the conventional and revised Monte Carlo simulation.

Table 1 Example 1: Analysis Results

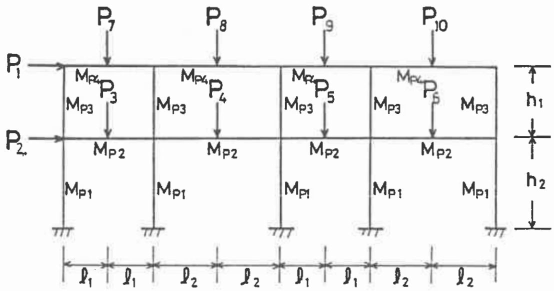
	P_f	CPU time
Proposed Method	$0.403*10^{-3}$	45 sec.
Yonezawa's Method	$0.410*10^{-3}$	---
Conventional Monte Calro Simulation	$0.440*10^{-3}$	1.7 min.
Revised Monte Calro Simulation	$0.440*10^{-3}$	3.2 min.

4.2 Example 2: 2-Story 4-Span Frame

The 2-story 4-span frame as shown in Fig.6 is analyzed in order to illustrate the application of the approach to the large structure. The value of demarcating correlation ρ_0=0.95 is herein used.

It is identified by the degree of freedom of joint displacement that the elementary modes with their relative rotation are shown in Fig.7. Performing the reliability analysis by the proposed method, the 1st, 2nd,....,6th representative modes and their collapse probabilities are found to be shown in Fig.8. Therefore, the collapse probability of this frame is evaluated as $P_f=0.73*10^{-3}$ shown in Table 2. It should be noted from Table 2 that this value is fairly close to the one obtained by the revised Monte Carlo simulation (150 000 trial) using collapse load analysis. It is also noted from Table 2 that the CPU time by the proposed method is extremely less than the one by the revised Monte Carlo simulation.

It is recognized that this large frame can not be analyzed by the conventional Monte Carlo simulation, because all possible modes can not be identified. Therefore, the only proposed method can be applied to the large structure by the simple and efficient calculation.



M_{P1}= N (80,8) t·m	P_1 = N (100,16) t
M_{P2}= N (60,6) t·m	P_2 = N (120,18) t
M_{P3}= N (40,4) t·m	P_3 = P_5= N (40,4) t
M_{P4}= N (30,3) t·m	P_4 = P_6= N (60,6) t
h_1 =2.0 m , h_2= 2.4 m	P_7 = P_9= N (20,2) t
ℓ_1 =1.2 m , ℓ_2= 1.8 m	P_8 = P_{10}= N (30,3) t

Fig. 6 Example 2: 2-Story 4-Span Frame

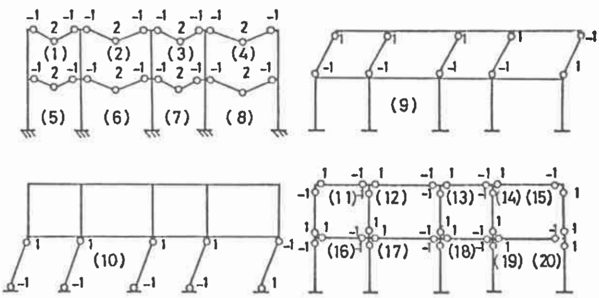


Fig. 7 Example 2: Elementary Modes

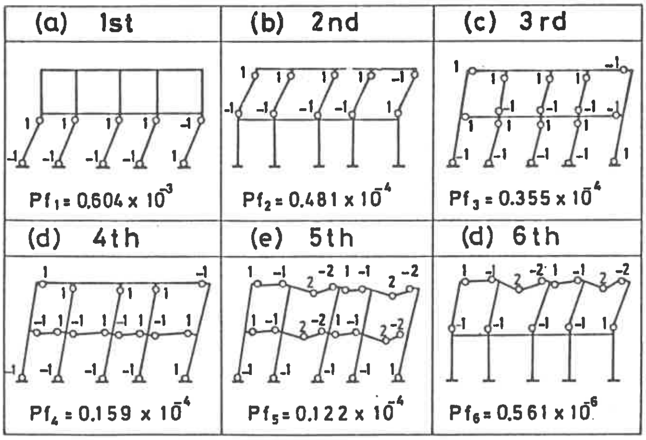


Fig. 8 Example 2: Representative Modes

Table 2 Example 2: Analysis Results

	P_f	CPU time
Proposed Method	$0.73*10^{-3}$	160 sec.
Revised Monte Calro Simulation (150 000 trial)	$0.82*10^{-3}$	13.5 hour

5 CONCLUSIONS

The following conclusions are drawn from this study.

(1) The proposed method makes the best use of an optimization technique and, as such, the method can be applied to the large-scaled structures in which all possible collapse modes can not be easily identified by hand task.

(2) The proposed method can take into account for the correlation of collapse modes without considering all possible collapse modes and, furthermore, its calculation is very efficient, because only elementary modes are required as input data.

(3) The validity of this proposed method has been confirmed numerically by comparing with the conventional and the revised Monte Carlo simulations and the other available results.

(4) The revised Monte Carlo simulation by collapse load analysis is also verified by the conventional Monte Carlo simulation which needs to consider all possible collapse modes. This approach needs only elementary modes and, therefore, the simulation can apply to the highly redundant structure, but its calculation requires more CPU time consuming.

(5) The proposed method is extremely efficient in computer time and in simple calculation rather than the revised Monte Carlo simulation and, therefore, the proposed method can be easily extended to the optimal reliability-based design for the large-scaled structures.

(6) This study dealts with only flexural frames, but the proposed method will be applied to the skeletal structures such as offshore platform or transmission tower in the future.

ACKNOWLEDGEMENTS

Authors are indebted to Professor D. E. Grierson of the University of Waterloo for his kind comment. The use of the HITAC M-200H Computer of the National Defense Academy is greatly appreciated.

REFERENCES

Ang, A.H-S., Abdelnour, J. and Chaker, A.A. 1975. Analysis of Activity Network under Uncertainty. Proc. of ASCE, EM4, 101:373-383.

Ang, A.H-S. and Ma, H-F. 1979. On the Reliability Analysis of Framed Structures, Proc. of the Speciality Conference on Probability, ASCE, Tuscon, Arizona, 106-111.

Ang, A.H-S. and Ma, H-F. 1981. On the Reliability of Structural Systems, Moan, T. and Shinozuka, M.(Ed.), Structural Safety and Reliability, Elsevier, 295-314.

Cohn, M.Z., Ghosh, S.K. and Parimi, S.R. 1972. Unified Approach to Theory of Plastic Structures, ASCE, EM5, 98:1133-1158.

Ditlevsen, O. and Bjerager, P. 1985. Reliability of Highly Redundant Plastic Structures, Proc. of ASCE, EM5, 110:671-693.

Mihara, T., Iizuka, M., Ishikawa, N. and Furukawa, K. 1986. Optimal Plastic Design of Large Scaled Structures under the Constraint of Safety Index, Proc. of Structural Engineering, 32A:475-483. (in Japanese)

Moses, F., and Kinser, D. 1967. Analysis of Structural Reliability, Proc. of ASCE, ST4, 83:147-164.

Murotsu, Y., Okada, H., Yonezawa, N. and Grimmelt, M. 1983. Reliability Analysis of Plane Frames, Proc. of JSME(A), 483:230-238. (in Japanese)

Shiraishi, N., Furuta, H. and Nakano, M. 1980. Some Considerations on Application of Safety Index to Reliability Analysis, Proc. of JSCE, 301: 13-22 (in Japanese)

Yonezawa, M., Murotsu, Y., Ohba, F. and Niwa, I. 1978. Reliability Analysis of Structures (in the Case of Normal Distribution), Proc. of JSME(A), 395:2936-2945. (in Japanese)