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THE USE OF THE SEMILOOF SHELL ELEMENT IN THE BERSAFE FINITE ELEMENT SYSTEM

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SUMMARY

The difficulties in formulating shell finite elements which are capable of reliable performance in a wide range of different shell structures and loading situations are well known. However, the importance of developing such elements for the analysis of present and future energy systems is such that research in this area is quite extensive. The present work is concerned with the semiloof shell element which has been planned to satisfy the requirements of good versatile performance. It is the culmination of years of research by Prof. Irons, during which the element has developed from the solid three-dimensional isoparametric elements. The element has been adapted in the BERSAFE finite element system, both as a six-noded triangle and an eight-noded quadrilateral, and modified for general mechanical and thermal load systems. In appearance, the element resembles the well-known quadratic isoparametric two-dimensional elements, for which many data generation and post-processing procedures are available. Simple extensions of these processes have been made in BERSAFE for semiloof, thereby maintaining user convenience for shell analysis. The nodal degrees of freedom are of relatively low order, there being only local normal rotations apart from the usual three displacement components, which minimise problems of defining complex boundary conditions.

A brief resume of the theoretical derivation of the element is given, including the treatment of both mechanical and thermal loading. Since the element is derived from three-dimensional isoparametric elements, it uses numerical integration techniques during evaluation of the element stiffness matrices. Consequently, it is important to establish suitable orders of integration. For the quadrilateral form, the use of 2 x 2 reduced integration is compared to the more complete 3 x 3 integration. As observed in other types of element, the reduced rule is very accurate and therefore superior to the 'complete' rule, although in certain circumstances this rule can break down. A full discussion of this is given and illustrated using a cylindrical structure.

The versatility and accuracy of the element is demonstrated for a number of cases. These include the well-known cylindrical shell roof problem and, compared with other elements of the literature, the reduced integration results are of unparalleled accuracy. A simple but effective way of combining the semiloof elements in the three-dimensional isoparametric elements is demonstrated by a simple example. Despite a slight incompatibility of degrees of freedom, good results are shown, which is of significance for large structures where shells form solid attachments such as welds of flanges.

1. Introduction

The semiloof general purpose thin shell element has been developed by Irons [1] as a direct stiffness displacement element. This format enables its inclusion in existing finite element systems and in particular has been adopted in the BERSAFE system [2]. The shells may be deeply curved, examples of elements subtending 60° having shown good results. Thus the performance resembles the solid 20-node isoparametric elements which, when used as thin shells, allow comparable angles of subtension [3]. In fact, the semiloof element is non-conforming, derived from the Ahmad element [4] which in turn is derived from the solid elements.

The versatility of the more familiar isoparametric elements has been passed on to semiloof, so that sharp corners, curved sides and surfaces, and multiply-connected regions are all allowed. Discontinuities in element thickness are permitted from element to element. The problems of a third in-plane rotation or of slope conformity at corners in shells are avoided. The degrees of freedom are all low-ordered, the only first derivatives being rotations, so that discontinuity incompatibilities inherent in higher order shell elements are avoided. Kirchhoff assumptions of shell theory, that normals to the mid-surface plane remain normal after deformation, are made at the integration points.

The patch tests [5] is passed for plane elements with straight sides, and rigid body motions are satisfied exactly for any combination of elements of any geometry. Extensions of this result to general shapes can be argued.

The elements exist in BERSAFE either as a 6-noded triangle (termed CS24) or an 8-noded quadrilateral (CS32). Numerical integration is used with various Gaussian quadratic-type rules. Stresses are produced at specific Gauss points within each element where they are most accurate.

2. The Semiloof Element

The semiloof element is intended for use in thin shell situations. The element has nodes and degrees of freedom which are intended for ease of use and similar to those of the isoparametric elements. Two versions of the element exist, one being triangular, with 6 nodes and termed CS24 in the BERSAFE system, the other quadrilateral, with 8 nodes, termed CS32. The vertex and midside nodes all have (u, v, w) degrees of freedom in global directions. In addition, local normal rotations $(\frac{\partial W}{\partial n})$ exist at loof nodes along the element sides. These nodes are situated in pairs at strategic positions on either side of each midside node, and from a user point of view the two rotational degrees of freedom are identified with that midside node. Hence, at vertex nodes the degrees of freedom are (u, v, w), whilst at midside nodes they are $(u, v, w, (\frac{\partial W}{\partial n})_1, (\frac{\partial W}{\partial n})_2)$. W here is the deflection along the local normal to the element surface.

Apart from the rotations, degrees of freedom similar to the isoparametric elements exist, with quadratic membrane and bending behaviour. Sides may be curved into quadratic shapes.

At the Gaussian integrating point, discrete Kirchhoff assumptions are made so that normals to the midsurface plane remain approximately normal after deformation. This is simply effected by imposing zero lateral strains, i.e. $\gamma_{xz} = \gamma_{yz} = 0$ at these points. Thus, the Kirchhoff hypothesis holds in the limit as the size of each element tends to zero.

3. Thermal Loading

The types of loading available again resemble those for the isoparametric elements, such as body forces, facial pressures, point forces and moments. Thermal effects can be admitted by specifying temperatures at each node, which produce membrane thermal loads, and temperature gradients acting through the shell thickness at each node, which produce thermal bending loads. As usual in finite element theory, the thermal effects are transmitted into the structure as equivalent nodal loads $\{F\}_o$, and the resulting stresses have to have a thermal stress quantity $\{\sigma\}_o$ subtracted out.

The thermal load vector $$\{F\}_o = \int_V [B]^T [D] \{\varepsilon_o\} \, dV \qquad (1)$$

for each element, where V is the volume of the element and the terminology is standard [6], therefore

$$\{F\}_o = \sum_{\text{Gauss points}} A_i t [B_i]^T [D] \{\varepsilon\}_o \, dV \qquad (2)$$

where A_i is the area associated with Gauss point i, and t is the element averaged thickness. Now

$$[D]\{\varepsilon\}_o = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t^2}{12} & \frac{\nu t^2}{12} & 0 \\ 0 & 0 & 0 & \frac{\nu t^2}{12} & \frac{t^2}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{t^2(1-\nu)}{24} \end{bmatrix} \begin{bmatrix} \alpha Ti \\ \alpha Ti \\ 0 \\ \alpha\delta T_i/t \\ \alpha\delta T_i/t \\ 0 \end{bmatrix} \qquad (3)$$

where Ti and δTi are the Gauss point temperature and temperature difference, respectively. δt_i acts through the thickness t, but in BERSAFE loads are expressed per unit thickness, hence we choose to use $\delta T_i/t$ in the thermal strain vector $\{\varepsilon\}_o$. Hence,

$$[D]\{\varepsilon\}_o = \frac{E\alpha}{1-\nu} \left(T_i \; T_i \; 0 \; \frac{t\delta T_i}{12} \; \frac{t\delta T_i}{12} \; 0\right)^T \qquad (4)$$

and the calculation of $\{F\}_o$ is a straightforward extension to the semiloof stiffness routine.

The thermal stress vector, $\{\sigma\}_o$, is given by

$$\{\sigma\}_o = [D]\{\varepsilon\}_o \qquad (5)$$

For the membrane stresses,

$$\{\sigma\}_o^m = \frac{Et}{1-\nu^2} \begin{bmatrix} 1 & \nu & o \\ \nu & 1 & o \\ o & o & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \alpha T_i \\ \alpha T_i \\ o \end{bmatrix} = \frac{E\alpha T_i}{1-\nu} \begin{bmatrix} 1 \\ 1 \\ o \end{bmatrix} \qquad (6)$$

and so the quantity $\frac{E\alpha T_i}{1-\nu}$ is stored, to be subtracted from the calculated direct membrane stress components in the stress subroutine.

The bending stresses are expressed as the difference in the stress at the surface and the membrane stress (which acts on the reference surface). Hence the thermal bending stress is as above with the actual temperature T_i replaced by the temperature difference between the surface and the reference surface, $\delta T_i/2$. Hence, $\{\sigma\}_o^b = \frac{E\alpha\delta T_i}{2(1-\nu)} \begin{bmatrix} 1 \\ 1 \\ o \end{bmatrix}$

and the quantity $E\alpha\delta T_i/2(1-\nu)$ is stored away for the stress routine.

Simple tests have shown identical performance between semiloof and quadratic solid elements for both types of thermal situation.

4. Numerical Integration Rules

The numerical integration rules for evaluating the stiffness matrices are defined by the BERSAFE input quantity NGAUS. For the triangle, CS24, a special 3-point rule exists, the Gauss points lying at the three midside nodes. Alternatively, NGAUS = 2 and = 3 specifies standard 2 x 2 and 3 x 3 Gaussian quadrature in the element, respectively, with a total of 4 and 9 points in each case. These last two rules also apply to the quadrilateral element. A special rule, NGAUS = 5, with 5 points per element, is also available.

For either element shape, the NGAUS = 3 rule is 'complete' in the sense that the stiffness matrix is correctly integrated for a geometry of simple shape. The NGAUS = 2 rule is theoretically insufficient but often produces very accurate results because excessive stiffness effects are avoided. The 5-point rule for CS32 lies between reduced and complete integration, whilst the special 3 point rule for the triangle appears to be less accurate than either of the 2 x 2 or 3 x 3 rules.

The stiffness matrix for CS32 should ideally have a rank not less than 26, = 32 (the number of degrees of freedom) minus 6 (the allowable rigid body motions). For a triangle, CS24, the rank should not be less than 18. Each integrating point contributes six (the rank of the modulus matrix).

For CS24, therefore, with each of the 3 point special and the 2 x 2 and 3 x 3 rules, the rank obtained is theoretically adequate and no spurious mechanisms exist.

For CS32, if using the 2 x 2 rule, a maximum rank of 24 is available. This is two or more less than the requirement, and so at least 2 spurious mechanisms, or zero energy modes, exist, one representing bending and the other membrane action. The mechanisms can theoretically propagate through neighbouring elements and eventually throughout the structure. One must therefore use reduced integration with care. The higher rules should be used if in doubt since they both contribute sufficient rank to the stiffness matrix.

5. Stresses

The element stresses are calculated at the Gauss points corresponding to the 2 x 2 rule for each element. These points are well known to be where the stresses are at their most accurate. The stresses are produced in local element co-ordinates and as principal stresses. Both membrane and bending components are calculated.

6. Cylinder under Internal Pressure

As an example of the use of different numerical integrating rules in the CS32 element, the stresses and deformations of an internally pressurised cylinder, encastré at one end and open at the other, have been investigated. The length, radius and thickness of the cylinder are 38mm, 9.975mm and 0.05mm respectively, with a pressure of 1000 N/mm^2. The mesh (figure 1) consists of 48 elements, and was designed using previously recommended element grading away from axial discontinuities [3], thereby yielding results within 1% of the analytical solution. Thus, attention could be paid to the effects of other discretizational parameters.

Three such parameters are considered, all being problems which the user may have to encounter in potential shell applications. Firstly, different numerical integration rules for evaluating the element stiffness matrix are compared, as the complete integration rule NGAUS = 3 and the reduced rules NGAUS = 2, or 5. For the case of a cylinder, the mechanisms produced by reduced integration can be removed by moving the circumferential midside nodes, i.e. those nodes at angles 15^o, 45^o, 75^o, 105^o, 135^o, 165^o around the circumference, radially inwards by a very small amount (by .9980564 x radius). This was proposed by Irons and Hellen [7] to remove spurious forces due to the parabolic sides inherent in isoparametric elements. This is the second problem parameter study, entitled 'midside node moving'. The third parameter concerns the precision of the input data. The mesh was generated automatically using the program SAFECT, and initially produced geometry on punched cards, to 4 significant digits. This was thought to be insufficient for an adequate discription of the cylinder, so later a magnetic tape was produced with geometry defined to higher precision (6 or 7 significant digits).

The various computer runs and principal results over the whole structure are shown in Table 1, whilst the displacements plotted around 90^o of the free end are shown in figure 2, on magnified ordinate scales. The results for NGAUS = 5 and NGAUS = 2 without midside node moving are so bad that they do not appear in the plotted range. The card-input form of the NGAUS = 3 run is seen to oscillate quite badly. The other oscillations, e.g. the NGAUS = 2 run with midside node moving, are only of the order of 2% or less and are not therefore important.

The hoop stresses around the end section are given at the 2 x 2 Gauss points and show excellent agreement with theory, except for the NGAUS = 5 runs. All other stress components, including bending stresses, never exceed more than a few per cent of the hoop stress in these cases.

It is concluded, therefore, that the semiloof elements are somewhat sensitive to the precision of the geometry data, so that for curved shell surfaces, high precision geometry must be specified together with NGAUS = 3. When using NGAUS = 2, good results are only obtained if the midside node moving facility is used.

7. Cylindrical Shell Roof

Most problems have sufficiently complicated loading as to deform without inducing mechanism, and one typical example is that of a cylindrical shell roof under self weight (figure 3). This has been used many times for testing shell elements, and is currently used to compare the triangular and quadrilateral semiloof elements with solid 20 node brick and Ahmad thick shell elements.

Because of symmetry, only one quarter of the roof requires a mesh. For each combination of element type and integrating rule, several mesh refinements have been investigated, in each case using an evenly distributed array of elements.

The vertical displacement at the centre of the free edge (point A in figure 3) is compared in figure 4 between the different types of elements as a function of mesh refinement. For this problem, with an 80^{o} sector, the deep shell solution given by Forsberg [8] is considered more accurate than the shallow shell solution of Scordalis and Lo [9]. Additional points include the FS12 element [10], which is a constant moment triangular facet, and the QUAD 4 element [11] which is a low ordered plate element including reduced integration effects. For a given NGAUS value, the CS32 element is seen to be an improvement on the Ahmad element which in turn is better than the brick elements. The CS24 element is less accurate, with its special 3 point rule, but improves with the standard NGAUS = 2 and 3 rules. However, the NGAUS = 2 rule does not show the dramatic improvement as in the quadrilateral elements, where it is extremely accurate. Its special NGAUS = 5 rule is also accurate. The result using FS12 elements is good but required a very fine mesh. The QUAD 4 results are also good too, although not as much as the NGAUS = 2 cases in the (effectively) higher elements.

Hence, with any of the solid brick, Ahmad or semiloof elements, in quadrilateral form, results of good accuracy can be achieved for this sort of problem with meshes of modest refinement provided reduced integration is used. Although not shown, these conclusions apply to displacements and stresses throughout the roof.

8. A Problem Combining Solid and Semiloof Elements

In order to assess the performance of meshes containing both EZ60 (solid isoparametric) and CS32 (shell) elements, the problem of a diving board supported at one end by a solid square based pedestal and loaded at the other was considered. The mesh was such as to contain two EZ60 elements in the pedestal. The board comprised three thin elements, one lying on the top of the pedestal. Two computer runs were performed, one with semiloof elements, the other with EZ60s, as the three board elements (figure 5). The loading was applied along the free edge of the board by a distributed line load P, approportioned 2P/3 at the centre node and P/6 at each corner node (the top line of nodes in the case of EZ60 elements). The bottom eight nodes of the pedestal were restrained from deformation. In the case of the EZ60 board, there were three nodes through the depth of the board at corners and two at midsides, and full compatability with the pedestal is available. For the semiloof board, the nodes lie in the centre plane and again coincide with the top eight nodes of the pedestal for the semiloof element on it. For those eight nodes, there is compatability of displacement degrees of freedom u, v, w, in total 24 degrees of freedom, but the semiloof degrees of freedom $\frac{\partial W}{\partial n}$, totalling eight, are spare. It should generally be sufficient not to constrain these degrees of freedom in any way, but just to leave them free. Then, most of the compatability is taken up by the u, v, w type degrees of freedom. However, in the present example, to prevent a pin-fixed mechanism at the join of the board and the element on the pedestal, the rotational degrees of freedom at the midside nodes A and B were restrained to zero.

The deformation of the board for the two cases is also shown in figure 5. The outline deformation results from using EZ60 elements in the board, whilst the centre line deformation is due to the semiloof board. The displacements are magnified the same amount and negligible deformation of the pedestal is observed. The agreement between the use of the two types of element is seen to be excellent.

The only other result of any significance in this problem is the stress in the upper and lower surface of the board in the longest direction. In the free part of the board, the upper and lower surface stresses are equal and opposite, reflecting the pure bending state, with negligible membrane stress. This stress is shown in figure 6, plotted at Gauss points for semiloof and Gauss points and node points for the solid elements. An extrapolation was required for the latter Gauss points since they are within the thickness of the board. Since no stress variation occurs across the board, the results were taken in suitable planes for Gauss points or the edge face for the nodes. The results are seen to be very consistent, particularly for semiloof. The nodal stresses agree very well except at the free, loaded, end of the board. The solid elements' Gauss point stresses are not as accurate as one would have expected. However, in view of the courseness of the mesh, all results can be considered satisfactory.

Thus, element mixing in a way such as here, with some care exercised over the incompatible degrees of freedom, produces results which are as accurate as using the individual elements on their own. This has useful implication for more complicated structures where shells and solid attachments may have to be considered.

9. Conclusions

The semiloof element, in both its triangular and quadrilateral form, has been described and examples of its use in the BERSAFE system have been presented. The element shows good signs of being unique in the field of shell finite elements for generality of use. Indeed, its resemblance to the powerful family of isoparametric elements not only reinforces this claim, but also renders the element readily insertible into the existing mesh generation and results presentation programs.

The examples demonstrate some interesting properties of the element. The use of suitable numerical integrating rules is important, and the use of reduced integration generally gives results of very high accuracy in structures where bending modes are important. However, the example of the cylinder shows that, in simple membrane modes, the rule breaks down because mechanisms arise. These mechanisms are due to geometric sensitivities which can be removed by a simple artifice. For the triangular element, the special 3-point node is seen to be inferior to the standard Gaussion rules.

The element has been successfully mixed with solid brick elements in the simple case of a diving board.

Acknowledgement

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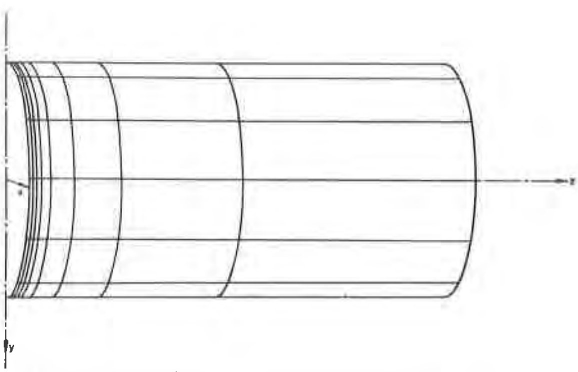

FIG.1 Mesh Used for Cylinder Under Internal Pressure with CS32 or EZ60 Elements

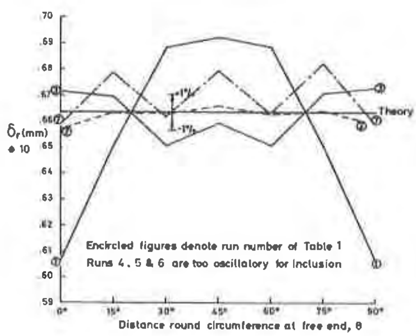

FIG.2. Radial Displacement Round Free End of Cylinder for CS 32 Elements

TABLE 1

Various Runs for Cylinder Under Internal Pressure

Run	NGAUS Rule	No. of points per element	Midside Node Moving Used?	Input Geom on Cards(C) or Tape (T)	Quality of Results		
					δ_r axially	δ_r round end section	σ_h round end section
1	3	9	No	C	S	S	E
2	3	9	No	T	E	E	E
3	3	9	Yes	T	O	G	E
4	5	5	No	T	O	O	O
5	5	5	Yes	T	O	O	O
6	2	4	No	T	O	O	E
7	2	4	Yes	T	G	G	E

Symbols

E Excellent results (within 1% of theory)

G good results (within 1-5% of theory)

S Satisfactory (within engineering tolerances)

O oscillatory, showing mechanism dominance.

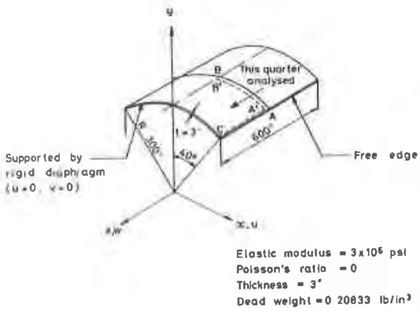

FIG. 3. Self-Loaded Circular Cylinder on Diaphragm.

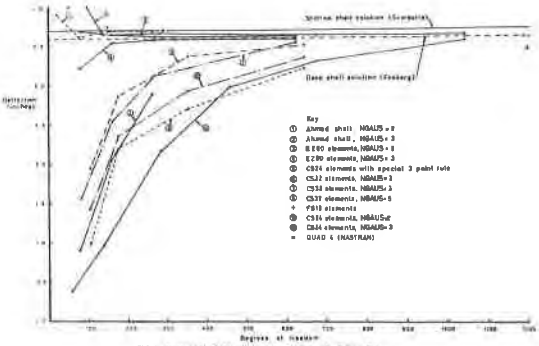

FIG.4. Vertical Deflection at Centre of Free Edge.

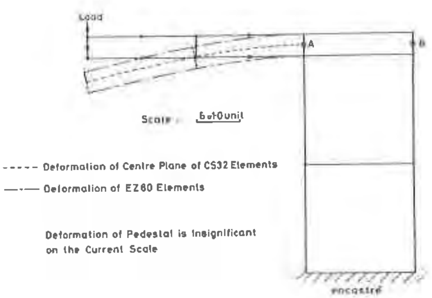

FIG.5. Deformation of Board for Both Semiloof and Solid Elements.

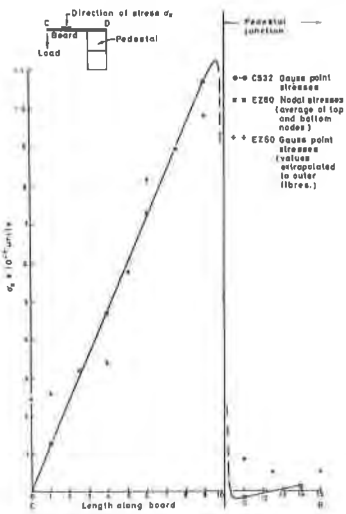

FIG.6. Bending Stress σ_x Along Top of Board.