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Decision-Feedback Detector
for Flat Rayleigh Fading
Synchronous CDMA Channels

H-Y. Wu
A. Duel-Hallen

Center for Communications and Signal Processing
Department of Electrical and Computer Engineering
North Carolina State University

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by Hsin-Yu Wu and Alexandra Duel-Hallen

Department of Electrical and
Computer Engineering
North Carolina State University
Raleigh, NC 27695-7911
phone/fax: (919) 515-7352 / 5523
internet: hwu@eos.ncsu.edu, sasha@eos.ncsu.edu

Abstract

We consider interference rejection for synchronous CDMA frequency non-selective channels using multiuser decision-feedback detector. The average error rate of the ideal two-user decision feedback detector is derived in a closed form and the asymptotic multiuser efficiency is proved to be unity. The effect of error propagation on the actual decision-feedback detectors is investigated through simulation.

Comparison among the decision-feedback detector, the conventional detector, and the linear decorrelating detector is undertaken for several two-user and four-user bandwidth-efficient synchronous CDMA flat Rayleigh fading channels. We find that the decision-feedback detector outperforms other detectors and its average bit error rate is close to the single user bound under diverse channel conditions.

I. Introduction

The major limitation of the current CDMA proposals is the multi-access interference (MAI) in the reverse link signal (i.e., the signal traveling from the mobile unit to the base station receiver). The interference can easily mask the weaker user in the 'near-far environment' in which the powers received from different users are dissimilar due to propagation losses (i.e., the near-far effect). In the presence of fading, the interference is enhanced and can severely degrade the receiver's detection ability [1]. The interference due to the presence of other users can be successfully canceled if the conventional single-user correlation receiver is replaced by a more sophisticated receiver structure. A number of interference cancellation methods were proposed recently. These approaches include the optimal [2], linear [3], multistage [4] and decision-feedback detectors [5]. In [3-5], the performance of these detectors was analyzed for additive white Gaussian noise synchronous CDMA channel. In [5-6] it was shown that the decision-feedback detectors solve the near-far problem, improve upon linear detectors, and are simpler and at least as reliable as the multistage detectors. In this paper, we are studying the ability of decision-feedback detectors to combat flat fading present in the mobile environment. The performance of the decision-feedback detectors will be evaluated and compared with the linear decorrelating detectors and the conventional detectors for flat Rayleigh fading synchronous CDMA channel.

In section II, we review the synchronous CDMA channel model and the decision-feedback detector. In section III, we analyze the average bit error rate (BER) of the ideal and actual two-user decision-feedback detectors. Asymptotic multiuser efficiency (AME) of a two-user decision-feedback detector is derived in section IV. Performance evaluation and numerical comparison among the conventional detector, the linear decorrelating detector and the decision-feedback detector are presented in Section V.

II. Synchronous CDMA System Model and the Decision Feedback Detector

Consider the synchronous CDMA channel with K users. Each user employs Binary Phase-Shift Keying (BPSK) modulation and is subject to flat Rayleigh fading. The noiseless baseband complex signal of the k -th user is

$$u_k(t) = C_k(t) b_k(t) s_k(t),$$

where $C_k(t)$ is a complex fading coefficient, $b_k(t)$ is the information bit drawn from $\{+1, -1\}$, and $s_k(t)$ is the signature waveform. The signal received at the receiver is the sum of K users' signals and noise:

$$r(t) = \sum_{k=1}^K u_k(t) + n(t),$$

where $n(t)$ is the additive white Gaussian noise with power N_0 .

The front-end of the decision-feedback multiuser detectors consists of a bank of matched filters. Each matched filter is designed to correlate each individual user's signature waveform. The output of the matched filter bank can be represented by the column vector \mathbf{y} of length K :

$$\mathbf{y} = \mathbf{R}\mathbf{W}\mathbf{b} + \mathbf{z}, \quad (1)$$

where the input vector \mathbf{b} consists of the information bits of K users drawn from the antipodal binary alphabet, the

matrix \mathbf{R} has components $R^{k,l} = \int_{-\infty}^{\infty} s_k(t) s_l(t) dt$, ($k, l = 1, \dots, K$), the cross-correlations between signature waveforms of users k and l , the diagonal matrix \mathbf{W}

represents the complex channel gains: $W^{k,k} = |C_k| \exp(-j\theta_k)$, $k=1, \dots, K$, and \mathbf{z} is a K -dimensional complex Gaussian noise vector with spectrum N_0R .

For a flat Rayleigh fading channel, the channel coefficients $|C_k| \exp(-j\theta_k)$, are constant during the transmitting interval, and are given by independent

complex Gaussian random variables $a_k + jb_k$, where a_k and b_k are i.i.d. real Gaussian random variables with variances N_0 . Thus the amplitude $|C_k|$ is Rayleigh distributed, and the phase θ_k is uniform on $[0, 2\pi]$. We assume that the fading varies sufficiently slowly so that the amplitude and phase of each user can be reliably estimated.

The output of the feed-forward filter of the decision-feedback detector is the white noise model equivalent to (1) [5]. As shown in Figure (1), the decision-feedback detector first arranges the users in the descending energy order ($|C_1| \geq |C_2| \geq \dots \geq |C_K|$), and then performs Cholesky factorization of the matrix \mathbf{R} to find the noise-whitening filter $(\mathbf{F}^T)^{-1}$ (i.e., $\mathbf{R} = \mathbf{F}^T \mathbf{F}$). If this filter is applied to the matched filter output \mathbf{y} , the resulting output vector is

$$\tilde{\mathbf{y}} = \mathbf{F}\mathbf{W}\mathbf{b} + \mathbf{n}, \quad (2)$$

where \mathbf{F} is a left lower triangular matrix with its entry denoted by f_{ij} , and \mathbf{n} is white complex Gaussian noise with spectrum $N_0\mathbf{I}$. The lower triangular structure of \mathbf{F} leads to the successive interference cancellation. The signal of the first user ($\tilde{y}_1 = f_{11}C_1b_1 + n_1$) does not contain any interference from the other users, therefore the information bit can be estimated by computing $\text{sgn}[\text{Re}(\tilde{y}_1 C_1^*)]$. The decision of the first user can be fed back and used for subtracting the interference term in the second user:

$$\tilde{y}_2 - f_{21}C_1\hat{b}_1 = f_{21}C_1(b_1 - \hat{b}_1) + f_{22}C_2b_2 + n_2.$$

The decision for the second user is $\text{sgn}[\text{Re}((\tilde{y}_2 - f_{21}C_1\hat{b}_1)C_2^*)]$. Similarly, the decision for the k -th user is

$$\begin{aligned} \hat{b}_k &= \text{sgn}\left(\text{Re}\left[\left(\tilde{y}_k - \sum_{i=1}^{k-1} f_{ki}C_i\hat{b}_i\right)C_k^*\right]\right) \\ &= \text{sgn}\left(\text{Re}\left[\left(f_{kk}C_kb_k + \sum_{i=1}^{k-1} f_{ki}C_i(b_i - \hat{b}_i) + n_k\right)C_k^*\right]\right). \end{aligned} \quad (3)$$

The multi-user interference in the signal of the k -th user can be completely deleted if decisions of the users 1 to $k-1$ are correct. To increase the probability that the previous decisions are correct, reordering the received signals according to their strengths ($|C_i|$) is desirable. Note that the path energy estimation and the reordering process are not needed for the decorrelator, but they are required for the decision-feedback detector.

III. Bit Error Rate Analysis for Multiuser Decision-Feedback Detector

The general approach to computing the ideal average error rate for the Rayleigh fading channel is to first calculate probability of error for the Gaussian channel in terms of the signal-to-noise ratio (SNR), and then to integrate it over the probability density function of the SNR. However, since the decision-feedback detector orders the users according to their strengths, the ideal average error rate will be calculated using the conditional

densities of the fading coefficients (i.e., the signal-to-noise ratios) given this order. In the following analysis, we fix the index of each user, and let the order vary as a function of the Rayleigh fading parameters. We will compute the ideal average error rate of the decision-feedback detector for the Rayleigh fading channel for first user (index=1).

Let γ_i be the signal-to-noise ratio of user i , defined by $\gamma_i = |C_i|^2 E_b / N_0$, where E_b is the energy per symbol. Since $|C_i|^2$ has a chi-square probability density function with two degrees of freedom, the probability density function of γ_i is [7]

$$f_{\gamma_i}(x) = \frac{1}{\bar{\gamma}_i} e^{-\frac{x}{\bar{\gamma}_i}},$$

where $\bar{\gamma}_i = E(\gamma_i) = \frac{E_b}{N_0} E(|C_i|^2)$. Let E_k be the event that user 1 is the k -th strongest user among the total K users. Then the average error rate of decision-feedback detector can be expressed as

$$PE_{df} = \sum_{k=1}^K \Pr(E_k) \int_0^{\infty} Pe_k(x) f_{\gamma_1}(x | E_k) dx, \quad (4)$$

where $\Pr(E_k)$ is the probability of event E_k , Pe_k is the probability of error when user 1 is the k -th strongest user for Gaussian channel, $f_{\gamma_1}(x | E_k)$ is the conditional probability density function of γ_1 given E_k .

As an example, consider a two-user channel.

(1) BER of Ideal Two-User Decision Feedback Detector:

Suppose that the normalized correlation matrix of signature waveforms \mathbf{R} in (1) is $\mathbf{R} = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$, where the auto-correlations of user 1 and user 2 are normalized to 1 and their cross-correlation is denoted by r ($0 \leq |r| \leq 1$). By Cholesky factorization, the matrix \mathbf{F} is

$$\mathbf{F} = \begin{bmatrix} \sqrt{1-r^2} & 0 \\ r & 1 \end{bmatrix}.$$

Two situations need to be considered: (i) E_1 : user 1 is the strongest user ($\gamma_1 > \gamma_2$) and (ii) E_2 : user 1 is the second strongest user (or the weakest in this case) ($\gamma_1 < \gamma_2$).

(i) When user 1 is the strongest user, the probability of error Pe_1 is

$$Pe_1(\gamma_1) = Q(\sqrt{2\gamma_1 f_{11}^2}) = Q(\sqrt{2\gamma_1(1-r^2)}). \quad (5)$$

(ii) When user 1 is the weakest user, the probability of error Pe_2 is

$$Pe_2(\gamma_1) = Q(\sqrt{2\gamma_1 f_{22}^2}) = Q(\sqrt{2\gamma_1}). \quad (6)$$

Equation (6) is obtained under the assumption that the previous decision is correct, i.e., $\hat{b}_2 = b_2$.

Note that Pe_1 is the same as the probability of error of user 1 for the decorrelator, and Pe_2 is identical to that of a single user system for AWGN channels [5]. Conditioning on these two situations, from (4), the

average error rate of the ideal two-user decision-feedback detector is

$$PE_{df} = \Pr(\gamma_1 > \gamma_2) \int_0^{\infty} Pe_1(x) f_{\gamma_1}(x | \gamma_1 > \gamma_2) dx + \Pr(\gamma_1 < \gamma_2) \int_0^{\infty} Pe_2(x) f_{\gamma_1}(x | \gamma_1 < \gamma_2) dx \quad (7)$$

The probabilities and the conditional probability density functions, can be evaluated as follows:

$$\Pr(\gamma_1 > \gamma_2) = \int_0^{\infty} \Pr(\gamma_1 > y) f_{\gamma_2}(y) dy = \frac{\bar{\gamma}_1}{\bar{\gamma}_1 + \bar{\gamma}_2} \quad (8)$$

$$\Pr(\gamma_1 < \gamma_2) = \frac{\bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \quad (9)$$

$$f_{\gamma_1}(x | \gamma_1 > \gamma_2) = \frac{d}{dx} \Pr(\gamma_1 < x | \gamma_1 > \gamma_2) = \frac{d}{dx} \frac{\Pr(\gamma_2 < \gamma_1 < x)}{\Pr(\gamma_1 > \gamma_2)} = \frac{\bar{\gamma}_1 + \bar{\gamma}_2}{\bar{\gamma}_1^2} [e^{-\frac{x}{\bar{\gamma}_1}} - e^{-\frac{x}{\bar{\gamma}'}}] \quad (10)$$

$$f_{\gamma_1}(x | \gamma_1 < \gamma_2) = \frac{1}{\bar{\gamma}'} e^{-\frac{x}{\bar{\gamma}'}} \quad (11)$$

and where $\frac{1}{\bar{\gamma}'} = \frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}$. Substituting these terms into (7)

leads to the average error rate of the ideal two-user decision-feedback detector,

$$PE_{df} = \frac{1}{2} [1 - \sqrt{\frac{\bar{\gamma}_1}{\bar{\gamma}_1 + 1/(1-r^2)}}] - \frac{1}{2} \left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \right) [1 - \sqrt{\frac{\bar{\gamma}'}{\bar{\gamma}' + 1/(1-r^2)}}] + \frac{1}{2} \left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \right) [1 - \sqrt{\frac{\bar{\gamma}'}{\bar{\gamma}' + 1}}] \quad (12)$$

The first two terms in (12) are the average error rate when user 1 is stronger, and the third term is the average error rate when user 1 is weaker. It will be shown in section IV that the asymptotic average BER of the ideal decision-feedback detector is the same as the asymptotic average BER of BPSK single user system.

(2) BER of Actual Two-User Decision-Feedback Detector

In practice, incorrect decisions made by stronger interferers can affect the correctness of the current decision for the desired user. Reconsider (5) and (6) for the two-user case. When user 1 is the stronger user, there is no change in its probability of error (Pe_1), because user 1 is

demodulated first, independently of the previous decisions. However, when user 1 is the weaker user, the actual decision \hat{b}_1 depends on \hat{b}_2 . From Figure 1, the input to the decision device is

$$\text{Re}[(\bar{y}_1 - r|C_2|e^{-j\theta_2}\hat{b}_2)|C_1|e^{j\theta_1}] = r|C_1||C_2|(b_2 - \hat{b}_2)\cos(\theta_1 - \theta_2) + |C_1|^2 b_1 + N_1,$$

where $N_1 = \text{Re}[n_1|C_1|e^{j\theta_1}]$. The decision error of the stronger user (user 2), $b_2 - \hat{b}_2$, is a random variable with the following probability mass function:

$$b_2 - \hat{b}_2 = \begin{cases} 0 & \text{with prob. } 1 - P_2 \\ 2 & \text{with prob. } \frac{P_2}{2} \\ -2 & \text{with prob. } \frac{P_2}{2}, \end{cases}$$

where P_2 is the probability that the decision for the stronger user (user 2) is incorrect, i.e., $P_2 = Q(\sqrt{2\gamma_2(1-r^2)})$. The probability of error when user 1 is the weaker user is then the function of γ_1 and γ_2 :

$$Pe_2(\gamma_1, \gamma_2) = (1 - P_2)Q(\sqrt{2\gamma_1}) + \frac{P_2}{2}Q(\sqrt{2\gamma_1} - 2r\cos\phi\sqrt{2\gamma_2}) + \frac{P_2}{2}Q(\sqrt{2\gamma_1} + 2r\cos\phi\sqrt{2\gamma_2}) \quad (13)$$

where $\phi = \theta_1 - \theta_2$ is the phase difference between the two received signals. The average error rate of the actual decision-feedback is in the form of

$$PE_{df} = \int_0^{\infty} \left\{ \int_y^{\infty} Pe_1(x) f_{\gamma_1, \gamma_2}(x, y) dx + \int_0^y Pe_2(x, y) f_{\gamma_1, \gamma_2}(x, y) dx \right\} dy \quad (14)$$

where $f_{\gamma_1, \gamma_2}(x, y) = \left(\frac{1}{\bar{\gamma}_1} e^{-x/\bar{\gamma}_1}\right) \left(\frac{1}{\bar{\gamma}_2} e^{-y/\bar{\gamma}_2}\right)$ is the

joint probability density function of γ_1 and γ_2 . We evaluate the integral numerically using Monte-Carlo simulation techniques. For more than two users, it is not trivial to analyze the average bit error rate for the actual decision-feedback detector, since every error pattern has to be considered [5]. These cases are also studied using simulations.

IV. Asymptotic Multiuser Efficiency of Decision-Feedback Multiuser Detectors

The Asymptotic Multiuser Efficiency (AME) is a measurement of the performance degradation of a multiuser detector due to the multiple-access interference. It was introduced in [2],[3], and was further generalized for the flat Rayleigh fading channel in [8]. The AME is defined as the ratio of the effective average SNR required by a single-user system and the actual average SNR

required by the multiuser detector to achieve the same asymptotic error rate.

Consider the ideal two-user decision-feedback detector. The asymptotic error rate of (12) is (as $\bar{\gamma}_1$ become large)

$$\begin{aligned} PE_{df} &\approx \frac{1}{4} \frac{1/(1-r^2)}{\bar{\gamma}_1} - \frac{1}{4} \left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \right) \frac{1/(1-r^2)}{\bar{\gamma}'} \\ &+ \frac{1}{4} \left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \right) \frac{1}{\bar{\gamma}'} \\ &= \frac{1}{4} \frac{1}{\bar{\gamma}_1}. \end{aligned} \quad (15)$$

The effective average SNR $\bar{\gamma}_0$ of the single-user system is expressed in terms of single user bound as [7]

$$PE_{su} = \frac{1}{2} \left[1 - \sqrt{\frac{\bar{\gamma}_0}{1 + \bar{\gamma}_0}} \right] \approx \frac{1}{4\bar{\gamma}_0}. \quad (16)$$

Therefore, for a given average error rate, i.e., $PE_{su} = PE_{df}$, the asymptotic multiuser efficiency is

$$\eta_{df} = \lim_{N_0 \rightarrow 0} \frac{\bar{\gamma}_0}{\bar{\gamma}_1} = 1. \quad (17)$$

The unity AME implies that the ideal two-user decision-feedback detector is interference resistant and its performance is the same as that of the optimal single user system for large SNR. This result leads to the conjecture that the AME of ideal decision-feedback detector is unity [9].

As a comparison, consider the AME of the two-user decorrelator. Since the average error rate of the decorrelator for flat Rayleigh fading channel is [8]

$$\begin{aligned} PE_{dec} &= \frac{1}{2} \left[1 - \sqrt{\frac{\bar{\gamma}_1}{\bar{\gamma}_1 + [\mathbf{R}^{-1}]_{1,1}}} \right] \\ &= \frac{1}{2} \left[1 - \sqrt{\frac{\bar{\gamma}_1}{\bar{\gamma}_1 + 1/(1-r^2)}} \right] \\ &\approx \frac{1}{4} \frac{1}{\bar{\gamma}_1} \left(\frac{1}{1-r^2} \right), \end{aligned} \quad (18)$$

the AME of the decorrelator is

$$\eta_{dec} = \lim_{N_0 \rightarrow 0} \frac{\bar{\gamma}_0}{\bar{\gamma}_1} = 1 - r^2. \quad (19)$$

Equation (19) and (17) implies that the asymptotic performance of the decorrelator degrades as the cross-correlation becomes large whereas the performance of the ideal decision-feedback detector is not affected by the cross-correlation.

V. Numerical and Simulation Result

Example 1: Two-User Channel

We compare the decision-feedback detector with the conventional detector and the decorrelator. The average error rate of the conventional detector is evaluated using the result for distribution of quadratic form in complex Gaussian variates [10]:

$$PE_{conv} = \sum_{\substack{m=(-1,1) \\ n=(-1,1)}} \sum_{i=1}^3 \left(\prod_{k \neq i, k=1}^3 \frac{\lambda_i}{\lambda_i - \lambda_k} \right) \Pr(b_1 = m, b_2 = n)$$

The variable λ_i is the negative (positive) eigenvalue of matrix $Q\Sigma$ when b_1 is +1 (-1), where

$$Q = \begin{bmatrix} b_1 & rb_2/2 & 1/2 \\ rb_2/2 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$$

and
$$\Sigma = N_0 \begin{bmatrix} \bar{\gamma}_1 & 0 & 0 \\ 0 & \bar{\gamma}_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Figures 2 and 3 show the result for two users with equal average path strength or SNR ($\bar{\gamma}_1 = \bar{\gamma}_2$). The gap between the decorrelator and the single user bound increases as cross-correlation (r) increases. This means that the performance of the decorrelator degrades as cross-correlation (r) increases, as expected in (19). However, the cross-correlation does not affect the performance of the decision-feedback detector significantly. The very large r ($r=0.9$) used here may not be realistic for the two-user case. The purpose of using heavy cross-correlation ($r=0.9$) is to investigate the performance of the decision-feedback detector for a constrained bandwidth-efficient channel. With many interferers in such channel, the cross-correlation may accumulate to this order. In both cases, the performance of the decision-feedback detector approaches the single user bound, showing that the asymptotic efficiency of the decision-feedback detector is unity, as shown in (17). The deviation between the curve of the decision-feedback detector and the single user bound occurs in the lower average SNR region ($\bar{\gamma}_1 < 20$ dB). In this low SNR region, the first two terms in (12) are significant and contribute to the average BER. In the high SNR region, these terms are negligible, therefore the asymptotic error rate of the ideal decision-feedback detector agrees with the single user bound. The error rate of the actual decision-feedback detector is quite close to the ideal case, showing that the error propagation does not degrade the system performance significantly.

In Figures 4 and 5, we study the case when the average path strengths of the two users are not equal. When the interfering user (user 2) is stronger than user 1 on average, the average BER of user 1 is almost the same as single user bound (Figure 4). With higher probability, user 1 becomes the weaker user who benefits from the correct previous decision, and the probability of error when user 1 is the weaker user is the same as that of the single-user case. On the other hand, when the interfering user is weaker on average (Figure 5), the curve of the decision-feedback detector is driven away from the single user bound since user 1 is more likely to be the stronger user, whose probability of error is the same as the error rate of the decorrelator.

Example 2: Four-User Channel

The simulation results for the four-user case are shown in Figures 6 and 7. The set of signature waveforms is derived from Gold sequences of length seven. The correlation matrices \mathbf{R} and \mathbf{F} are

$$\mathbf{R} = \frac{1}{7} \begin{pmatrix} 7 & -1 & 3 & 3 \\ -1 & 7 & 3 & -1 \\ 3 & 3 & 7 & -1 \\ 3 & -1 & -1 & 7 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 0.69 & 0 & 0 & 0 \\ -0.32 & 0.9 & 0 & 0 \\ 0.495 & 0.41 & 0.99 & 0 \\ 0.43 & -0.14 & -0.14 & 1 \end{pmatrix}$$

The performance of the actual decision-feedback detector is still close to the single user bound, and the decision-feedback detector outperforms the decorrelator by 3 dB.

VI. Conclusion

We have evaluated the performance of the multiuser decision-feedback detector and compared it with the decorrelator and the conventional detector for the flat Rayleigh fading, synchronous CDMA channel. The average error rate of the ideal decision-feedback detector is analyzed in closed form for a simple two-user case and simulation results are provided for a four-user channel. These results indicate that the decision-feedback detector consistently gives near single user system performance, even in the bandwidth-efficient channels where the signature waveforms have strong cross-correlations. These results also verify the fact that asymptotic multiuser efficiency of the decision-feedback detector is unity.

The performance gain of the decision-feedback detector is at the cost of computational complexity due to rearrangement of the input signals, factorization of the cross-correlation matrix of the signature waveforms and successive cancellation of the multi-access interference. In addition, the decision-feedback detector requires path energy estimation, which is not needed for the decorrelator. The effect of estimation errors on the decision-feedback detector will be considered in our future work.

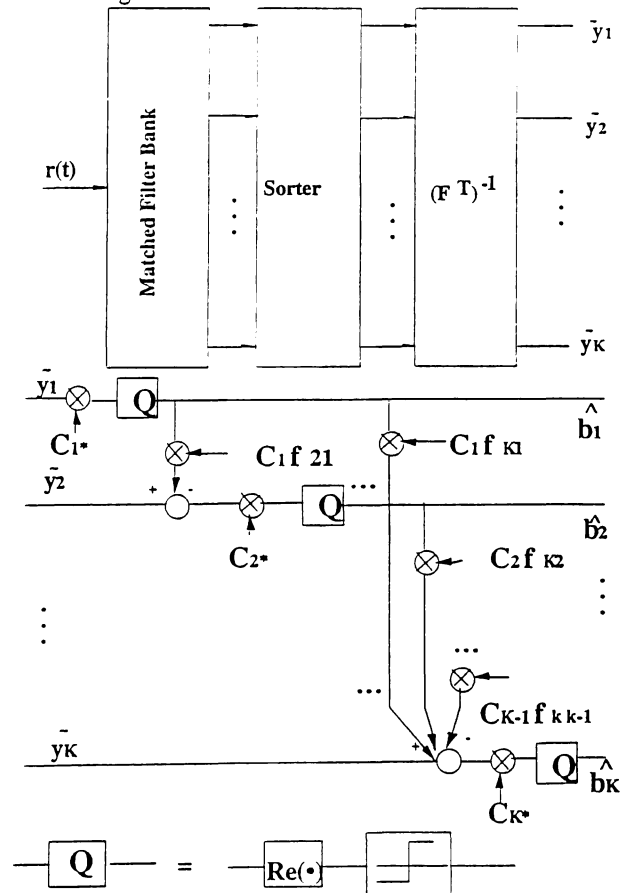
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Figure 1: The Decision-Feedback Detector



Ideal Decision-Feedback:
Actual Decision-Feedback: +++

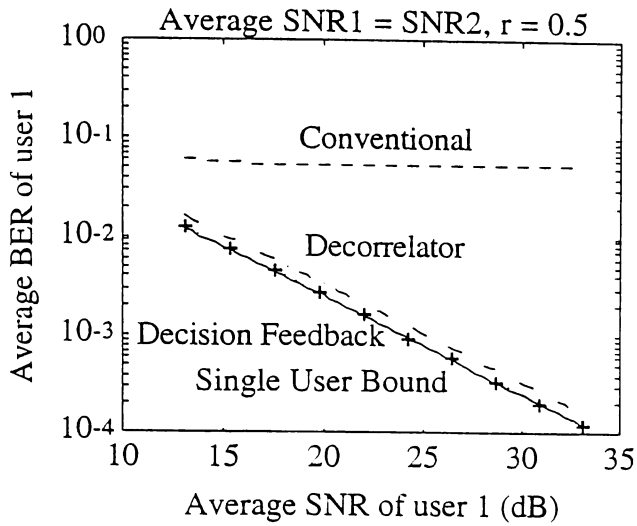


Figure 2: Two-user channel with equal path strengths

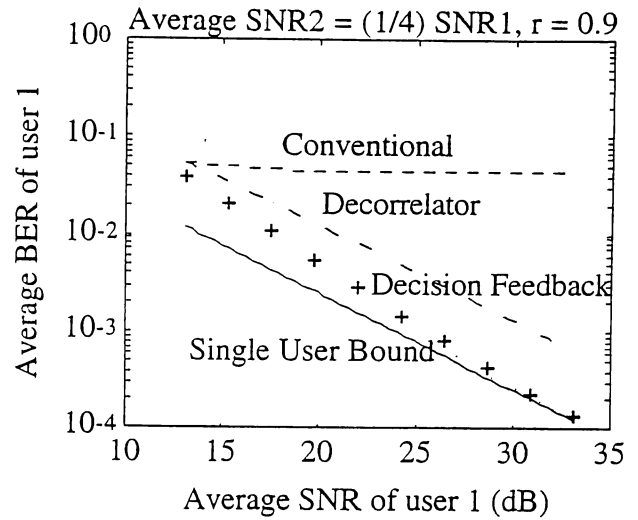


Figure 5: Two-user channel with weak interferer

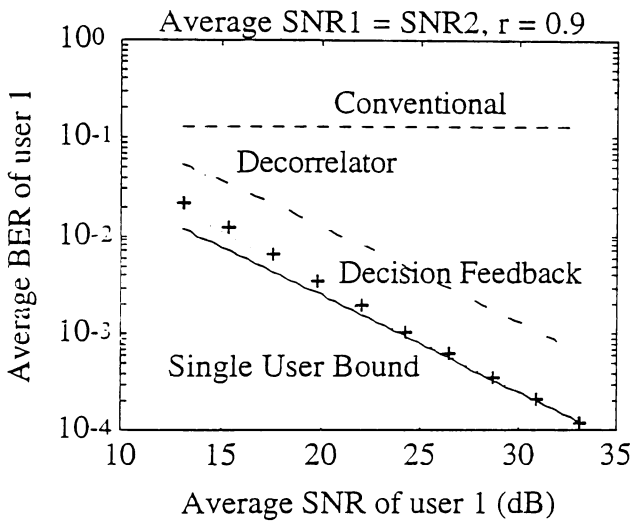


Figure 3: Two-user channel with equal path strengths

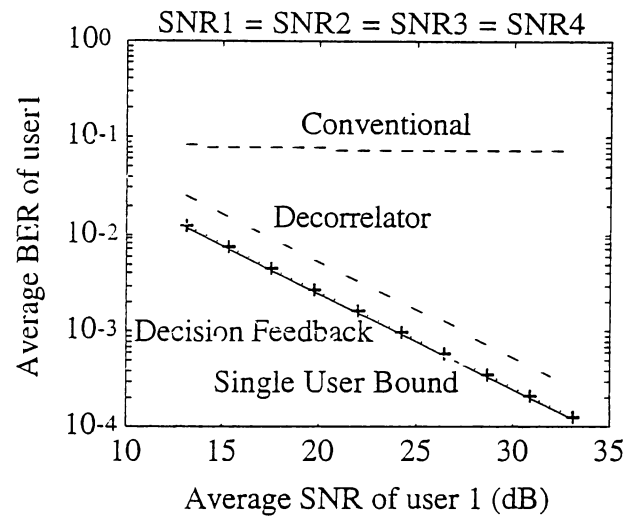


Figure 6: Four-user channel with equal path strengths

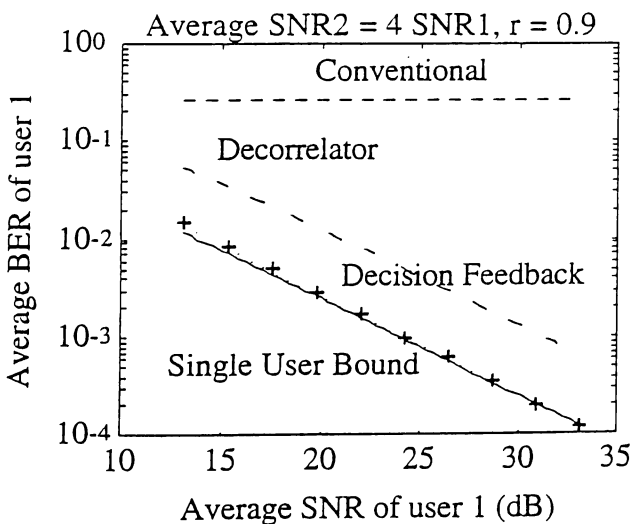


Figure 4: Two-user channel with strong interferer

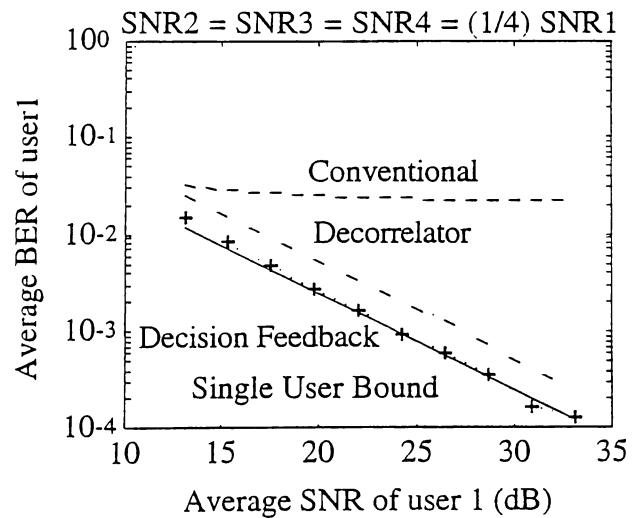


Figure 7: Four-user channel with unequal path strengths