ABSTRACT

YANG, BO. On The HCM's Treatment of Trucks on Freeways. (Under the direction of Dr. George List.)

Trucks play an important role in freight transportation and the significance of evaluating truck performance on freeways has recently received attention. In the Highway Capacity Manual (HCM), the concept of passenger car equivalency (PCE) is presently used to estimate the influence of trucks on the mixed traffic stream. However, the HCM has limitations in its ability to predict truck PCE values for various weight-to-horsepower ratios, grades, and truck types. Moreover, there is no explicit accounting for the quality of service experienced by the trucks themselves. To further explore truck influences on the traffic stream along a freeway along freeway segment, this research focused on three topics: predicting truck speeds, estimating PCE values, and predicting auto speeds in mixed flows. For truck speeds, an adaptation of prior research was employed. For the PCE values, a predictive regression model has been developed based on grade percentage, truck type, weight-to-horsepower ratio, and truck percentage. For auto speed prediction, a new model was developed that is sensitive to the percentage of trucks in the mixed traffic stream. The model uses a logistics equation based formulation, the V/C ratio and two calibration parameters to predict auto speed for a given traffic flow condition.

On the HCM's Treatment of Trucks on Freeways

by
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BIOGRAPHY

Bo Yang was born on May 2nd, 1988 in Beijing. He finished his study from primary school to high school in Beijing and spent four years his undergraduate study in Tongji University in Shanghai from 2006 to 2010.

In August, 2011, he attended North Carolina State University for his master degree in Civil Engineering in Raleigh, NC. His research focused on trucks' effect on traffic stream along freeways. He is advised by Dr.George List.

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1. PROBLEM, PURPOSE, AND CONTEXT

1.1. Problem Statement

Truck traffic has increased significantly over the past decades. It affects congestion, crash severity, air pollution, pavement damage, and delay because truck sizes and weights are greater than those of passenger cars. Furthermore, limits on truck performance during lane changing, acceleration and deceleration, have significant impacts on freeway traffic flow.

The impact of heavy vehicles on freeway operation is addressed in the HCM 2000 (1). The HCM 2000 uses passenger car equivalent values to convert trucks, buses, and recreational vehicles (RVs) into passenger cars so that an adjusted auto-only flow predicts a similar V/C ratio, and hence density which has performance equivalent to that of the mixed traffic flow. It does not address truck speeds or LOS and it does not indicate how auto speeds are affected by trucks, except through the PCEs.

Insofar as truck speeds are concerned, the HCM 2010 (2) provides only one case (Exhibit 11-21) and it is based on a truck with a weight-to-horsepower ratio of 200lbs/hp. This is a typical of the trucks in use today. Moreover, this exhibit is for only one type of truck with a specific weight-to-horsepower ratio and the starting speed is the same for upgrades and downgrades.

Insofar as PCEs are concerned, the current HCM methodology has three shortcomings. Firstly, in order to make it easy for users to apply PCEs, the HCM heavy vehicle classes are only trucks, buses, and recreational vehicles. Secondly, the HCM applies a PCE look-up table in which interpolation is required. Thirdly, the HCM PCE values do not take into account truck weight to horsepower ratios, truck classification, and length.

Insofar as auto speeds are concerned, there is no explicit calculation of auto speeds in the mixed traffic stream. Trucks are converted to autos through the PCE values and then an auto-only speed is computed according to the adjusted flow rate under the assumption that trucks can be treated as autos.

1.2. Purpose of this Study

This research focuses on three items related to the HCM treatment of trucks on freeways: 1) estimates of truck speeds in mixed traffic, 2) estimates of truck PCE values for trucks with various weight-to-horsepower ratios, and 3) estimates of auto speeds in mixed traffic. For truck speed prediction, a prior model is updated so it can predict truck speeds for a range of weight-to-horsepower ratios and truck percentages along different grades. For PCEs, instead of using a look up table, an equation has been developed to estimate truck PCEs based on grade, truck type, volume, and weight-to-horsepower ratio. For auto speed prediction, a new relationship has been developed for settings with high truck percentages and a predictive model is built around the resulting trends.

1.3. Context of the Study

This research work is related to the NCFRP-41 “Incorporating Truck Analysis into the Highway Capacity Manual” project effort. The author is the principal graduate student involved in that effort. The research work further advances the preliminary findings in that effort to refine and enhance the models used to predict PCEs and auto speeds. To avoid duplication of text, some work from the NCFRP-41 project is cited. This research aims to estimate the influences of trucks on freeway based on the results of VISSIM (3) in three ways:

PCE value prediction, truck speed prediction, and auto speed prediction in mixed traffic stream.

1.4. Simulation-Based Data

All of the predictive models are based on outputs (speed, flow, density, etc.) from VISSIM models. The principal VISSIM runs upon which the research is based are for settings involving trucks of FHWA Type 5 and Type 9. For each of these types, four weight-to-horsepower ratios have been considered: 50lbs/hp, 100lbs/hp, 150lbs/hp, and 200lbs/hp.

The truck acceleration-versus-speed curves for each of these truck type, weight-to-horsepower ratio combinations are derived from an enhanced version of the truck performance model described in NCHRP Report 505 (4) and a separate model developed by Hesham et al. 2001 (5). The VISSIM model consists of level part and grade part of which all have three lanes and speed limit of 70mph. The level part has a length of 8 miles with a grade of 0% while the graded part has a length of 5 miles. Thirteen separate models consider the grade to be -6%, -5%, …. 6%. The input volume ranges from 0 vph to 7200vph and the truck percentages are 10%, 20%, 30%, 40%, and 50%. Each model has one type of truck with a single weight-to-horsepower ratio.

The truck PCE values model are developed based on the capacity difference of base traffic stream and mixed traffic stream. The truck speeds are developed based on both the truck acceleration characteristics and VISSIM outputs.

The auto speeds are estimated from auto-only speeds, truck-only speeds, and the auto speeds observed for the various mixed flow scenarios (truck percentages, grades, etc.).

2. LITERATURE REVIEW

Since the research described here focuses on truck speeds, truck PCEs, and auto speeds, literature reviews are provided on all three of these topics.

2.1. Truck Speeds

The HCM as well as the Green Book (2001) (6) use truck speed profile plots (plots of truck speed versus distance along a grade) to predict truck speed on a specific composite lengths and grades. These plots are not as useful as they might be because they assume a single initial truck speed for upgrade or downgrade and a single truck weight-to-power ratio (200lbs/hp).

To address this problem, Harwood et al. (4) provide a truck speed profile model spreadsheet which allows user both roadway characteristics and truck characteristics. The road characteristics include the percent grade and elevation above sea level. The truck characteristics include desired speed, initial speed, weight-to-power ratio, and weight-to-front-area ratio. The truck acceleration consists of three parts: coasting acceleration during gear shifts, horsepower-limited acceleration, and effective acceleration including an allowance of 1.5s for gear shift delays. The limitations of this model are that the effect of grade is only included in coasting acceleration but not in the resistance equations and there is no upper bound on tractive effort due to the weight on the powered axles.

Rakha et al. (2001) (5) present a vehicle dynamics models in which the tractive effort is based on engine power, truck speed, and transmission efficiency. Equations 2.1 and 2.2 present the tractive effort and maximum tractive force.

$$F_t = 3600\eta \frac{P}{V} \qquad 2.1$$

$$F_{\max} = 9.8066 M_{ta} \mu \qquad 2.2$$

where, F_t: tractive effort (N);

P: engine power (kW);

V: truck speed (km/h);

η: transmission efficiency

F_{max}: maximum tractive force (N);

M_{ta}: vehicle mass on tractive axle (kg) such that M_{ta}=M.perc$_{ta}$;

$perc_{ta}$: percent mass acting on tractive axle;

μ: coefficient of friction between tires and pavement; and

F: tractive effort effectively acting on truck (N).

Equation 2.2 indicates that the maximum tractive force is a function of the proportion of the vehicle mass on the tractive axle. The tractive force of a truck is the minimum of Equation 2.1 and 2.2.

In addition, the resistance terms are discussed. Equation 2.3 shows the resistance equation.

$$R = R_a + R_r + R_g \qquad 2.3$$

Where, R: total resistance (N);

R_a: air drag or aerodynamic resistance (N);

R_r: rolling resistance (N); and

R_g: grade resistance (N).

The total resistance consists of aerodynamic resistance, rolling resistance, and grade resistance. The maximum acceleration was then calculated by equation 2.4.

$$a = \frac{F - R}{M} \qquad 2.4$$

Where, a: maximum truck acceleration (m/s^2);

F: tractive effort (N);

R: total resistance force (N); and

M: vehicle total mass (kg).

In this model, vehicle power is assumed to be a constant number while Rakha and Lucic (2002) (7) introduce the concept of a linearly increasing variable power. Equation 2.1 is replaced by Equation 2.5.

$$F_t = 3600 \beta \eta \frac{P}{V} \qquad 2.5$$

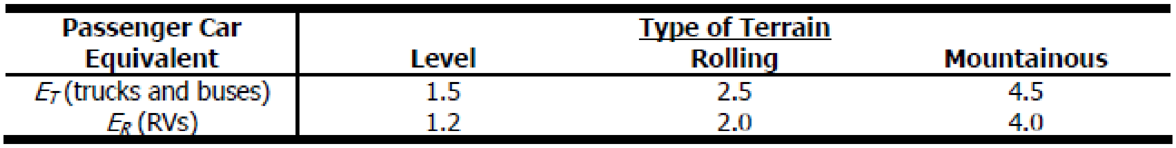
The added variable β as a function of vehicle speed takes the engine characteristics into consideration.

In the research work reported here, the model created by Harwood et al. (4) is enhanced by adding grade resistance term and a tractive effort limit; and some logic errors have been corrected.

2.2. Truck PCEs

The HCM 2010 uses the passenger car equivalency concept that Webster and Elefteriadou 1999 (8) created to represent the number of passenger cars that would have an equivalent effect on the quality of the traffic using the freeway. These passenger car equivalents were estimated based on the flow-density curves developed through the output of numerous simulation runs. For purposes of simplifying the procedure used to evaluate level of service along freeway, a brief look up table of these PCE values was created. Table 1 shows the suggested PCE values for the analysis of freeways in level, rolling, and mountainous terrain.

Table 1: Passenger Car Equivalents for Heavy Vehicles in General Terrain Segments in the HCM2010 (2)



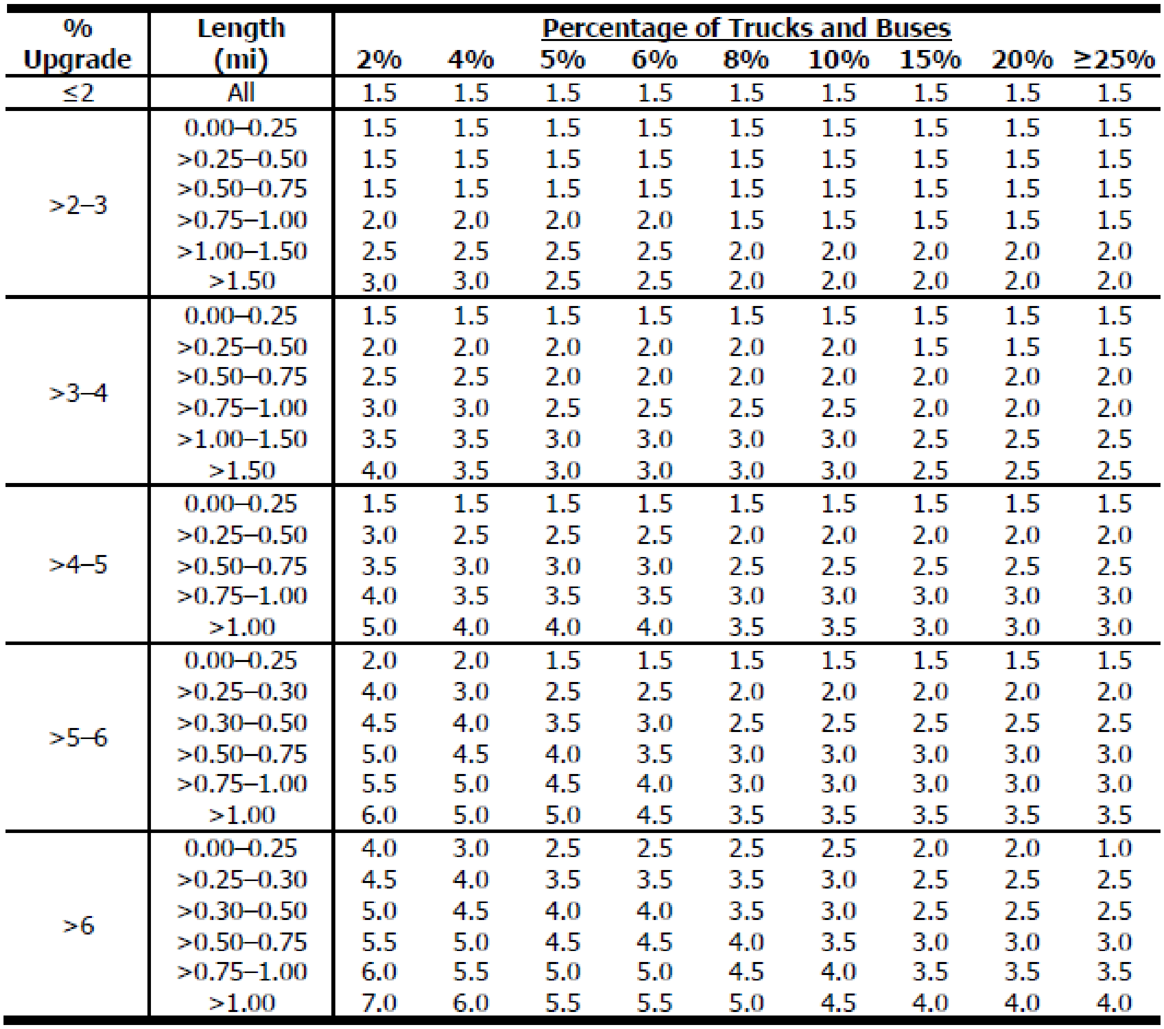
Passenger Car Equivalent	Type of Terrain		
	Level	Rolling	Mountainous
E_T (trucks and buses)	1.5	2.5	4.5
E_R (RVs)	1.2	2.0	4.0

Table 1 is presented for users to use in general terrain segments. Heavy vehicles are classified as trucks, buses, and RVs. As stated in the HCM 2010, any freeway grade between 2% and 3% and longer than 0.5 mile or 3% or greater and longer than 0.25 mi, is to be considered a separate segment. For further analysis in terms of freeway segments with composite grades, a more detailed exhibit is introduced for users to look up PCE values of different upgrades, lengths of grade, and percentage of heavy vehicles up to 25%.

Table 2 shows the comprehensive passenger car equivalents for trucks and buses on upgrades in the HCM 2010.

From the description above, it is obvious that the HCM uses exhibits presenting PCE values in a simple and user friendly way. However, interpolation is required for situations not matching those described in the table. In addition, the look up table has a limitation in terms of grade and truck type. To estimate PCE values in a more sophisticated way, it is necessary to be aware the current status of researches in developing PCE values.

Table 2: Passenger Car Equivalents for Trucks and Buses (E_T) on Upgrades in the HCM 2010 (2)



% Upgrade	Length (mi)	Percentage of Trucks and Buses								
		2%	4%	5%	6%	8%	10%	15%	20%	≥25%
≤2	All	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
>2–3	0.00–0.25	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.25–0.50	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.50–0.75	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.75–1.00	2.0	2.0	2.0	2.0	1.5	1.5	1.5	1.5	1.5
	>1.00–1.50	2.5	2.5	2.5	2.5	2.0	2.0	2.0	2.0	2.0
	>1.50	3.0	3.0	2.5	2.5	2.0	2.0	2.0	2.0	2.0
>3–4	0.00–0.25	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.25–0.50	2.0	2.0	2.0	2.0	2.0	2.0	1.5	1.5	1.5
	>0.50–0.75	2.5	2.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0
	>0.75–1.00	3.0	3.0	2.5	2.5	2.5	2.5	2.0	2.0	2.0
	>1.00–1.50	3.5	3.5	3.0	3.0	3.0	3.0	2.5	2.5	2.5
	>1.50	4.0	3.5	3.0	3.0	3.0	3.0	2.5	2.5	2.5
>4–5	0.00–0.25	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.25–0.50	3.0	2.5	2.5	2.5	2.0	2.0	2.0	2.0	2.0
	>0.50–0.75	3.5	3.0	3.0	3.0	2.5	2.5	2.5	2.5	2.5
	>0.75–1.00	4.0	3.5	3.5	3.5	3.0	3.0	3.0	3.0	3.0
	>1.00	5.0	4.0	4.0	4.0	3.5	3.5	3.0	3.0	3.0
>5–6	0.00–0.25	2.0	2.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	>0.25–0.30	4.0	3.0	2.5	2.5	2.0	2.0	2.0	2.0	2.0
	>0.30–0.50	4.5	4.0	3.5	3.0	2.5	2.5	2.5	2.5	2.5
	>0.50–0.75	5.0	4.5	4.0	3.5	3.0	3.0	3.0	3.0	3.0
	>0.75–1.00	5.5	5.0	4.5	4.0	3.0	3.0	3.0	3.0	3.0
	>1.00	6.0	5.0	5.0	4.5	3.5	3.5	3.5	3.5	3.5
>6	0.00–0.25	4.0	3.0	2.5	2.5	2.5	2.5	2.0	2.0	1.0
	>0.25–0.30	4.5	4.0	3.5	3.5	3.5	3.0	2.5	2.5	2.5
	>0.30–0.50	5.0	4.5	4.0	4.0	3.5	3.0	2.5	2.5	2.5
	>0.50–0.75	5.5	5.0	4.5	4.5	4.0	3.5	3.0	3.0	3.0
	>0.75–1.00	6.0	5.5	5.0	5.0	4.5	4.0	3.5	3.5	3.5
	>1.00	7.0	6.0	5.5	5.5	5.0	4.5	4.0	4.0	4.0

Chitturi and Benekohal 2007 (9) presented an idea about PCEs based on using headways (time or space). This method has been used for freeway sections and rural highways by considering the space headways. Greenshields et al. (1947) (10) pioneered the headway ratio method and the basic formula of for a PCE using the headway approach as follows:

$$PCE_i = \frac{\overline{h_i}}{\overline{h_c}} \qquad 2.6$$

where, $\overline{h_i}$ =Average Time Headway for Vehicle of Class i.

$\overline{h_c}$ = Average Time Headway of Passenger Cars

In Equation 2.6, the average time headway is the time headway between the vehicles discharging from queue as they pass the stop-line.

Ahmed (2009) (11) used regression model to find equation computing headways based on speed. Individual speeds were divided into 10 speed ranges from 0mph to greater than 50mph. The mean headway for each of these speed ranges was calculated and then Equation 2.7 was developed to compute vehicle headway.

$$Vehicle\ Headway = 0.0018 * Speed^2 + (-0.14) * \text{Speed} + 4.94 \qquad 2.7$$

where vehicle headway is measured in seconds. Vehicle speed (the mid-point of speed range) is measured in mph. The PCE values are computed according to Equation 2.6.

A discussion of the lagging headway for trucks following trucks and cars following trucks to compute PCE values is presented by Krammes and Crowley (1986) (12). Equation 2.8 is presented in their research.

$$PCE = \frac{(1-p)(h_{pt} + h_{tp} - h_{pp}) + ph_{tt}}{h_{pp}} \qquad 2.8$$

where, p =percentage of trucks at a mixed traffic stream;

h_{pt} = Mean headway time in seconds for trucks following PC;

h_{tp} = Mean headway time in seconds for PC following trucks;

h_{pp} = Mean headway time in seconds for PC following PC;

h_{tt} =Mean headway time in seconds for trucks following trucks.

Huber (1982) (13) proposed that PCE-values should be related to the ratio between the volumes of mixed stream and auto only stream at some common level of impedance. In Huber's research, PCE equations were derived by three criteria (impedance): speed, density, and passenger car speed in the base and mixed streams. The model to estimate the impedance-flow relationship is deterministic. Huber's basic equation is as follows:

$$E_T = \frac{1}{P_T}\left(\frac{q_B}{q_M} - 1\right) + 1 \qquad 2.9$$

where, q_B =equivalent passenger car only flow rate

q_M =mixed flow rate

P_T =truck proportion in the mixed traffic flow

Sumner et al.(1984) (14) addressed the drawback of the assumption that the mixed vehicle flow contains passenger cars and only one type of truck by further developing an equation including more than one truck type in the traffic steam. The equation is as followed:

$$E_T = \frac{1}{\nabla P}\left(\frac{q_B}{q_S} - \frac{q_B}{q_M}\right) + 1 \qquad 2.10$$

where, q_S=additional subject flow rate

∇ P =proportion of subject vehicles

As discussed previously, Webster and Elefteriadou (1999) created the methods to calculate PCEs for trucks based on the flow versus density relationships provided by simulation model. The results in their research showed that PCEs increase with increasing traffic flow on freeway segments and decrease with increasing proportion of trucks and number of lanes. Due to the definition of truck type in the simulation model, length and weigh to power ratio is critical for determination of PCEs (15). Al-Kaisy, Jung and Rakha (2005) (16) stated that recent empirical evident suggests that the PCE factor for free-flow conditions largely underestimate the effect of heavy vehicles after the onset of congestion. The authors did a very similar to that of the HCM PCE factors for free-flow conditions to compute PCE factors for oversaturated conditions.

Werner and Morral (1976) (17) discussed a sensitivity testing of a recreational vehicle simulator model which was extended to include highway capacity. The authors find that there is a need of further refinement and updating of the present passenger car equivalent speed curves and adjustment factors in the 1965 HCM and estimated their correct placement by applying basic traffic engineering relationships.

Two other efforts are similar to Werner and Morral. St.John et al. (1978) (18) developed a nonlinear relationship for computing PCE values based the mean speed and the truck factor is calculated based on speed flow curve. Aerde and Yagar (1984) (19) examined PCE values derived for purposes of capacity, speed, and platooning analyses using literature sources and traffic data analyzed for 37 different two-lane rural highway sites. The authors found that the relative effects of trucks and recreational vehicles were much smaller than the corresponding equivalents for speed.

Kockelman and Shabih (2000) (20) examined the effect of light-duty trucks on the capacity of signalized intersections. The authors broke light duty trucks (LDTs) into detailed categories such as single large sport-utility vehicles and vans and intersection capacity reduction was discussed according to headways. PCE computed based on capacity represents the number of passenger cars that would use the same amount of highway capacity as the vehicle being considered under the prevailing roadway and traffic conditions.

Al Kaisy and Ahmed (2006) (21) found that HCM PCE factors are inappropriate for the analysis of queues and congestion. Applying queue discharge flow as an equivalency criterion in developing PCE factors for use in determining capacity and analyzing queues and congestion is proposed in their research.

Geistefeldt (2009) (22) presented a new empirical method for estimating PCE values for heavy vehicles along freeway based on stochastic capacities. The stochastic capacities concept is related to empirical capacity distribution functions for specific roadway, traffic, and control conditions.

In summary, PCE values can be computed multiple ways such as headway, density, speed, and capacity. In the research work that follows, PCE values are based on capacity equivalency. The 95th percentile of the flow on the network is defined as the capacity and equivalence is achieved through the PCE value between the 95th percentile flow for the mixed traffic stream and the 95th percentile flow for the auto-only traffic stream.

2.3. Auto Speeds

The HCM applies speed-flow curves for 5 types of freeway segments under base conditions. Figure 1 shows the HCM flow-speed curves. In the base condition, there is an assumption of no heavy vehicles (trucks, buses, recreational vehicles) in the traffic stream.

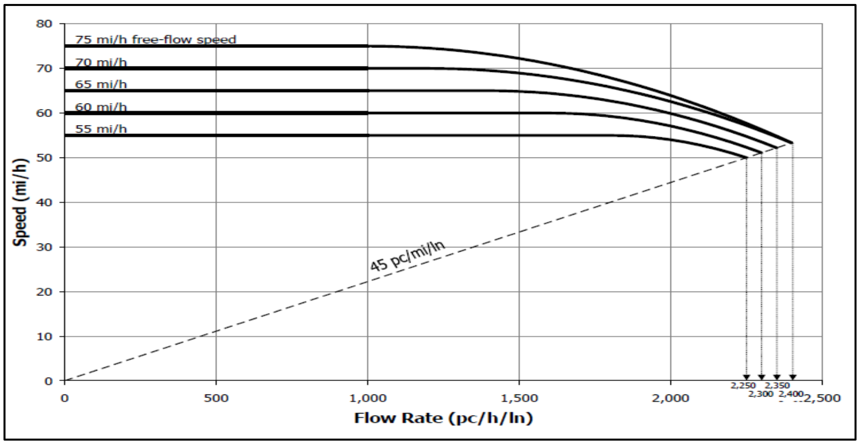


Figure 1: the HCM Speed-Curves for Basic Freeway Segments under Base Conditions

In the HCM procedure of evaluating of LOS, the heavy vehicles are first converted to passenger cars and the speed flow curves are then applied to compute auto speeds. The speed-flow curve under base condition is useful when PCE values apply.

However, due to the dissimilar performance between trucks and cars especially on upgrades, autos can be affected by trucks in a significant way. If the truck percentage is high, autos cannot overtake the trucks. In such situations, auto speed is limited by truck speed. To find the relationship between auto speed and truck speed, this thesis focuses on developing auto speed based on auto only speed and truck speed.

3. BASELINE TRUCK SPEEDS

As mentioned in Section 1, this research work is closely related to NCFRP-41. Therefore, there is overlap between that effort and this thesis. To be clear, the basic ideas for the development of models for truck speeds, truck PCEs, and auto speeds were developed as part of the project effort, for which the author was one of the principal researchers, and the work presented here, especially for truck PCEs and auto speeds, is an extension of that work. Figure 2 presents a flowchart of the NCFRP-41 effort. In this paper, Steps 2 to 5 are discussed in detail. Step 5, which is highlighted, is the extension work beyond the report which is the calibration of the coeffiecients in the auto speed estimation equation.

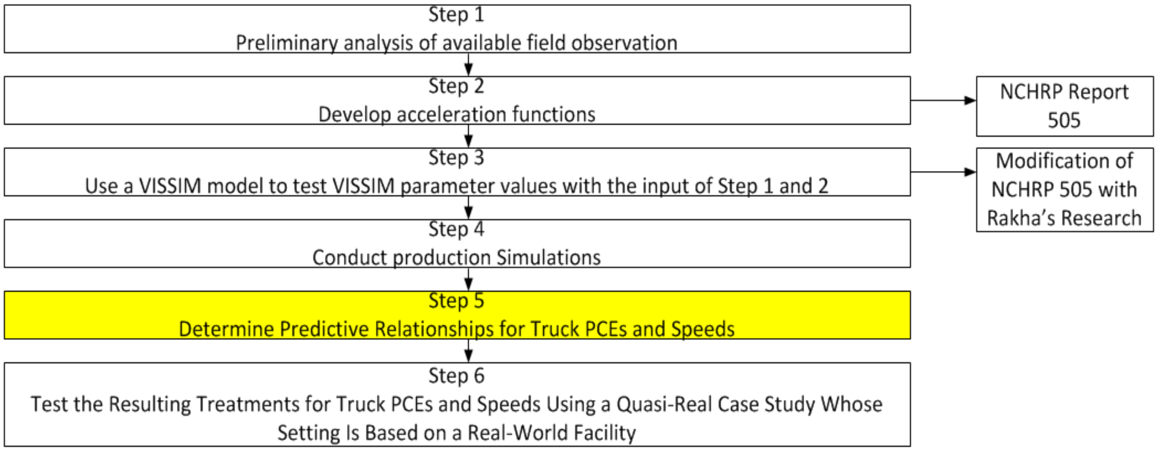


Figure 2: NCFRP-41 Report Steps

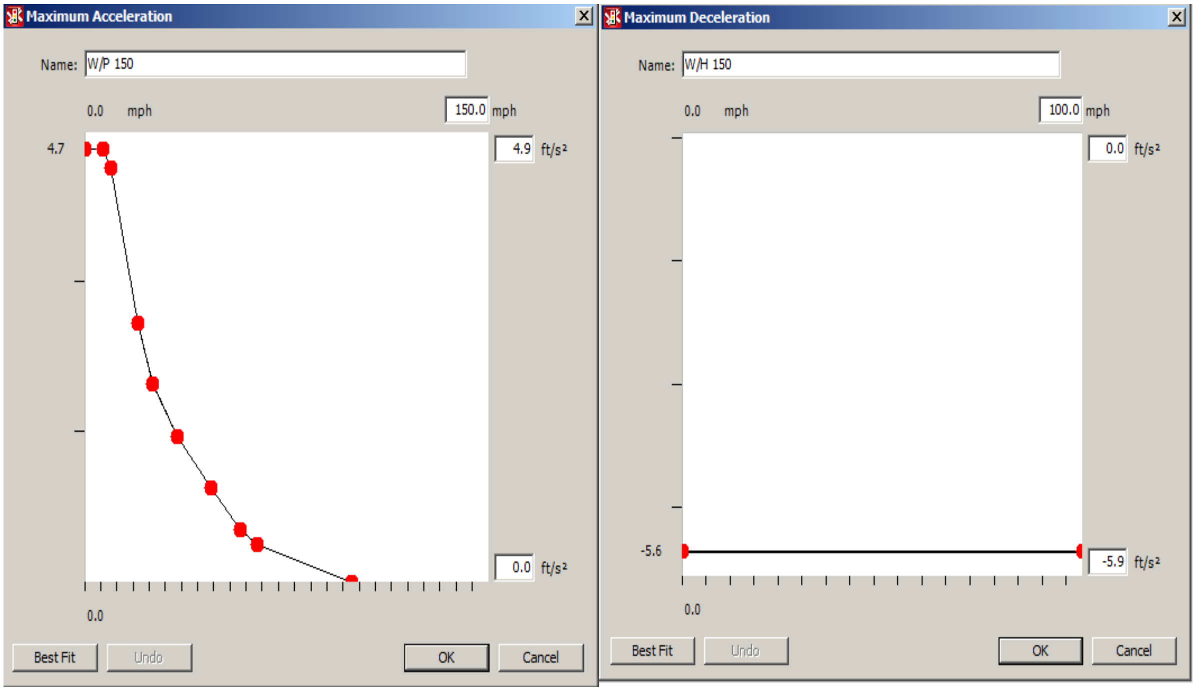
Steps 2 to 4 are described in detail in the NCFRP-41 report. To avoid overlap in content, these steps are discussed briefly in the text that follows.

3.1. Steps 2 and 3 in NCFPR-41 Project

Step 2 focused on developing truck acceleration functions and Step 3 aimed at coding the truck acceleration functions into Vissim. According to VISSIM 4.4 (23), the maxium acceleration curve is reduced by 0.1m/s^2 for every 1% increase in grade and increased by 0.1 m/s^2 for every 1% decrease in grade. This is close to the effect of gravity on the contribution of the truck weight to grae-related resistance. These two steps guaranteed that the VISSIM parameters were appropriately adjusted to reflect truck accelerations.

Insofar as the prediction of truck speeds is concerned, the starting point was the NCHRP 505 report (4) since it had a detailed description of truck acceleration characteristics and it had a model which was nominally able to predict truck performance on specific grades or sequences of grades for various weight-to-horepower ratios, weight-to-frontal-area, elevation, etc.. Furthermore, there was an excel spreadsheet with the coded procedure which was intended to generate the truck trajectories directly with the input of specific truck characteristics such as weigh-to-horsepower ratio.

Corrections had to be made to this spreadsheet to generate predictions consistent with the basic physics of the situation. The effect of grade was introduced in the resistance equations and a limit on the tractive effort was added based on the percentage of truck weight on the powered axles and the coefficient of friciton. In addition, the tractive axle weight percentage and weight to frontal area in Rakha's work was applied to produce modified results. These corrections are described more fully in the NCFRP 41 project report. The corrected model was checked against the separate model developed by Rakha et al.(2001). Figure 3 shows the modifed acceleration and maximum deceleration functions coded into VISSIM for a a weight-to-horsepower ratio of 150 lbs/hp truck.



Name: W/P 150

150.0 mph

4.9 ft/s²

0.0 ft/s²

Name: W/H 150

100.0 mph

0.0 ft/s²

-5.9 ft/s²

Figure 3: Maximum Acceleration and Deceleration Curves for a Weight-to-Horsepower Ratio of 150 lbs/hp Truck.

In VISSIM, a truck's maximum acceleration and deceleration curves consist of three curves: minimum, maximum and mean values. All heavy vehicles with a P/W ratio of 7kw/ton(235lbs/hp) or less use the minimum curve. Heavy vehicles with a P/W ratio of 30kw/ton(55lbs/hp) or more use the maximum curve and heavy vehicles with a P/W ratio of 18.5kw/ton (89lbs/hp) use the average curve. Figure 4 shows the default truck acceleration functions in VISSIM.

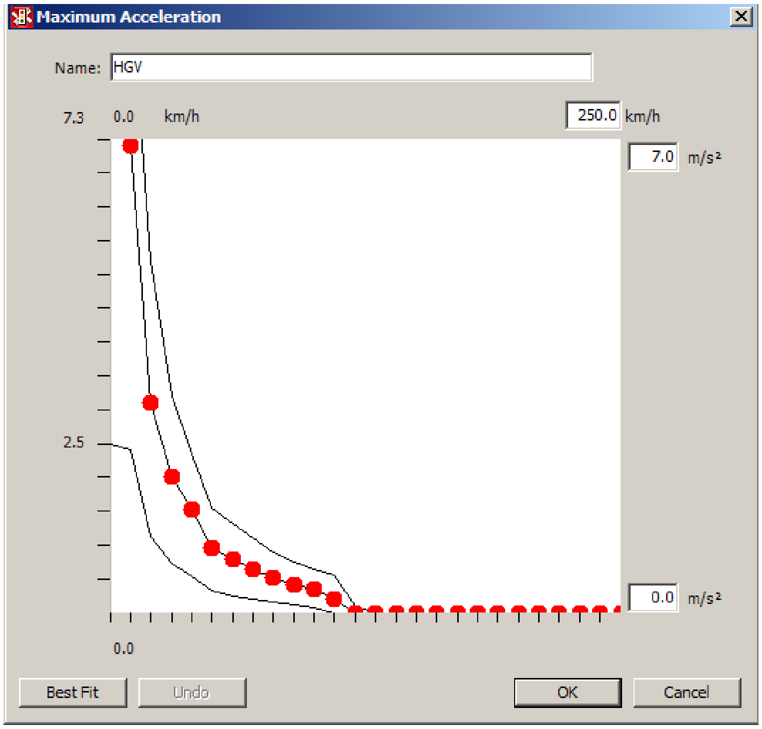


Figure 4: Default Truck Acceleration Function

To reduce the variation in the truck acceleration functions, a single truck acceleration function with minimum, maximum, and mean the same was employed for each weight-to-horsepower ratio.

Finally, Step 3 was excecuted again to see whether the trajectories predicted in VISSIM were consistent with the modified NCHRP 505 spreadsheet. Figure 5 shows the comparision of the two trajectories. That they agree is clearly evident.

3.2. Step 4 in the NCFRP 41 Project

As an aside, in step 4 of the NCFRP project effort, footprints for the AASHTO WB50 and single unit truck were coded into VISSIM. (The default truck 2D/3D models in VISSIM are for German trucks.) Exhibit 8 in NCFRP-41 report shows the AASHTO WB50 Tractor and Trailer which was used for FHWA Class 9 (24).

The main reason to create these models was to ensure that the footprints for truck length reflected the truck types being modeled. This affects car following and lane-changing behavior.

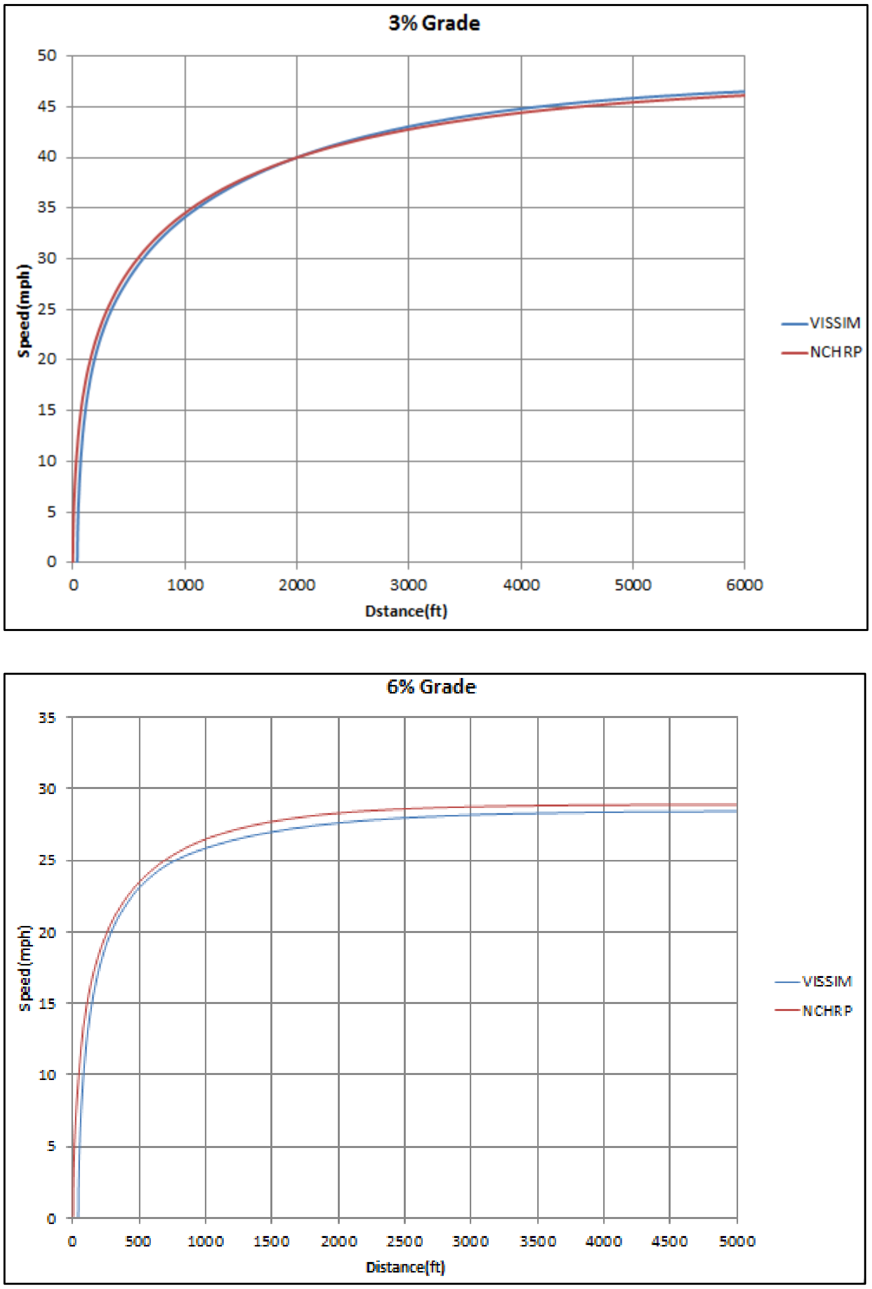


Figure 5: Trajectories Comparison of Modified NCHRP 505 Report and VISSIM

4. METHODOLOGY

Extensive work in this thesis focused on developing PCE values for mixed traffic streams. The PCE values were computed based on obtaining parity in the 95th percentile flow rate values for the auto-only and mixed-flow scenarios.

4.1. Truck PCE Methodology

Figure 6 shows the methodology used to develop the PCE values.

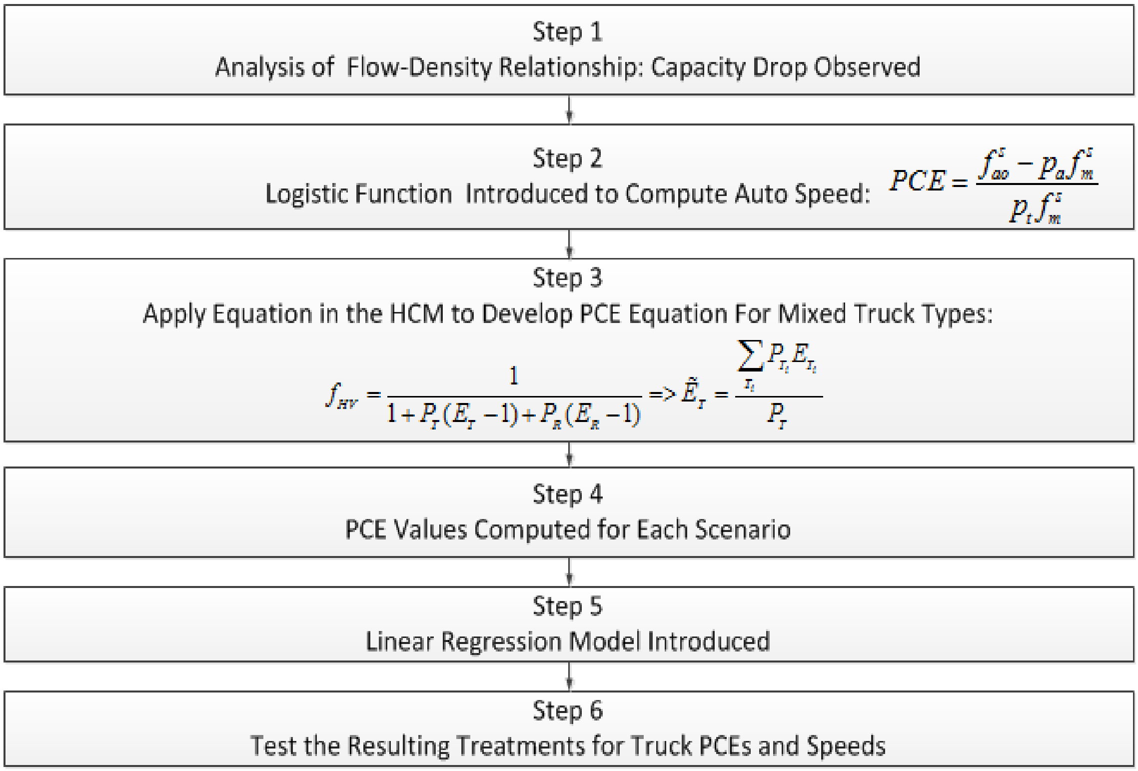


Figure 6: PCE Development Methodogy

The average one-minute density, flow rate, and speed data on the 5 mile grade section for cars, trucks, and mixed traffic was obtained from link evaluation output in VISSIM. From the

VISSIM ouput, a significant drop in maximum flow was observed when comparing the auto-only runs and mixed traffic runs. The red dots in Figure 7 shows a plot of one-minute flow-density data points for a mixed traffic stream on a 6% upgrade involving 50% Class 9 trucks at 150 lbs/hp.

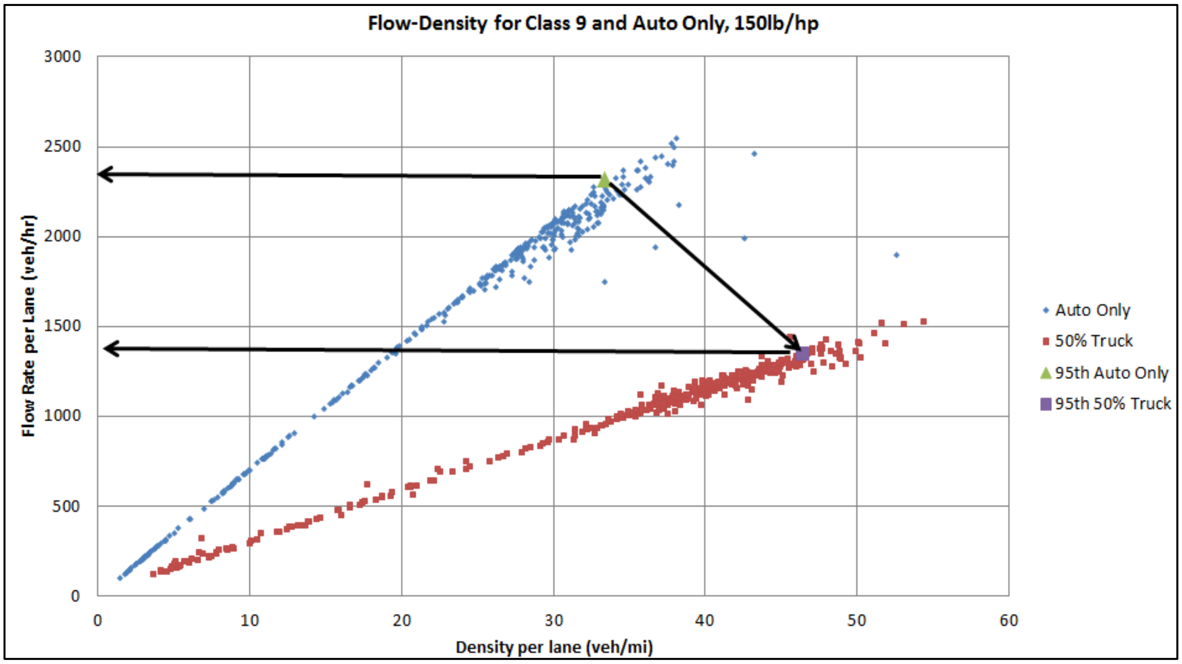


Figure 7: Flow-Density Relationship on a 6% Upgrade for Both 50% Class Trucks Traffic Stream and Auto-only Traffic Stream

In Figure 7, the blue dots represent a plot of one-minute flow-density data points for auto-only scenraio, the highest point on y-axis (flow rate per lane) indicates the maximum flow rate that the freeway network could achieve which its capacity. From Figure 7, it can be observed that the capacity of the auto-only scenario is about 2400 vphpl while the capacity of 50% truck scenario is about 1400 vphpl and the reason is that one truck could perform like more than one car in the traffic stream. This plot motivates a computation of PCE values based on capacity. The 95th percentile flow rate from the auto-only runs, f_{ao}^{s}, is used as an estimate of the facility's auto-only capacity. The 95th percentile flow rate from the mixed traffic runs, f_{m}^{s}, is used as the mixed traffic capacity. These two values ae then used in combination with the auto and truck percentages, p_a and p_t, to compute the PCE value.

$$f_{ao}^{s} = f_{m}^{s}(p_a + p_t PCE) \quad or \quad PCE = \frac{f_{ao}^{s} - p_a f_{m}^{s}}{p_t f_{m}^{s}} \tag{4.1}$$

In this research, the main concept of developing PCE values focuses on the projection of base flow-density curve onto mixed flow-density curve, in which a critical process is to find the corresponding points to process the projection. This thesis selects the capacities of these on these two curves as corresponding points. Figure 8 displays another thought. In Figure 8, the PCE values would be computed based on the flow rates of both scenarios when the density at capacity of auto-only is reached.

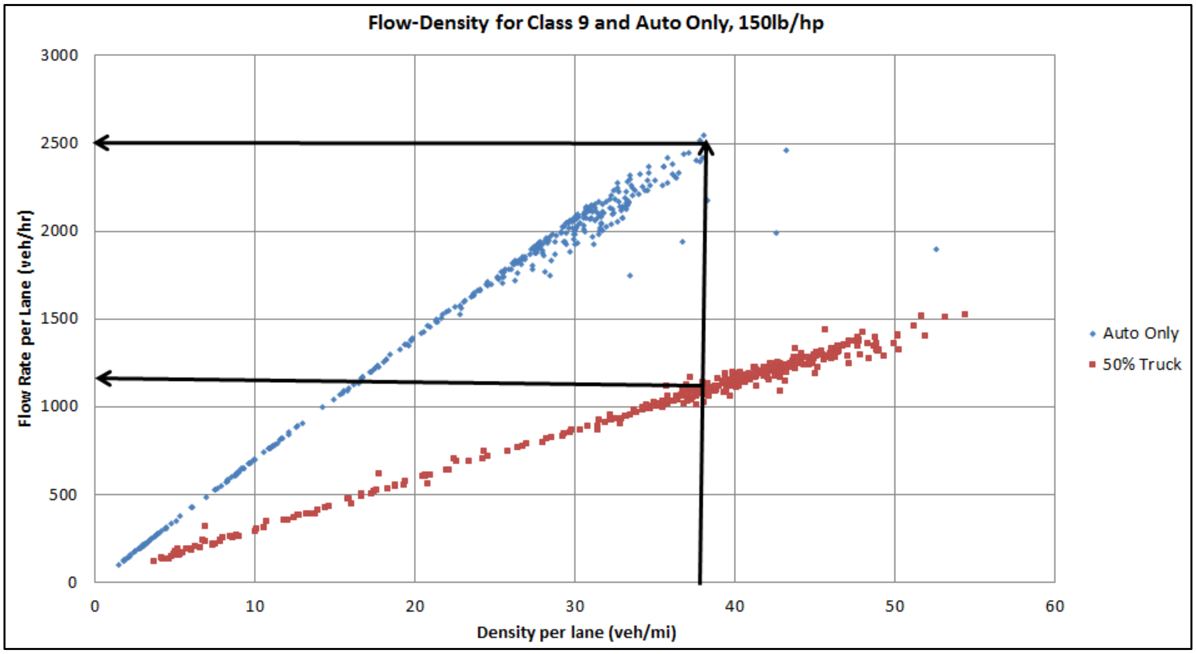


Figure 8: Capacity-Density Relationship Thought on a 6% Upgrade for Both 50% Class Trucks Traffic Stream and Auto-only Traffic Stream

In this case, f_m^s in Equation 4.1 is then defined as the flow rate reached when mixed flow reaches the maximum density of auto only scenario. The problem of this thought is that the auto-only scenario has already reached its capacity while the mixed scenario has not when the density is greater than 40veh/mi, which leads to the situation that the projection from auto only scenario to mixed scenario could not be found at flow rate greater than 40veh/mi.

A PCE value was estimated for each one of all the mixed flow scenarios. The important independent variables proved to be weight-to-horsepower ratio, percent trucks, grade, and truck type. The reason why the independent variables include truck type is that truck type determines the truck length which affects the car-following and lane changing decisions in traffic stream. A linear regression model was prepared for computing PCE values based on the variables mentioned above.

The methodology above focuses on developing PCE values for single truck type in a traffic stream. Since there are multiple truck types, it is necessary to go further to estimate a mixture trucks from FHWA class 4-13. The HCM uses Equation 4.2 to compute the heavy vehicle adjustment factor (Equation 11-3 of the HCM 2010):

$$f_{HV} = \frac{1}{1 + P_T(E_T - 1) + P_R(E_R - 1)} \qquad 4.2$$

Where, PT: Percentage of trucks and buses in the traffic stream

PR: Percentage of trucks and buses in the traffic stream

ET: PCE for trucks and buses

ER: PCE for recreational vehicles

If the trucks and recreational vehicles were regarded as vehicles of type Ti, then the heavy vehicle adjustment factor could be rewritten as Equation (9):

$$f_{HV} = \frac{1}{1 + \sum_{Ti} P_{Ti}(E_{Ti} - 1)} \qquad 4.3$$

In equation 4.3, *Ti* represents different types of trucks. If there are only trucks and recreational vehicles, then equation 4.3 is the same as equation 4.2. In this thesis, a composite truck which is able to represent all the mixture trucks in the traffic stream were created according to equation 4.3. The objective of a composite truck PCE is to create the following equivalence:

$$P_T(\tilde{E}_T - 1) = \sum_{T_i} P_{T_i}(E_{T_i} - 1) \qquad 4.4$$

Where $\tilde{E}_T$ is the composite truck PCE. Solving Equation 4.4 for $\tilde{E}_T$ results in the following:

$$\tilde{E}_T = \frac{\sum_{T_i} P_{T_i}(E_{T_i}-1)}{P_T}+1 \quad = \frac{\sum_{T_i} P_{T_i}(E_{T_i}-1)}{\sum_{T_i} P_{T_i}}+1 \qquad 4.5$$

However, since $\sum_{T_i} P_{T_i} * 1 = P_T$, then Equation 4.5 can be simplified to:

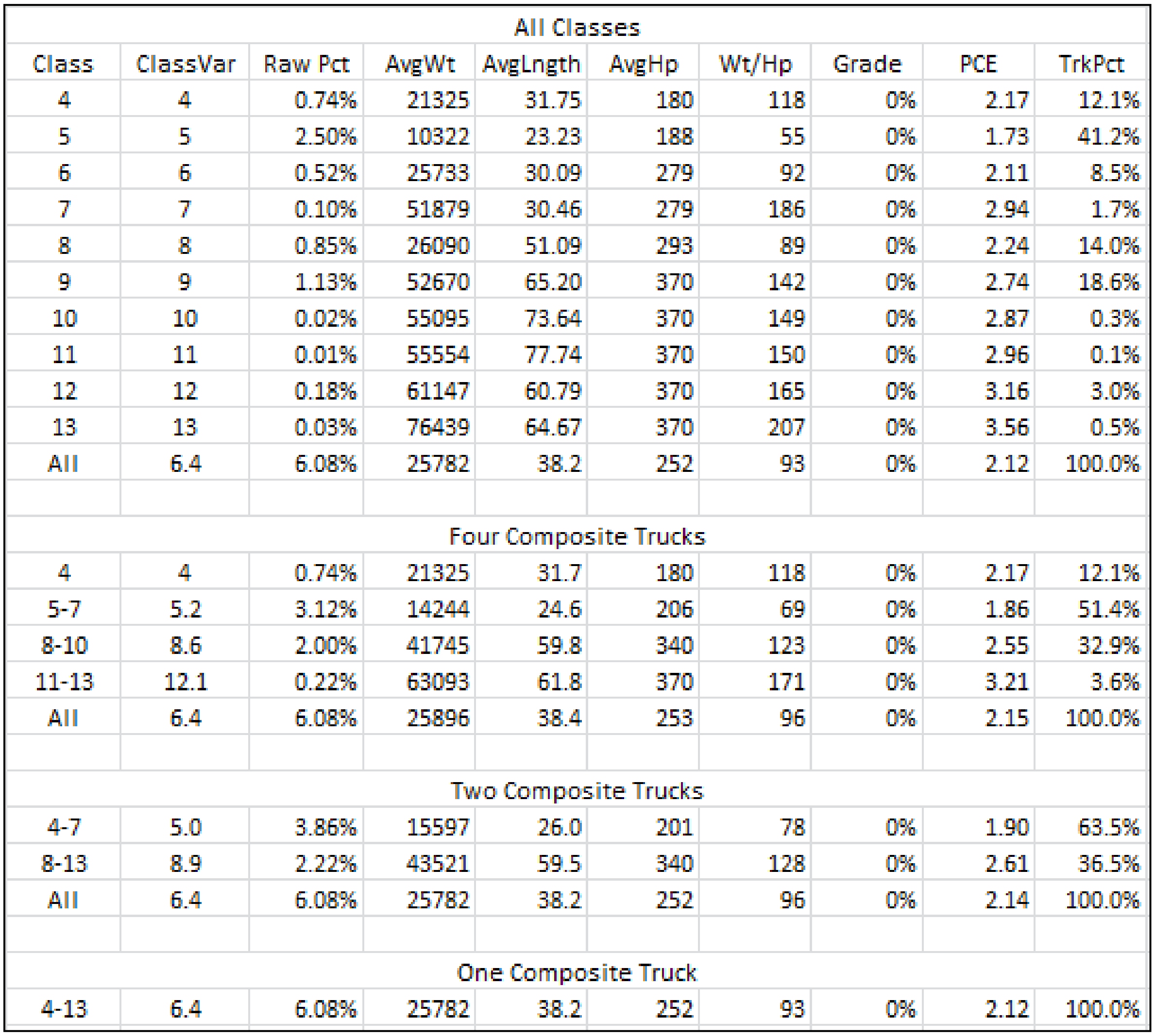
$$\tilde{E}_T = \frac{\sum_{T_i} P_{T_i}(E_{T_i}-1)}{\sum_{T_i} P_{T_i}}+1 = \frac{\sum_{T_i} P_{T_i}E_{T_i}}{\sum_{T_i} P_{T_i}} - \frac{\sum_{T_i} P_{T_i}}{\sum_{T_i} P_{T_i}}+1 \quad = \frac{\sum_{T_i} P_{T_i}E_{T_i}}{\sum_{T_i} P_{T_i}} -1+1 = \frac{\sum_{T_i} P_{T_i}E_{T_i}}{P_T} \qquad 4.6$$

Hence, $\tilde{E}_T$ is given by the percentage weighted average of the E_{T_i} values.

An illustration of this technique is shown in Table 3. It shows the distribution of trucks by class along a freeway traffic stream in which the total truck percentage is 5.1%.

The second column of the table shows the raw percentage of each truck type in the traffic stream while the last column of the table shows the relative percentages among trucks. Average weight, length are obtained from WIM data. From the table, four, two, and one composite truck scenarios are synthesized from the table on the top through weighting their characteristics by the relative truck percentages. In the four composite truck scenario, there are four types of synthesized trucks: 4 stands for buses, 5-7 stands for single unit trucks, 8-10 stands for single trailer trucks, and 11-13 stands for multiple trailors trucks. In the two composit truck scenario, there are two types of synthesized trucks: 4-7 stands for single unit trucks and 8-13 stands for trucks with trailers. In the one composite truck scenario, one type of composite truck stands for all trucks in the traffic steam.

Table 3: Distribution of Trucks by Class along a Freeway Traffic Stream



Class	ClassVar	Raw Pct	AvgWt	AvgLngth	AvgHp	Wt/Hp	Grade	PCE	TrkPct
All Classes									
4	4	0.74%	21325	31.75	180	118	0%	2.17	12.1%
5	5	2.50%	10322	23.23	188	55	0%	1.73	41.2%
6	6	0.52%	25733	30.09	279	92	0%	2.11	8.5%
7	7	0.10%	51879	30.46	279	186	0%	2.94	1.7%
8	8	0.85%	26090	51.09	293	89	0%	2.24	14.0%
9	9	1.13%	52670	65.20	370	142	0%	2.74	18.6%
10	10	0.02%	55095	73.64	370	149	0%	2.87	0.3%
11	11	0.01%	55554	77.74	370	150	0%	2.96	0.1%
12	12	0.18%	61147	60.79	370	165	0%	3.16	3.0%
13	13	0.03%	76439	64.67	370	207	0%	3.56	0.5%
All	6.4	6.08%	25782	38.2	252	93	0%	2.12	100.0%
Four Composite Trucks									
4	4	0.74%	21325	31.7	180	118	0%	2.17	12.1%
5-7	5.2	3.12%	14244	24.6	206	69	0%	1.86	51.4%
8-10	8.6	2.00%	41745	59.8	340	123	0%	2.55	32.9%
11-13	12.1	0.22%	63093	61.8	370	171	0%	3.21	3.6%
All	6.4	6.08%	25896	38.4	253	96	0%	2.15	100.0%
Two Composite Trucks									
4-7	5.0	3.86%	15597	26.0	201	78	0%	1.90	63.5%
8-13	8.9	2.22%	43521	59.5	340	128	0%	2.61	36.5%
All	6.4	6.08%	25782	38.2	252	96	0%	2.14	100.0%
One Composite Truck									
4-13	6.4	6.08%	25782	38.2	252	93	0%	2.12	100.0%

4.2 Auto Speeds Methodology

Figure 9 shows the methodology used to develop a model for predicting auto speeds.

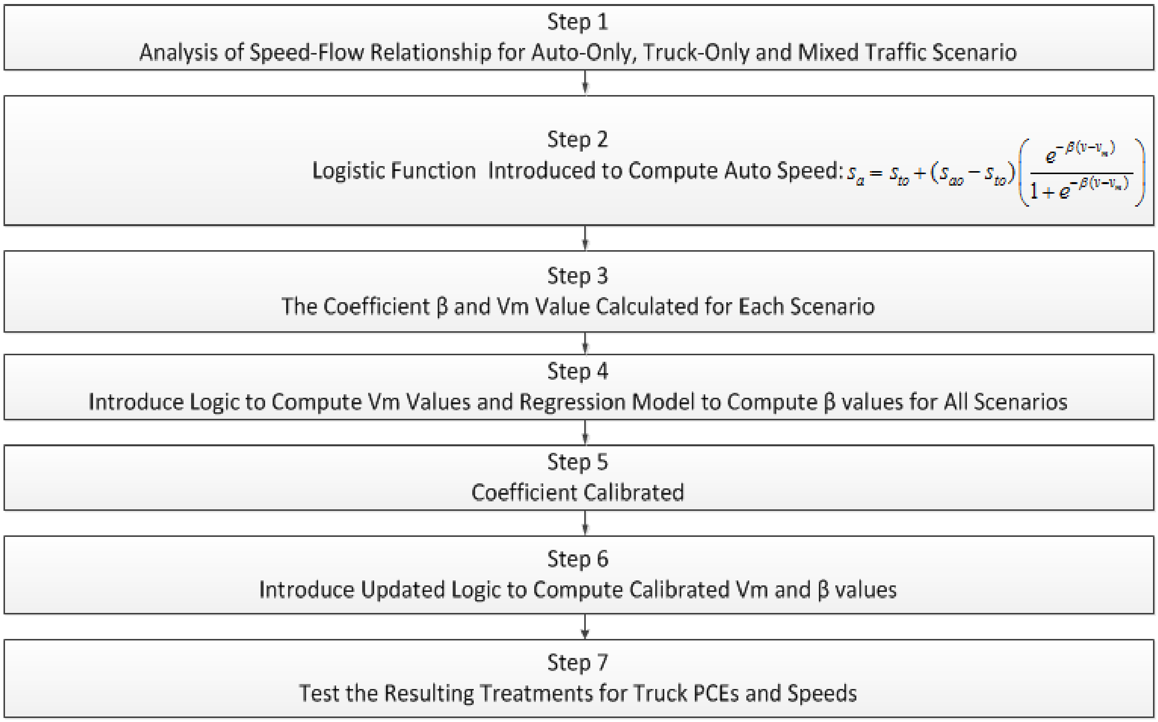


Figure 9: Auto Speed Prediction Methodology

During simulation runs, truck speed and auto speed in mixed traffic stream as well as average truck speed and auto speed in truck-only and auto-only traffic stream on the network were recorded. All the speeds of one vehicle type are the one-minute average speeds of all that vehicle type on the 5 mile long grade segment. The mixed traffic speeds are the average speeds of passenger cars and trucks on the grade segment. Figure 10 shows speed-flw plot for this 6% grade with 30% class9 trucks with 150lbs/hp and truck only and auto only traffic stream. The data points marked "Auto Only" are for an all-auto condition. To make the V/C ratios match, the auto-only flow rates can be adjusted downward so that the auto-only maximum flow rate matches that of the mixed scenario. The auto only speeds stay at or near 70 mph until capacity is reached. The data points marked "truck only" are from the simulation of a traffic stream involving 100% trucks. To make the V/C values match in this instance, the flow rates have been adjusted upward so that the actual maximum flow rate in the all-truck circumstance (855 veh/hr/lane) maps to the maximum flow rate in the case under study (again, 1500 veh/hr/lane).

The data marked "Scenario" are the average mixed speeds for the 30% truck condition. It could be seen that the average mixed speeds are scattered from 30mph to 60 mph when the flow rate is lower than 400vphpl. They are bounded from 25mph to 40mph when the flow rate is greater than 400vphpl. The auto speeds in the mixed stream follow a similar pattern as the average mixed speed.

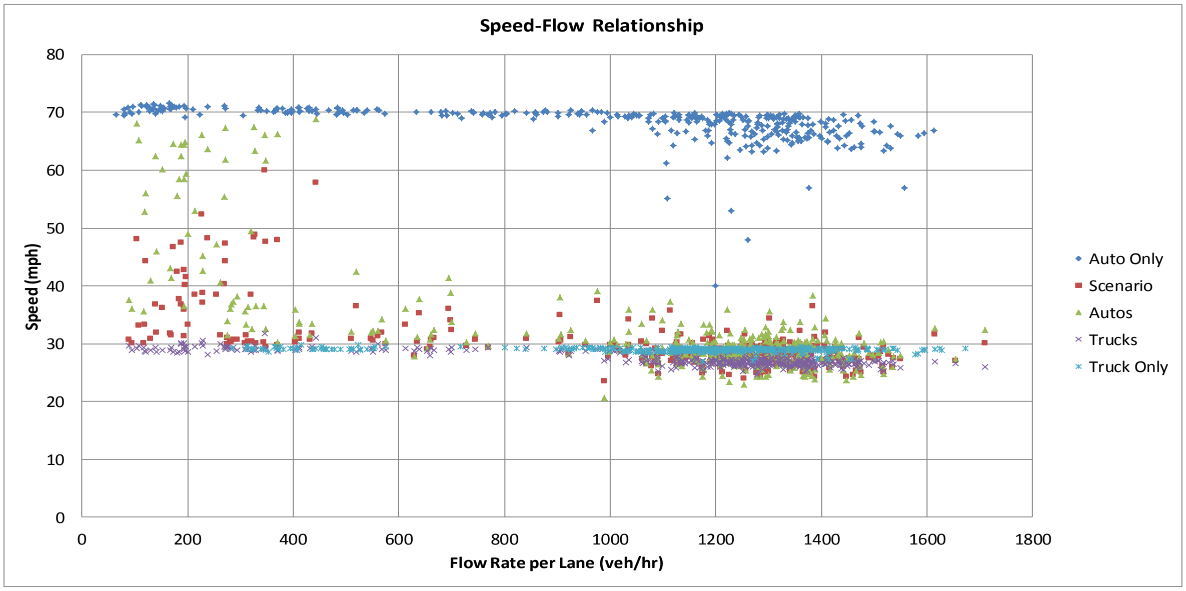


Figure 10: Speed-Flow Relationship on a 6% Upgrade for 30% Class Trucks Traffic Stream, Auto-only, and Truck-only Traffic Stream

The data points marked “Trucks” represent the trucks speeds in the mixed traffic stream and the “Trucks Only” represent the trucks speeds in the truck only traffic stream. Since this is on 6% upgrade, class9 truck with 150lbs/lb would have a crawl speed around 30mph. Unlike the trend of average mixed speed and auto speed in mixed traffic stream, either truck speeds in mixed traffic stream or truck speeds in truck only traffic stream stay concentrated from 28mph to 30mph, which shows that the trucks are not affected by flow rates obviously. This observation indicates that truck speeds are mostly determined by truck’s characteristics such as acceleration and grade of the roads. The plot marked as “Auto Only” is the plot of auto speeds in auto only traffic stream. Most of Auto only speeds stay around 70mph especially when the flow rates are lower than 1000vphpl. If we look at the auto only speeds, auto speeds in mixed stream, and truck only speeds, it could be observed that when the flow rates are low, the auto speeds in mixed traffic stream are driven by auto only speed while when the flow rates are high, the auto speeds in traffic sream driven by truck only speeds.

As discussed in the analysis of Figure 10, auto speeds in the mixed traffic stream is driven by truck speeds when the flow rates are high. When the truck percentage is high, the grades are steep, and flow rates are high, the autos are not able to overtake trucks easily. The auto speeds are constrained by truck speeds. From all the individual simulation runs, the following trends could be summarized:

- When the truck percentage is low and grades are less than or equal to 0%, the autos speeds in mixed traffic stream were able to follow closely to the speeds in all-auto scenario.

- When the truck percentage is high, and grades are greater than 0%, the autos speeds in mixed traffic tended to follow the truck speeds when flow rates are high.
- The pattern of auto speeds decrease when flow rates grow followed that of a logistics curve. As the flow rates increased, the auto speeds decreased slowly at first, then more rapidly to reach the speed close to truck speeds, then more slowly as the trucks speeds are reached.

Equation 4.7 shows the logistics function proved to be effective in fitting auto trends.

$$s_a = s_{to} + (s_{ao} - s_{to})\left(\frac{e^{-\beta(v-v_m)}}{1+e^{-\beta(v-v_m)}}\right) \qquad 4.7$$

Where, s_a: Auto Speed at Flow Rate v

s_{ao}: Auto-only Speed that Would Arise at Flow Rate v (taking into account the PCE value)

s_{to}: Truck-only Speed that Would Arise at Flow Rate v (taking into account the PCE values for the mixed flow case and the all-truck case)

v_m: Flow Rate at Which the Auto Speed Has Accomplished Half of its Transition From s_{ao} to s_{to}

β : Calibration Coefficient that Ensures the Following Holds True:

$$-\beta(v_m - \frac{\Delta V}{2}) = 5 \quad and \quad -\beta(v_m + \frac{\Delta V}{2}) = -5 \qquad 4.8$$

ΔV: Range of Flow Rates over Which the Transition Occurs

This ensures that the logit term within the large parentheses is approximately equal to 1 when $v = v_m - \Delta V/2$ and equal to 0 when $v = v_m + \Delta V/2$. In Figure 11, v_m is approximately 250 veh/hr/lane and ΔV is about 500 (from 100 to 600).

Figure 11 shows the predicted auto speeds which were fitted to the auto speeds in mixed traffic stream in Figure 10. The logistic function-shaped line represents the estimated auto speed in mixed traffic stream as predicted by by Equation 4.7. The marked "Auto90" plot is the 90^{th} auto speed in mixed traffic stream. The model focuses on predicting 90^{th} auto speed of every 11 minutes auto speed in mixed traffic stream. In this case, the transition period which is the scale of the logic curve along x-axis is from the flow rate where 90^{th} value of every 11 miutes auto speeds in mixed traffic reaches 98^{th} auto-only to the flow rate where 90^{th} value of every 11 miutes auto speeds in mixed traffic reaches 10th percentile of truck only speed. In Figure 11, the transition period is from 180vphpl to 1150vphpl from which the ΔV could be computed as around 970vphpl.

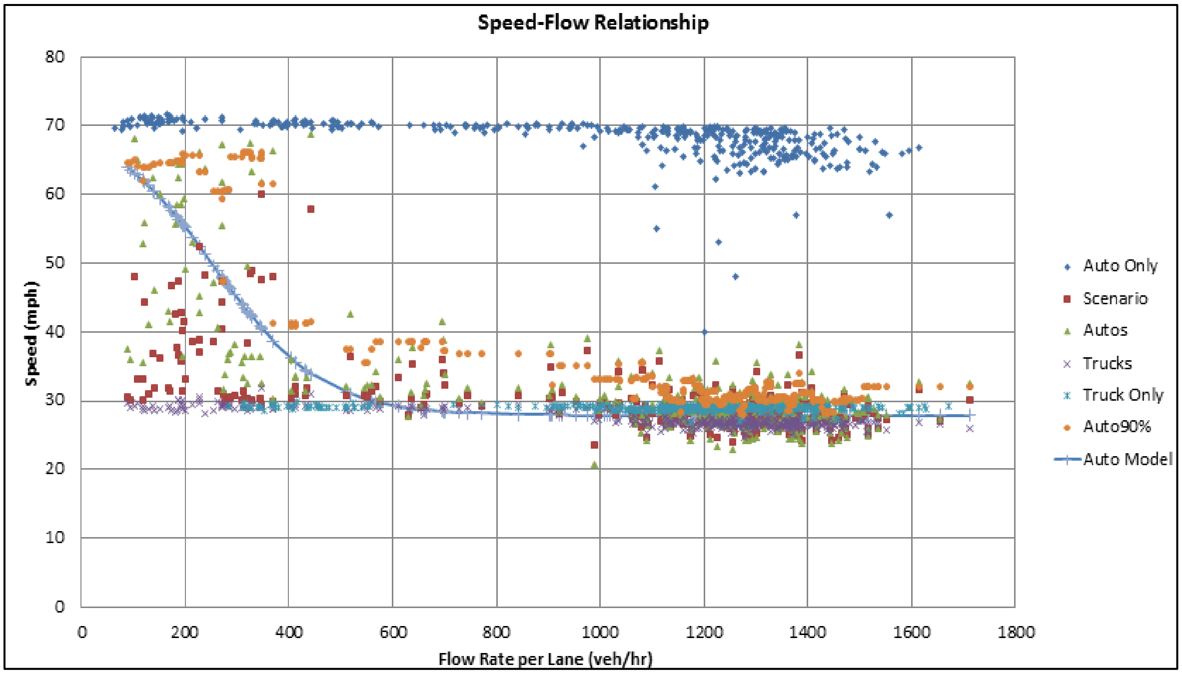


Figure 11: The Estimated Auto Speeds in Mixed Stream Illustion in the Speed-Flow Relationships on a 6% Upgrade for 30% Class Trucks Traffic Stream, Auto-only, and Truck-only Traffic Stream

Figure 11 shows the estimated auto speeds for only one scenario among all the simulation runs. To further estimate v_m and ΔV using a single logic, V_m and ΔV estimates for all scenarios were first obtained through statistical analysis. Then, the resulting estimates were placed in a database and curve-fitting techniques were used to develop estimates of the three parameters. The logic in Figure 12 worked well:

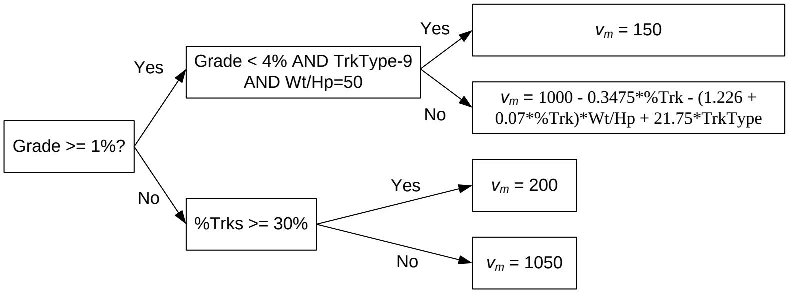


Figure 12 : Logic for Determining v_m values

Figure 13 shows the correspondence between this estimate and the observed values for v_m.

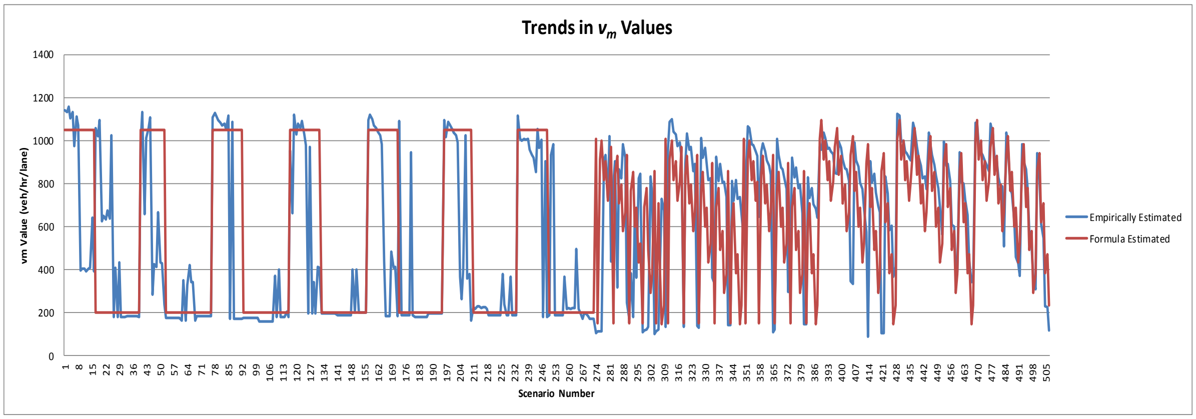


Figure 13: Emprical Estimates and Formula-Based Estimates of v_m

The sorted order from left-to-right is first by grade (from -6% on the left to 6% on the right, hence there are 13 patterns), then by percent trucks (the descending sets of 5 seen at right) and then weight-to-horsepower ratio (descending sets of four, almost visible) and then truck type (class 5 then class 9, not particularly visible). It is obvious that there are two major trends in the plot. When the grades are less than 1% and truck percentages are less than 30%, vm is around 1000vphpl while when the grades are less than 1% and truck percentages are greater than 20%, vm is around 200vphpl. For grades greater than 1%, v_m decreases when truck percentages increase and weight-to-horsepower ratios increase. There is no obvious change in v_m as the grades change when grades are greater than 1% or below 1%. This is analysis was translated into the logic in Figure 12. The blue line shows the v_m derived from the simulation data and the red line shows the estimated v_m based on the logic in Figure 12. It could be observed that when the truck grades are getting steeper, the estimated v_m is getting closer to actual v_m, which shows that this model performs better when the situation is worse.

The analysis below demonstrates extension work done subsequent to the above results for calibrating the α (v_m),β values in Equation 4.7.

From Figure 11, the 98th percentile auto speed and 10th percentile auto speed were used to determine v_m and β values. The motivation to calibrate v_m and β came from Figure 14.

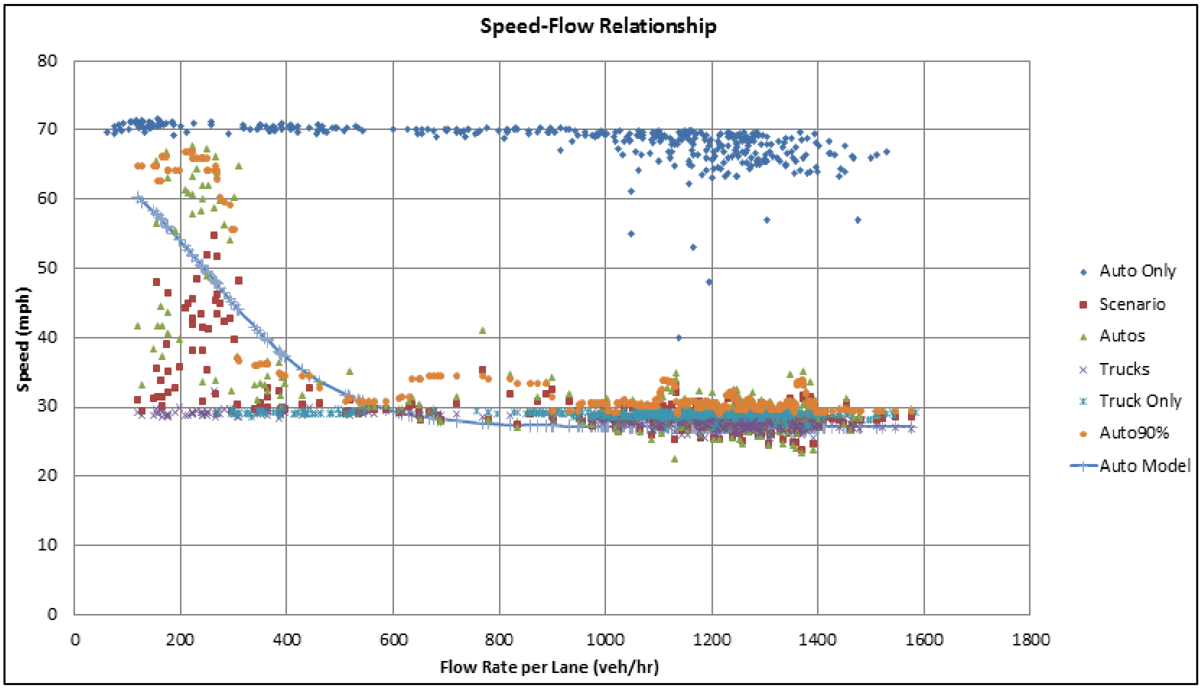


Figure 14: Estimated Auto Speed Example on the Speed-Flow Plot for this 6% Grade with 40% Class9 Trucks with 150lbs/hp

The line marked “Auto Model” was calculated using Equation 4.7 where the v_m and β value were determined by 98th auto speed and 10th auto speed in mixed traffic stream. From Figure 14, it can be seen that this model can be improved since the difference between the predicted model and 90th auto speed is easy to see. Therefore, the thought to calibrate v_m and β values to improve fitting quality was motivated. As discussed previously, the auto speeds in traffic stream were estimated using a logistics function. To take a further observation of the marked “Auto Model” in Figure 14 which is the estimated auto speeds in mixed traffic stream, it can be seen that even though the model could fit the observed auto speeds in mixed traffic stream in principle, the curve is not fitting accurately. When the flow rates are from 400vphpl to

1000vphpl, the estimated auto speeds are well below the observed 90th percentile auto speeds. Therefore, it is necessary to find the best α (vm),β values to make the model to be the most accurate.

Equation 4.9 shows the basic logistic function form:

$$y = \frac{e^{-\beta(x-\alpha)}}{1+e^{-\beta(x-\alpha)}} \qquad 4.9$$

In Equation 4.9, α determines the center of the curve and β determines the scale of the curve. Figure 15 shows the shape of logic curve when changing β with α at 0.5.

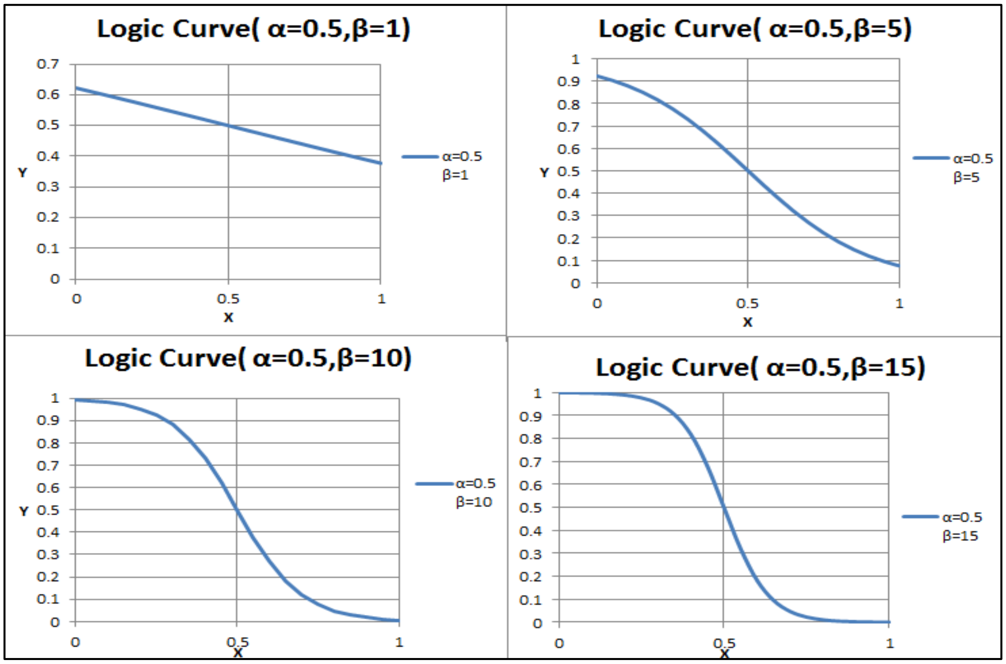


Figure 15: The Display of Logic Curve as β Changes

From Figure 15, it can be seen that when β is small as 1, the logic curve looks like a straight line and the y value changes from 0.62 to 0.38. When equals to 5, it starts to have head and tail part in which the y value changes slower as the x value changes comparing to the middle part. When β equals to 10, y is 1 correspondent to x at 0 and y is 0 correspondents to x at 1. When β equals to 15, y stays at one until x becomes larger than 0.2. Since the observed auto speed in mixed traffic stream would soon drop from the highest value, this is the reason the logic function with the scale coefficient up to 10 was selected. The β here is the scale coefficient while the β in equation 4.10 is the coefficient which ensures that the scale coefficient equal to 10. Equation 4.10 shows the relationship.

$$-\beta(v_m - \frac{\Delta V}{2}) = 5 \quad and \quad -\beta(v_m + \frac{\Delta V}{2}) = -5 => \quad \beta\Delta v = 10 => \beta = \frac{10}{\Delta v} \qquad 4.10$$

Equation 4.11 shows the methodology to compute ΔV in step 5.

$$\Delta v = MixFlw(98\%Spd) - MixFlw(10\%Spd) \qquad 4.11$$

Where, MixFlw(98%Spd): The flow rate when 90^{th} value of every 20 minutes auto speeds in traffic stream reaches 98^{th} auto-only speeds in mixed traffic stream

MixFlw(10%Spd): The flow rate when the 10^{th} auto speeds in mixed traffic stream

From equation 4.10 and equation 4.11, equation 4.12 can be developed. It focuses on computing β.

$$\beta = -\frac{10}{MixFlw(98\%Spd) - MixFlw(10\%Spd)} \qquad 4.12$$

The analysis above describes the method to compute β value. It is clear that the values of MixFlw(98%Spd) and MixFlw(10%Spd) determine β value. The 98^{th} and 10^{th} were selected so that it covers most of the speed range. However, sometimes the speed could be noisy when the percentile is either too high or too low. This situation can be shown in Figure 14 where the marked “Auto90%” plot is noisy when the flow rates are greater than 1100vphpl or less than 200vphpl. Therefore, the major exploration on calibrating β value is to find the best percentiles.

The calibration of α value is not as straightforward as β. An inspiration could be obtained from the derivation of Equation 4.7.

$$s_a = s_{to} + (s_{ao} - s_{to})\left(\frac{e^{-\beta(v-v_m)}}{1+e^{-\beta(v-v_m)}}\right) => \qquad \frac{s_a - s_{to}}{s_{ao} - s_{to}} = \frac{e^{-\beta(v-v_m)}}{1+e^{-\beta(v-v_m)}}$$

$$=> e^{-\beta(v-v_m)} = \frac{s_a - s_{to}}{s_{ao} - s_a} \quad => \; -\beta(v-v_m) = \ln\left(\frac{s_a - s_{to}}{s_{ao} - s_a}\right) => v_m = v - \frac{1}{10}\ln\left(\frac{s_a - s_{to}}{s_{ao} - s_a}\right)$$

$$=> v_m - v = -\frac{1}{10}\ln\left(\frac{s_a - s_{to}}{s_{ao} - s_a}\right) \qquad 4.13$$

Equation 4.13 is derived from Equation 4.7. From Equation 4.13, it can be seen that (v-v_m) is proportional to $\ln\left(\frac{s_a - s_{to}}{s_{ao} - s_a}\right)$. Since v_m defined to be the flow rate that the auto speed has finished half of its transition, it is then the flow rate correspondent the center of 98th Auto speeds and 10th Auto speeds. Figure 16 shows the detailed information about α based on the speed-flww plot for this 6% grade with 30% class9 trucks with 150lbs/hp.

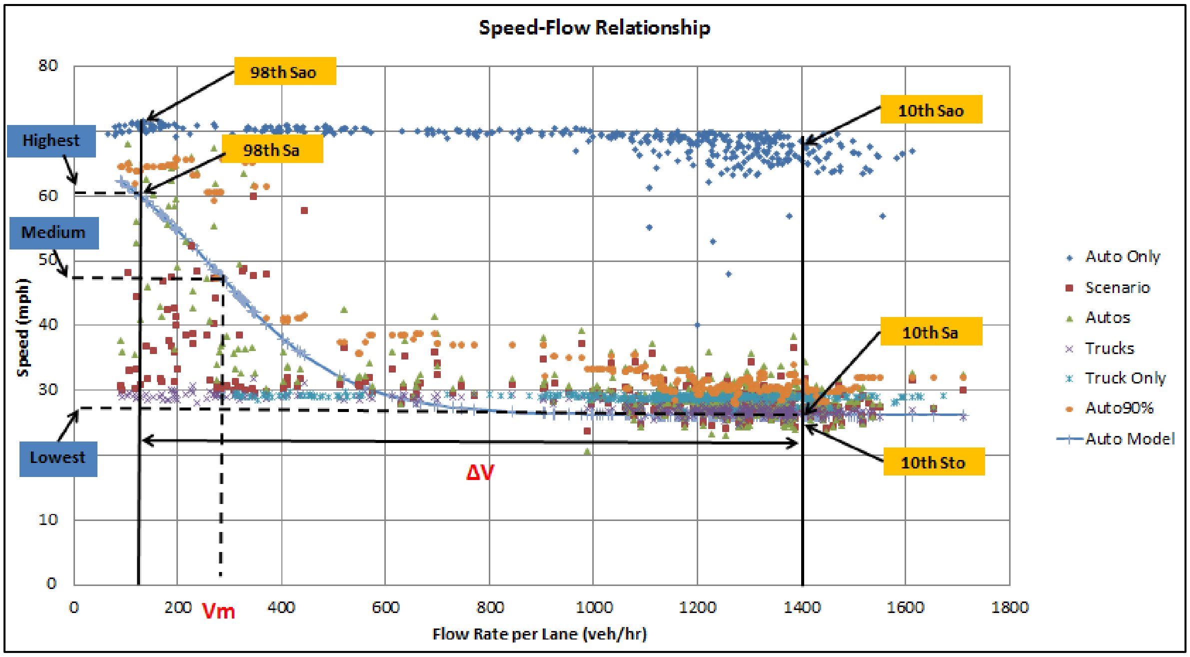


Figure 16:Detailed Information for Computing α Based on the Speed-Flow Plot for the 6% Grade with 30% Class9 trucks with 150lbs/hp

From Figure 16, it can be seen that α is the flow rate correspondent the the 50% of the speed in the marked "Auto Model". In order to compute the value of α, the values of 98th auto speeds, truck-only speeds, auto-only speeds, and the flow rates correspondent to the 98th auto speeds as well as 10th auto speeds, truck-only speeds, auto-only speeds, and the flow rates correspondent to the 10th auto speeds should be know first. The 99% point of this logistic function at y-axis is the 98th auto-only speed in mixed traffic stream and 1% point of this logistic function at x-axis is the 10th truck-only speed in mixed stream. Then, equation 4.13 could be more developed to 2 equations. Equation 4.14 is the equation when the auto-only speed is at 99% point of the logistic function.

$$v_m - v_{98th} = -\frac{1}{10}\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right) \tag{4.14}$$

Equation 4.15 is the equation when the truck-only speed is at 1% point of the logistic function.

$$v_m - v_{10th} = -\frac{1}{10}\ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right) \tag{4.15}$$

Where, $S_{a,98th}$, $S_{a,10th}$: 98th and 10th Auto Speed in mixed traffic stream

$S_{ao,98th}$, $S_{ao,10th}$: 98th and 10th Auto-Only Speed (taking into account the PCE values for the mixed flow case and the all-truck case)

$S_{to,98th}$, $S_{to,10th}$: 98th and 10th Truck-Only Speed (taking into account the PCE values for the mixed flow case and the all-truck case)

v_m: Flow Rate at Which the Auto Speed Has Accomplished Half of its Transition From s_{ao} to s_{to}

By applying both Equation 4.14 and 4.15, Equation 4.16 is developed as follows:

$$\frac{v_m - v_{98th}}{v_m - v_{10th}} = \frac{-\frac{1}{10}\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right)}{-\frac{1}{10}\ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right)} = \frac{\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right)}{\ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right)} \tag{4.16}$$

v_m in Equation 4.16 is α and V98th and V10th are MixFlw(98%Spd) and MixFlw(10%Spd). Then Equation 4.17 can be further developed as:

$$\frac{v_m - v_{98th}}{v_m - v_{10th}} = \frac{\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right)}{\ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right)} \Rightarrow \frac{v_m - MixFlw(98\%Spd)}{v_m - MixFlw(10\%Spd)} = \frac{\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right)}{\ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right)}$$

$$\Rightarrow v_m = \frac{\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right) \times MixFlw(10\%Spd) - \ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right) \times MixFlw(98\%Spd)}{\ln\left(\frac{s_{a,98th} - s_{to,98th}}{s_{ao,98th} - s_{a,98th}}\right) - \ln\left(\frac{s_{a,10th} - s_{to,10th}}{s_{ao,10th} - s_{a,10th}}\right)} \quad 4.17$$

The V_m value could then be computed from Equation 4.17. In Equation 4.17, the 98^{th} and 10^{th} speeds are chosen so that all the values can be covered, which follows the same principle when computing β. The same concern aroused: sometimes the speed could be noisy when the percentile is either too high or too low. Therefore, the major exploration on calibrating α value is to find the best percentiles to locate the highest point (99%) and lowest point (1%) of the logistics function.

At each flow rate, there is a difference between estimated auto speeds and observed auto speeds. Then, the sum squared prediction error could be computed as:

$$\text{SSPE} = \sum (\tilde{S}_{a,i} - S_{a,i})^2 \quad 4.18$$

The goal to find the best percentiles is to minimize the squared prediction error. The solver tool in excel was selected to minimize the sum squared prediction error. There are three engine choices in solver:

- GRG Nonlinear engine for Solver Problem that are smooth nonlinear
- LP Simplex engine for linear Solver Problems
- Evolutionary engine for Solver problems that are non-smooth

Since this logistic function is a smooth nonlinear curve, GRG Nonlinear engine was chose to minimize the goal equation 4.18.

If we set that the highest percentage of the logistic function is determined by X_1^{th} auto speeds in the traffic stream and the lowest percentage of the logistic function is determined by X_2^{th} auto speeds in the traffic stream. The logic in this minimization problem is as followed:

$$\text{Minimize SSPE} = \sum (\tilde{S}_{a,i} - S_{a,i})^2$$

$$S.\mathrm{T}. \quad \tilde{s}_{a,i} = s_{to,i} + (s_{ao,i} - s_{to,i})\left(\frac{e^{-\beta(v_i-\alpha)}}{1+e^{-\beta(v_i-\alpha)}}\right)$$

$$\beta = -\frac{10}{MixFlw(x_1\%Spd) - MixFlw(x_2\%Spd)}$$

$$\alpha = \frac{\ln\left(\dfrac{s_{a,x_1} - s_{to,x_1}}{s_{ao,x_1} - s_{a,x_1}}\right) \times MixFlw(x_2\%Spd) - \ln\left(\dfrac{s_{a,x_2} - s_{to,x_2}}{s_{ao,x_2} - s_{a,x_2}}\right) \times MixFlw(x_1\%Spd)}{\ln\left(\dfrac{s_{a,x_1} - s_{to,x_1}}{s_{ao,x_1} - s_{a,x_1}}\right) - \ln\left(\dfrac{s_{a,x_2} - s_{to,x_2}}{s_{ao,x_2} - s_{a,x_2}}\right)}$$

$$\ln\left(\frac{s_{a,x_1} - s_{to,x_1}}{s_{ao,x_1} - s_{a,x_1}}\right) > \ln\left(\frac{s_{a,x_2} - s_{to,x_2}}{s_{ao,x_2} - s_{a,x_2}}\right)$$

$$\mathrm{x}_1 <= 100$$

$$\mathrm{x}_1 > \mathrm{x}_2$$

$$\mathrm{x}_2 >= 0 \qquad 4.19$$

Where, $MixFlow(x_1\%Spd)$: the Flow Rate Reached at the x_1^{th} Auto-only Speed (99% point in Logistic Function) in Mixed Traffic Stream

$MixFlow(x_2\%Spd)$: the Flow Rate Reached at the x_2^{th} Truck-only Speed (1% point in Logistic Function) in Mixed Traffic Stream

V_i: ith flow rate during the transition from $MixFlow(x_1\%Spd)$ to $MixFlow(x_1\%Spd)$

$\tilde{S}_{a,i}$: the Estimated Auto Speeds at V_i

$S_{a,i}$, $S_{ao,i}$, $S_{to,i}$: the Observed Auto Speed, Auto-Only Speed, Truck-Only Speed at V_i

$S_{a,x1}$, $S_{ao,x1}$, $St_{o,x1}$: X_1^{th} Auto Speed, Auto-Only Speed, Truck-Only Speed

$S_{a,x2}$, $S_{ao,x2}$, $S_{to,x2}$: X_2^{th} Auto Speed, Auto-Only Speed, Truck-Only Speed

5. Results

5.1. Truck PCE Results

Figure 17 shows the result of PCE calculated by Equation 4.1 for all the mix traffic stream simulation runs. The sorted order from left-to-right is first by grade, then by percent and then weight-to-horsepower ratio and then truck type. It is obvious in Figure 17 that PCE values get greater when the grades get steeper and PCE values increase as the weight-to-horsepower increases while PCE values decrease as the truck percentage increases. PCE values stay stable around 1 to 2.2 when the grades were less than -4% (scenario number less than 81). When the grades get steeper, the trends become clear.

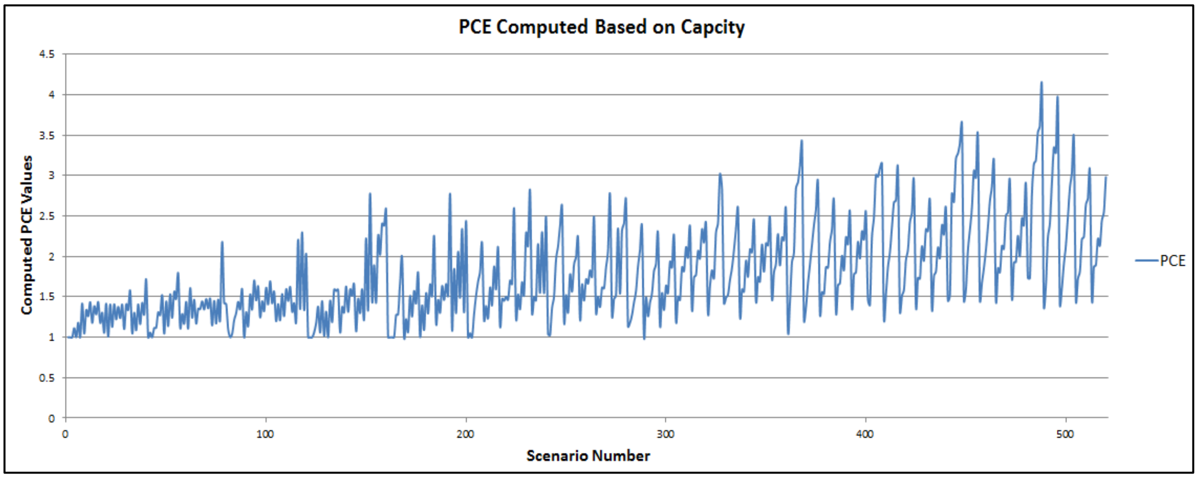


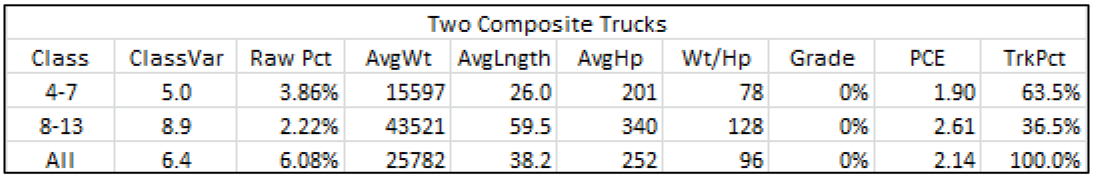
Figure 17: Computed PCE Values Based on Capacity-Density Relationship

Following the analysis above, a linear regression model was introduced as Equation 5.1.

$$PCE = \underset{3.95}{0.322} + \underset{9.40}{0.07632} TT + \underset{27.5}{0.00799} WtHp - \underset{-5.06}{.00582} T\% + \underset{16.01}{.1300} G\% \quad R^2 = 0.8976 \qquad 5.1$$

TT is the truck type (4-13), WtHp is the lbs/hp, T% is the truck percentage, and G% is the grade percentage. All the t statistic values are greater than the t-critical values which is 1.97. In order to test the ability of the developed method to evaluate the trucks' influence on traffic stream when there are more than one type of truck, the two composite trucks scenario in Table 3 was used as an example of composite trucks. Table 4 shows the two composite trucks scenario.

Table 4: Two Composite Trucks Scenario



Two Composite Trucks									
Class	ClassVar	Raw Pct	AvgWt	AvgLngth	AvgHp	Wt/Hp	Grade	PCE	TrkPct
4-7	5.0	3.86%	15597	26.0	201	78	0%	1.90	63.5%
8-13	8.9	2.22%	43521	59.5	340	128	0%	2.61	36.5%
All	6.4	6.08%	25782	38.2	252	96	0%	2.14	100.0%

From Table 4, the weight-to-horsepower ratios of the two composite trucks were calculated. For class 4-7, the weight-to-horsepower ratio is 78lbs/hp. The 78lbs/hp was then put into the modified NCHRP-505 report to acquire the acceleration-speed curve for class 4-7. Then, the acceleration-speed curve of class 4-7 will be codified into VISSIM. The process of codifying class 8-13 follows the same steps. The relative percentage which was 62.5% for light trucks and 36.5% for heavy trucks would stay the same during the simulation. In the simulation model, a 5 mile and 3 lanes segment with 3% upgrade was used to validate this method.

During the simulation, the network average travel time, speed, density, and flow rates were recorded. Figure 18 shows the comparison of travel time cumulative density functions of the composite truck scenario and adjusted auto only scenario of 30% trucks. There are two types of truck in the traffic stream and each of them has a PCE value computed based on Equation 5.1. When conducting the simulation for adjusted auto only scenario, the PCE values of these two types of truck were used to convert the number of trucks to passenger cars. For example, if the flow rate is 200vph with 10% class 4-7 truck and the PCE value for class 4-7 truck is 2. The flow rate of class 4-7 truck is 20vph and the adjusted amount of passenger cars is 40vph. Then the total flow rate is converted from 200vph to 220vphpl.

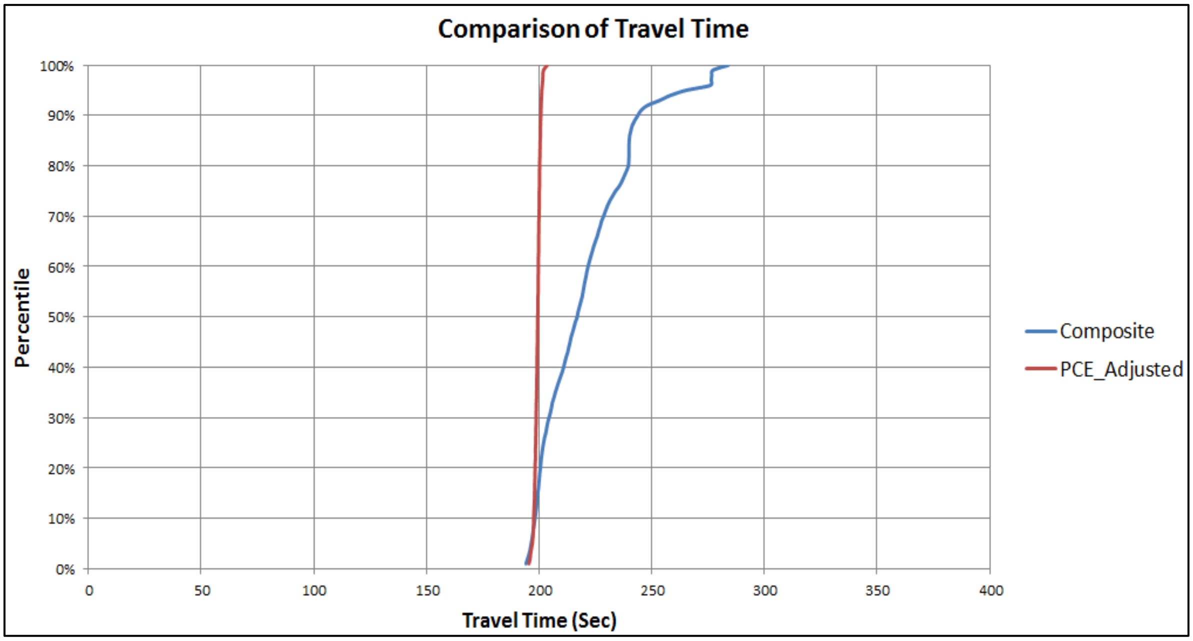


Figure 18: Comparison of Travel Time for Composite Truck and PCE Adjusted Auto-Only Scenario

The simulation was conducted with the flow rate increasing from 0vphpl to capacity across time. One-minute average travel time data of all vehicles on the grade segment was recorded during the simulation. The line marked "Composite" plot is the cumulative density function of 30% composite trucks travel time and the marked "PCE_Adjusted" is the correspondent cumulative density function of auto-only run with the volume adjusted according to Figure 7. From Figure 18, it can be observed that the travel times are almost the same at both low and high percentiles. For the composite truck scenario, the travel time is longer due to the effects of the grades of trucks. The largest difference of these two density functions at the same percentile was about 40%. To have a deeper comparison of these two scenarios, Figure 19 shows the flow-density relationships for both of them. It can be observed in Figure 19 that the composite truck scenario has a capacity of about 1000vphpl while the auto-only scenario has a capacity of 1300vphpl. The flow-density relationships of these two scenarios are very similar trend, which proves that the PCE adjusted auto-only scenario is able to perform close to the composite trucks scenario.

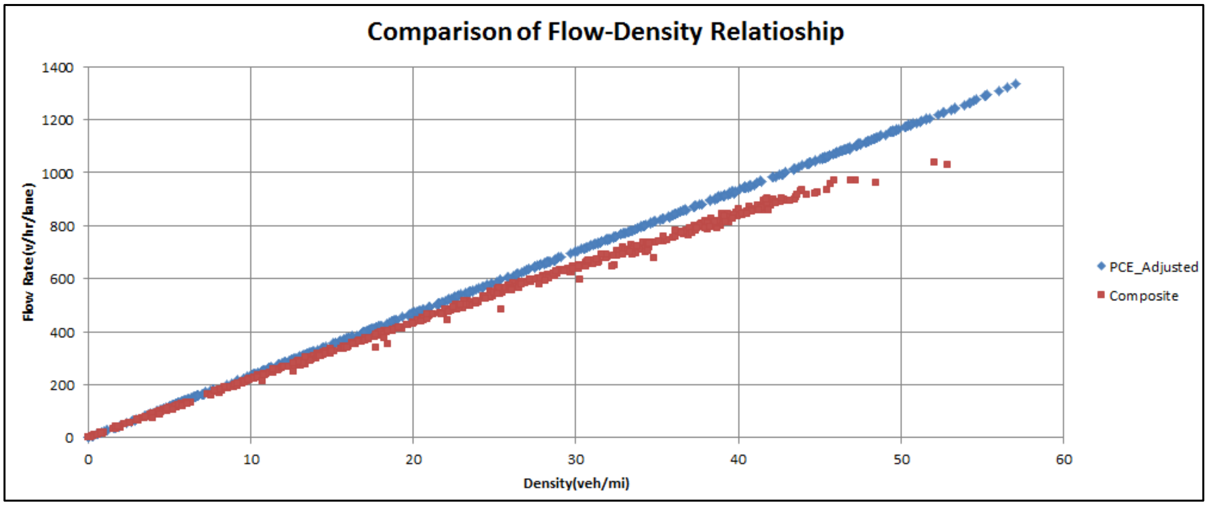


Figure 19: Comparison of Flow-Density Relationship for Composite Truck and PCE Adjusted Auto-Only Scenario

5.2. Auto Speeds Results

The best percentiles could then be found through this process. Figure 20 shows the calibrated estimated auto speed example on the speed-flow plot for this 6% grade with 30% class9 trucks with 150lbs/hp.

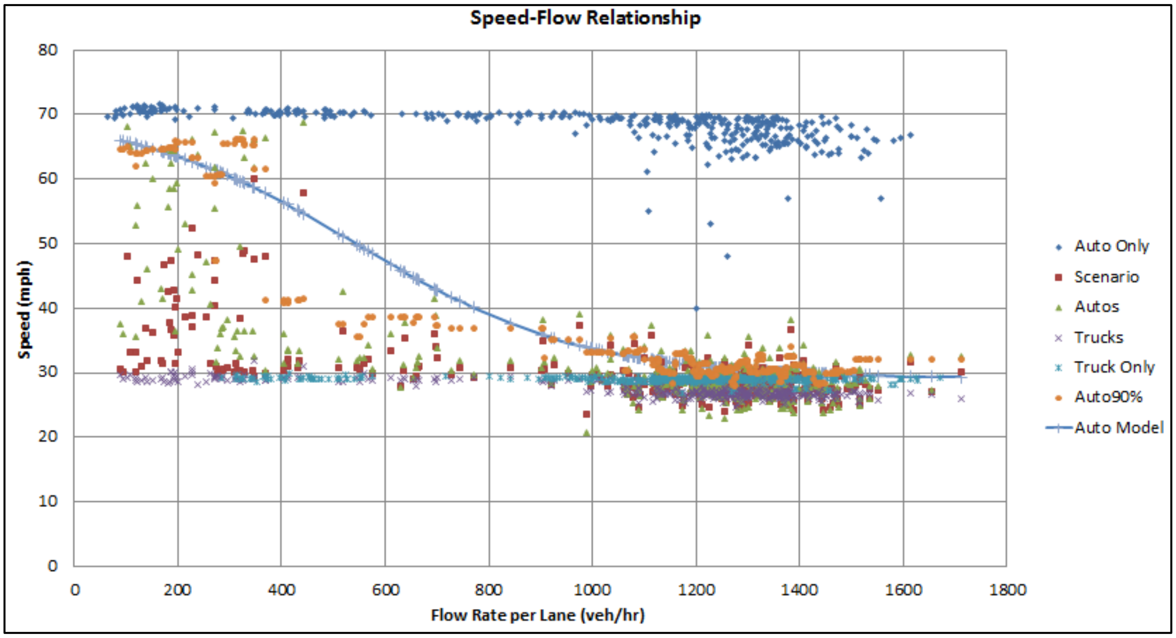


Figure 20: Calibrated Estimated Auto Speed Example on the Speed-Flow Plot for this 6% Grade with 30% Class9 Trucks with 150lbs/hp

Compared the calibrated estimate auto speeds ("Auto Model") in Figure 20 to that in Figure 14, it can be seen that this updated model is able to fit better than previous model with 98th auto-only speed and 10th truck-only speed chosen as the 99% and 1% points of the logistic function.

To review briefly, there are two coefficients in Equation 4.7, $V_m(\alpha)$ and β. In the methodological work described so far, one more variable μ has been added. Therefore, the logic in Figure 12 needs to be updated according to the trend of new V_m values.

Figure 21 shows the plot of the new empirically-estimatedd V_m .

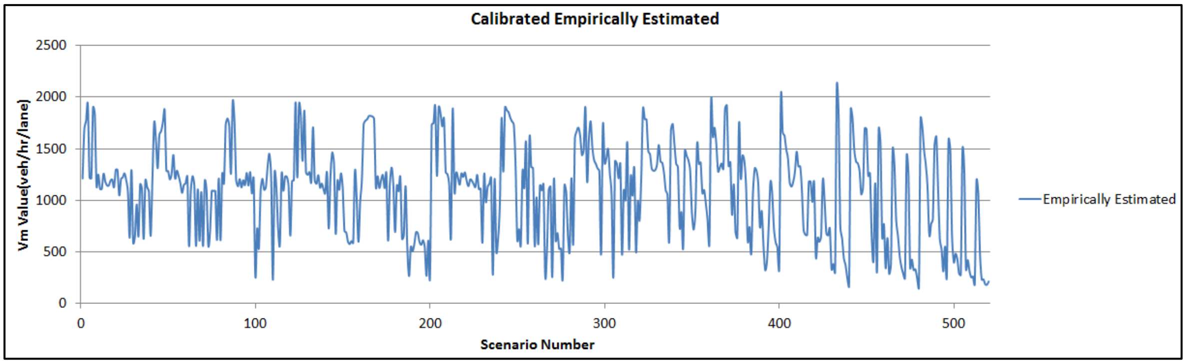


Figure 21: Comparison of Calibrated Empirically Estimated V_m and Formula Predicted V_m

The marked "Empirically Estimated" in Figure 21 is the V_m value calibrated. We cannot see the same trend of that in Figure 16 that when the grades are less than 1% and truck percentages are less than 30%, v_m is around 1000vphpl while when the grades are less than 1% and truck percentages are greater than 20%, v_m is around 200vphpl. From Figure 21,it can be seen that there are two patterns for v_m when the grades were less or more than 1% grade. Therefore, two regression models were used to fit the v_m trend.

Equation 5.2 shows the equation for predicting V_m values when grade is less than 1%.

$$v_m = 1989.217 - 39.797 \underset{-4.67}{TT} - 19.3267 \% \underset{-16.04}{Trk} \qquad \mathrm{R}^2 = 0.5 \qquad 5.2$$

Where, %Trk: Truck Percentage(%)

TT: Truck Type, 5 or 9 in this case

The t-statistic values of weight-horsepower ratio and percentage grade are smaller than 1.5, so Equation 5.2 only contains independent variables as truck type and percentage truck.

Equation 5.3 shows the equation for predicting V_m values when grade is greater than or equal to 1%.

$$v_m = \underset{24.91}{2609.877} - \underset{-2.31}{23.302\,TT} - \underset{-12.56}{16.3481\%Trk} - \underset{-11.45}{4.5376\,Wt/Hp} - 101.216\% \underset{-8.5643}{G} \qquad R^2 = 0.61 \qquad 5.3$$

Where, %Trk: Truck Percentage(%)

%Grade: Grade Percentage(%)

Wt/Hp: Weight-to-Horsepower Ratio (lbs/hp)

TT: Truck Type, 5 or 9 in this case

Figure 22 shows the plot comparing of calibrated empirically estimated v_m and formula predicted when the grade is greater than 1%.

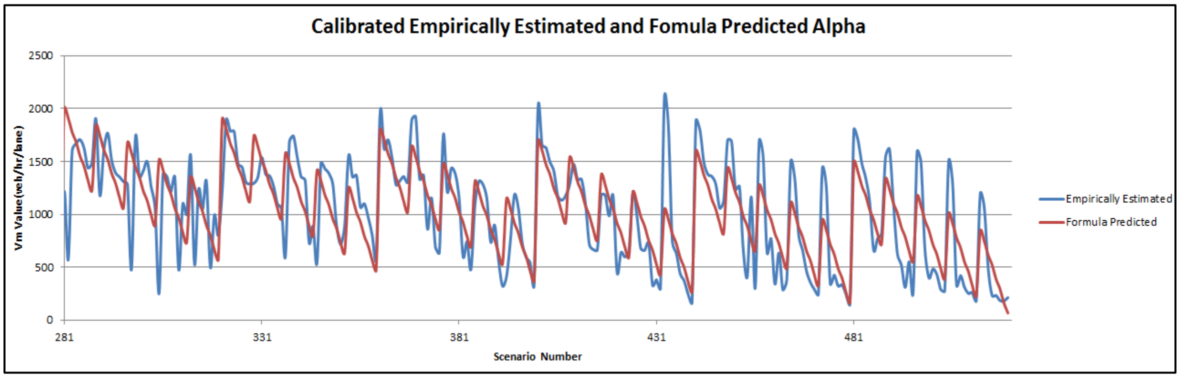


Figure 22: Comparison of Calibrated Empirically Estimated V_m and Formula Predicted V_m (Grade>1%)

Each pair of V_m and β values have a pair of correspondent higher percentile (previous 98th) and lower percentile (previous 10th). Figure 22 shows the plot of these calibrated empirically estimated percentiles. The marked "Higher Percentile" (previous 98th) stays restricted between 0.8 and 1. The marked “lower percentile” (previous 10th) is more scattered especially when the grade is greater than 1%. Both of these two plots of percentiles do not have obvious trend. For higher percentile the best fited regression model has independent variables of V_m and truck percentage. Equation 5.4 shows the model to predict higher percentiles.

$$HP = \underset{76.97}{1.085} - \underset{-8.22}{0.002}\%Trk - \underset{-17.2906}{0.00013} v_m \qquad \mathrm{R}^2 = 0.37 \qquad 5.4$$

Where, HP:Higher Percentile

%Trk: Truck Percentage(%)

v_m: v_m value

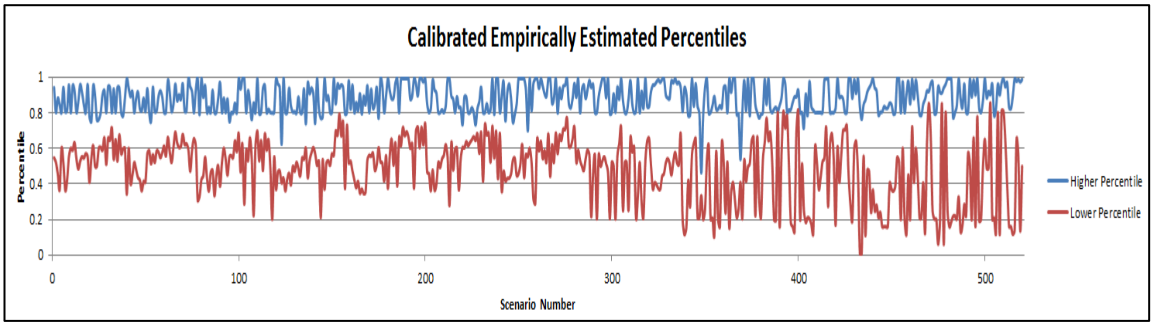


Figure 23: Calibrated Empirically Estimated higher and lower Percentiles

The R^2 in Equation 5.4 is less than 0.5, which is not as ideal as Equation 5.3. From Figure 23, a default value for higher percentile could be 0.9.

For lower percentile the best fited regression model has independent variables of V_m and grade percentage. Equation 5.5 shows the model to predict higher percentiles.

$$LPercentile = \underset{40.94}{0.678} - \underset{-12.00}{0.02}\%Grade - \underset{-12.90}{0.00018} v_m \qquad \mathrm{R}^2 = 0.34 \qquad 5.5$$

Where, LP:Lower Percentile

%Trk: Truck Percentage(%)

v_m: v_m value

From Figure 23, suggested values for grade less than 1% and greater than 1% are 0.55 and 0.4 separately.

Figure 24 shows the trend of β values.

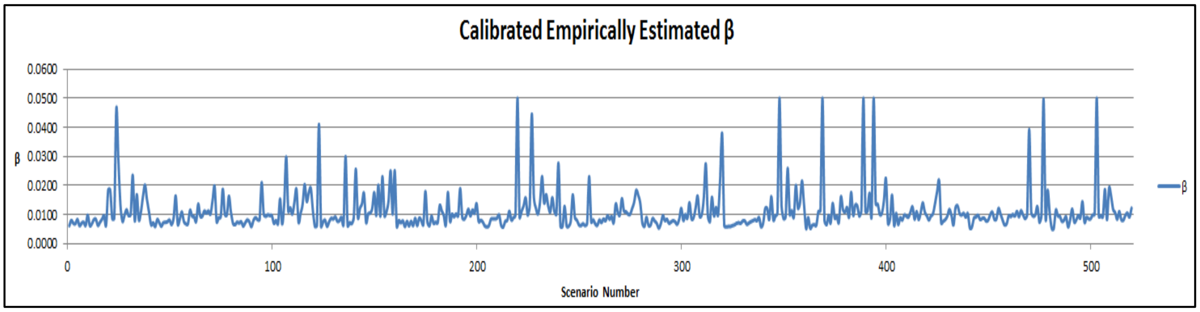


Figure 24: Calibrated Empirically Estimated β

It could be seen from Figure 24 that even though the β values trend is not clear, it stays in the range from 0.005 to 0.05. To estimate β value, a regression employing V_m,higher percentile, and lower percentile as the independent variable was applied in Equation 5.6.

$$\beta = \underset{13.28}{0.0986} - \underset{-12.00}{9\times10^{-6}}\, v_m - \underset{-13.3431}{0.09418}\, HP + \underset{4.17}{0.012616}\, LP \qquad R^2 = 0.28 \qquad 5.6$$

The two composite trucks scenario in Table 3 can be used as an example of a composite truck scenario. The relative percentage which is 62.5% for light trucks and 36.5% for large trucks can stay the same during the simulation. Then, a 6% grade network was employed as the worst situation and truck percentage varied from 10% to 50%. Figure 25 shows the speed-flow relationship of 30% mixed trucks scenario which has 19.1% light trucks and 10.9% large trucks. In Figure 25, the marked "Trucks" is the plot of speed-flow relationship of the composite trucks. The speeds and flow rates of the composite trucks in mixed traffic stream as well as the marked "Truck only" plot which is speed-flow relationship of the composite trucks only(62.5% light trucks and 36.5% large trucks) in have been synthesized according to their relative percentages. Equation 4.7 is used to compute estimated auto speed in mixed traffic stream and the marked "Auto Model" shows the estimated auto speeds in mixed traffic stream. When applying Equation 4.7, the truck-only speed in this case is the synthesized composite truck only speeds based on their relative percentage. From Figure 25, it can be concluded that this model is able to predict auto speeds in mixed truck classes stream.

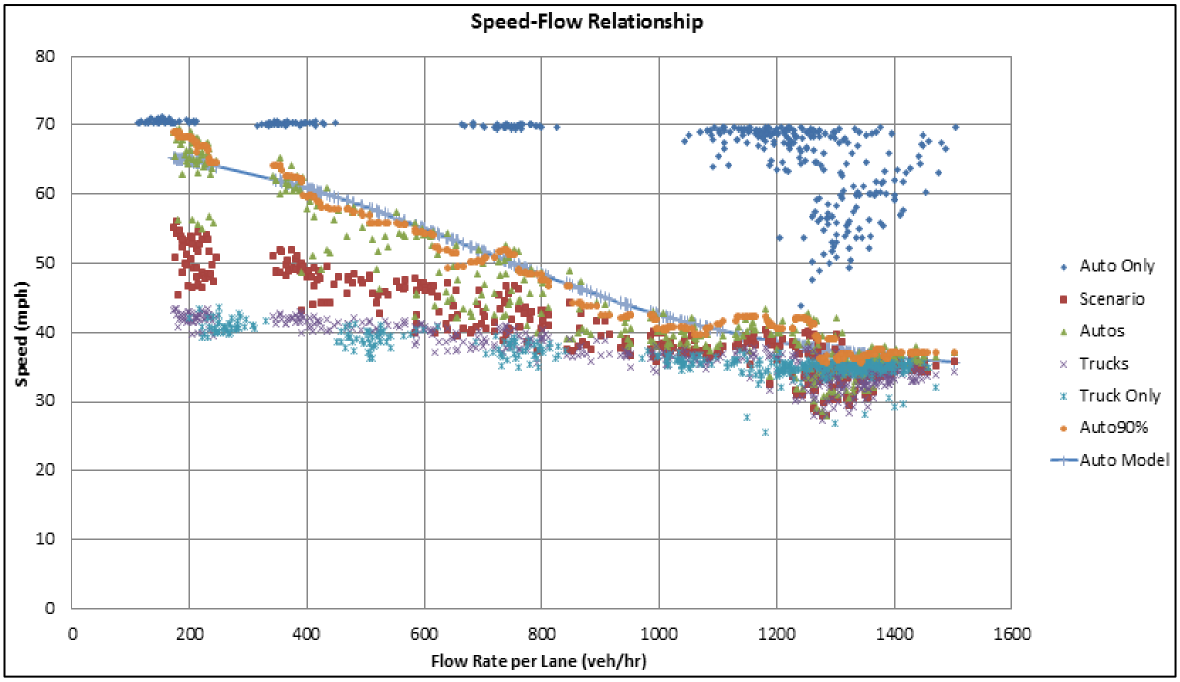


Figure 25: Speed-Flow Relationships for 30% Composite Trucks along 6% Grade

6. CONCLUSIONS AND RECOMMENDATIONS

6.1. Conclusions

This paper focused on further exploration of the HCM's methodology to estimate trucks' effect on freeways. Most of the work in this thesis was derived from for NCFRP-41 report. In the freeway section of NCFRP-41 report, the trucks effects were analyzed in three ways: PCE values prediction, truck speed prediction, and auto-speed prediction. To avoid the overlapping, this thesis focused mainly on PCE values prediction and auto-speeds prediction in mixed traffic stream prediction. 6552 combinations of truck mix, grade, and traffic flow rate simulation were as follows:

- FHWA class: 5 and 9;
- Weight-to-horsepower ratio: 50, 100, 150, and 200 lbs/hp;
- Grade: from -6% to 6% (This is 13 grades total.);
- Truck percentage: 0, 10%, 20%, 30%, 40%, 50% and 100%;
- Flow rate: 240, 600, 1200, 1800, 1920, 2040, 2160, 2280, and 2400 vphpl.

  The one-minute average density, flow rate, and speed on the grade section for each scenario were evaluated in VISSIM.

6.1.1. PCE Value Estimation

Comparing the flow-density plot of mixed traffic stream and auto-only traffic stream, a capacity reduction was observed for almost all the scenarios. Therefore, the idea to compute PCE values based on capacity was motivated. Equation 6.1 was then employed to estimate the PCE values for truck of all the simulations.

$$f_{ao}^{s} = f_{m}^{s}(p_a + p_t PCE) \quad or \quad PCE = \frac{f_{ao}^{s} - p_a f_{m}^{s}}{p_t f_{m}^{s}} \qquad 6.1$$

In VISSIM, we haven't found a way to get capacity values directly. Thus, the 95th flow rate of the network was used as the capacity of the network. The breakthrough of this technique was that it introduced a regression equation to predict PCE values. Equation 7.2 shows the PCE prediction regression model.

$$PCE = \underset{3.95}{0.322} + \underset{9.40}{0.07632} TT + \underset{27.5}{0.00799} WtHp - \underset{-5.06}{.00582} T\% + \underset{16.01}{.1300} G\% \quad R^2 = 0.8976 \qquad 7.2$$

This equation enables users to apply different truck types, weight-to-horsepower ratios, truck percentages, and various grades. Compared to the PCE values presented in HCM, the equation was able to provide continuous PCE values. From Figure 17, it could be concluded that grade was the major factor which drives the PCE values up. When grades are less than 0%, truck percentage's influence on PCE values was more obvious than weight-to-horsepower ratios and when grades are greater than 0%, weight-to-horsepower ratio's influence was more obvious.

Compared to the HCM look up table, this equation has a limitation of not being able to evaluate PCE based on length of grade. There is one critical point in this research is that even though the truck speed was calibrated based on the revised NCHRP 505 report, the PCE values were developed from VISSIM model in which some of the variables may not be treated exactly the same way in real world. For example, the truck type in VISSIM only determines the vehicle 3D model and vehicle length. The 3D model would make a difference in animation but not the performance of the vehicle. Therefore, the independent variable of truck type in Equation 7.2 is a reflection of truck length.

The coefficients of all the variables are positive except for truck a percentage, which means PCE values decrease as the truck percentages increase. This result indicates that individual truck has more impact on traffic steam when truck percentage is lower and it is consistent with the PCE trend in the HCM.

6.1.2. Auto-Speed Estimation

The speed-flow curves in the HCM only provide auto speed in base conditions in which there is an assumption of no trucks in traffic stream. In the current HCM procedure, traffic stream could be treated as base condition after adjusting the heavy vehicle by PCE values. However, from Figure 13, it could be seen that auto speed in mixed traffic stream did not always follow the auto speed under base condition. Actually, it was found that when the truck percentage is low along downgrades, the auto speeds in mixed traffic stream tended to follow the speeds in auto-only scenario. When both the truck percentage is high and flow rates are high along upgrades, the auto speeds in mixed traffic stream tended to follow the truck speeds. Eventually, the pattern that auto speeds decrease across flow rates was modeled as a logistics curve. Equation 6.3 was first used to estimate auto speeds in traffic stream.

$$s_a = s_{to} + (s_{ao} - s_{to})\left(\frac{e^{-\beta(v-v_m)}}{1+e^{-\beta(v-v_m)}}\right) \qquad 6.3$$

The V_m and β in Equation 7.3 were the critical coefficients which determine the quality of the prediction model since V_m locates the center this logistic curve and β masters the scale of this logistic function. Figure 16 illustrates that there were two percentiles of auto speeds in mixed traffic stream that determine V_m and β values. They were first defined as the 98th and 10th auto speed in mixed traffic stream. From the analysis in chapter 6, these two percentiles were not always the best since sometimes the data were noisy especially on the low percentiles. Therefore, it was necessary to find out the best percentiles for each pair of V_m and β.

Thus, a minimization equation was employed to find the best V_m and β for each scenario. The minimization process commenced in Microsoft Excel by solver tool and GRG Nonlinear searching algorithm was selected since the logistic function is nonlinear and smooth. V_m , β, and the correspondent speed percentiles were found for each scenario. The V_m showed two different patterns for upgrades and downgrades. β did not have clear trends but it stays from 0.005 to 0.05. The trend of pairs of percentiles did not clear as well but they were restricted in one range (0.8 to 1 for higher percentiles).

6.2. Recommendations for Future Research

6.2.1. Truck Class in Simulation Model

Truck class 5 and 9 models by FHWA definition were employed in the VISSIM models. Truck 5 is a single unit truck while truck 9 is tractor-trailer truck. In VISSIM, the different truck classes would have different 3D models. However, in the simulation runs, the only effect of the truck model in traffic stream is the length of model, which means it doesn't take the maneuverability, performance of different truck classes into consideration. In Equation 6.2, the coefficient of truck type is positive and this is based that class 9 truck is longer than class 5 truck. In this research, all types of trucks were supposed to be able to be synthesized into class 5 and class 9 trucks. It will be useful if the future research could focus on developing a more elaborate way which takes the maneuverability, performance, and driver behavior of different class of trucks into consideration to analyze methodology for different truck classes.

6.2.2. Trucks on Downgrades

In the logic to predict truck speed along freeway, the truck speed was equal to free flow speed when grades are lower than 0%. In aspect of developing truck profile, this research focused on developing truck acceleration functions on upgrades. For downgrades, the truck crawl speed was able to maintain at 70mph, which lead to a situation that when it is on downgrades, trucks look like autos except for the length and the time of acceleration. This did not take complete driver behaviors of truck drivers and auto drivers into consideration. For example, autos tend to overtake trucks or stay away from trucks even when the trucks' speeds are high. It will be of significance to further develop models that could take into account the driver behavior so that trucks and autos perform dissimilarly, especially on downgrades.

6.2.3. Capacity Definition

In this research, PCE values were computed based on capacity and the 95th flow rate on the network was used as capacity which proved to be an appropriate method. However, since capacity is the maximum hourly rate at which vehicles can reasonably be expected to cross a point on a roadway during a given time period under prevailing traffic roadway and control condition (25), the theoretical percentile of the flow rate should be at 100th. The 95th flow rate was selected in this research to avoid the noise due to the randomness in simulation. In fact, capacity could be computed in various ways. For example, Geistefeldt (22) predicted PCE values based he stochastic capacities concept is related to empirical capacity distribution functions for specific roadway, traffic, and control conditions. The PCE values computed based capacity could be improved if capacity can be calculated in a more accurate way.

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