Elasto-plastic response of multi-story shear wall structures

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1 INTRODUCTION

The nonlinear response of structures during strong motion earthquakes is one of the major problems to be carefully addressed in structural engineering. Especially for important structures such as nuclear power plant buildings, it is essential to estimate the inelastic behavior accurately as much as possible. However, the inelastic analyses and associated sensitivity studies require a large amount of computational effort. Hence, the nonlinear behavior is often estimated on the basis of some approximate relationship between the nonlinear response and linear response. For example, in the current Probabilistic Risk Assessment (PRA) procedure [1,2], the effect of the elasto-plastic response is considered using an energy absorption factor which is constructed from the relationship between the ductility factor and the maximum linear response. However, the validity of this procedure is limited, since the energy absorption factor originally developed for single-degree-of-freedom systems [3,4] is also used for estimating the elasto-plastic response of multi-degree-of-freedom (multi-DOF) systems, which may have so-called damage concentration due to the imbalance of the mass and stiffness distributions.

In this paper, a Monte Carlo simulation study is carried out in order to address to this question. The relationship between the elasto-plastic and linear response for multi-DOF systems is developed based on the results of the simulation study. Several 6-story shear wall structures are considered as structural models which represent typical nuclear power plant buildings. A bilinear force-displacement relationship is assumed for each story. A number of artificial earthquakes based on the Kanai-Tajimi power spectrum and a trapezoidal envelope function are used as the input ground motion. The least square method is introduced for the purpose of evaluating the median relationship between the ductility factor and linear response from the simulated data and also evaluating the deviation from this median relationship. This relationship derived for the 6-story buildings is compared with the currently used energy absorption factor, $\sqrt{2\mu-1}$, and the simulation results for Zion auxiliary building model.

2 METHOD OF ANALYSIS

2.1 Inelastic response analysis

The bilinear force-displacement relationship shown in Fig.1 is assumed

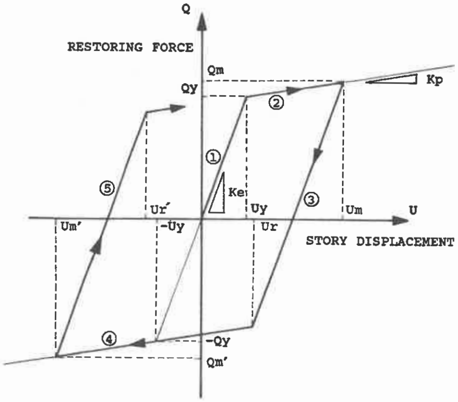

Figure 1. Bilinear force vs. displacement relationship

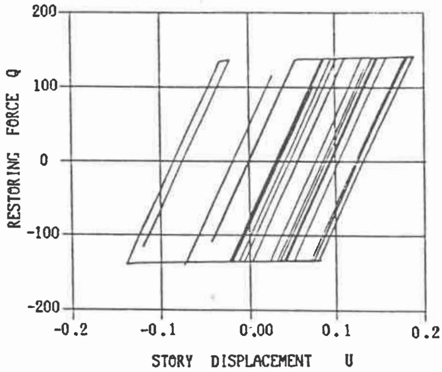

Figure 2. Sample bilinear hysteresis by simulation

as an inelastic behavior of each story. The shear beam model is composed of these bilinear shear springs and lumped masses. The equation of motion for this system is written as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{Q} = -\mathbf{M}\mathbf{r}\ddot{z} \quad \text{................. (1)}$$

in which **M** is the diagonal mass matrix, **C** is the Rayleigh damping matrix calculated from the first two natural frequencies of the linear system, **Q** is the restoring force vector, **r** is a vector whose components are all unity and $\ddot{z}$ is the base ground acceleration. The central difference method [5] is used for the time integration of Eq.1. When the overshooting of the yield restoring force occurs, $\ddot{\mathbf{X}}$ is modified such that the dynamic equilibrium by Eq.1 is satisfied. A sample bilinear relationship obtained by the response analysis is depicted in Fig.2.

2.2 Ground motion

The ground acceleration, $\ddot{z}(t)$, is generated as the product of a Gaussian process, $g(t)$, and a deterministic envelope function, $f(t)$, as follows:

$$\ddot{z}(t) = g(t) \cdot f(t) \quad \text{................. (2)}$$

The envelope function is assumed to have a trapezoidal shape with the total duration 15 *sec* including the rise time 2.5 *sec* and decay time 2.5 *sec*. The well-known Kanai-Tajimi spectrum is assumed as the power spectrum of $g(t)$ as

$$S(\omega) = S_0 \frac{1 + 4\zeta_g^2 (\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g^2 (\omega/\omega_g)^2} \quad \text{............... (3)}$$

where ω_g is the characteristic ground frequency and ζ_g and S_0 are constants related to the shape and intensity of the spectrum. The time series, $g(t)$, is generated by the following form:

$$g(t) = \sqrt{2} \sum_{k=1}^{N} \sqrt{G(\omega_k)\,\Delta\omega}\ \cos(\omega_k t + \phi_k) \quad \text{................. (4)}$$

with $\omega_k = k\Delta\omega$, $G(\omega_k) = 2S(\omega_k)$, N = the number of equally spaced frequencies to synthesize and ϕ's = random phase angles uniformly distributed between 0 and 2π. The different time series are generated by selecting different sets of the random phase angles.

In this study, the following values representing a typical hard rock ground condition are used to generate the ground motion: S_0 = 1.0, 2.5, 6.25 ft^2/sec^3 ; $\omega_g = 8\pi$ rad/sec ; ζ_g = 0.6.

2.3 Relationship between ductility factor and linear response factor

The ductility factor is defined by

$$\mu = U_b / U_y \qquad \text{(5)}$$

in which U_b is the maximum story displacement obtained by the bilinear response analysis and U_y is the story displacement at the yield point. The linear response factor is also defined by

$$m = U_e / U_y \qquad \text{(6)}$$

where U_e is the maximum story displacement which is obtained by the linear response analysis being carried out under the same condition as that for the corresponding bilinear response analysis.

In order to estimate the median relationship between the ductility factor μ and the linear response factor m, the following equation is assumed.

$$\breve{m} = (p\mu - p + 1)^r \qquad \text{(7)}$$

where $\breve{m}$ is the median of the linear response factor for each μ value and p and r are coefficients to be determined from the simulated data. Taking the logarithm of Eq.7 and linearizing using Taylor series expansion, one obtains

$$ln\ \breve{m} = f(p, r) \approx f(p_0, r_0) + (p - p_0) f_p(p_0, r_0) + (r - r_0) f_r(p_0, r_0) \qquad \text{(8)}$$

in which $f_p(p_0, r_0) = \partial f(p, r)/\partial p$ and $f_r(p_0, r_0) = \partial f(p, r)/\partial r$ evaluated at $p = p_0$ and $r = r_0$. The estimated values of the coefficients, p and r, are chosen to minimize the following expression:

$$D = \sum_{i=1}^{n} (ln\ m_i - ln\ \breve{m})^2 \qquad \text{(9)}$$

in which n is the number of data points (the sample size of Monte Carlo simulation). From the assumed initial values p_0 and r_0, p and r are obtained by the least square method. Replacing p_0 and r_0 by p and r, this step is iterated until p and r converge.

Assuming m to be log-normally distributed for each value of μ, an empirical relationship for the square of the deviation from the above median relationship, Δd, is also introduced as

$$(\Delta d)^2 = (ln\ m_i - ln\ \breve{m})^2 = s(\mu - 1)^t \qquad \text{(10)}$$

In the same manner as for the median relationship, the parameters, s and t, are estimated by the least square method with a weighting function $w_i = 1/\mu_i$.

It is noted that in the above equation, the deviation of m from its median μ-m relationship can also be expressed as a function of μ as well as this median relationship.

Table 1. Model structures

Mass ($k \cdot s^2/ft$)	Elastic Shear Stiffness (100,000 k/ft) ◇ #1	⊡ #2	⊙ #3	× #4	+ #5	✱ #6	✱ #7	Strain Hardening Ratio	Yeild Story Disp. (ft)
111.8									
	2.293	2.133	2.920	3.680	2.958	3.158	2.646	0.04	0.054
111.8									
	5.967	4.728	5.444	3.055	5.397	6.743	4.850	0.04	0.054
111.8									
	5.852	8.058	6.591	6.251	7.193	6.571	6.614	0.04	0.054
111.8									
	8.669	7.041	5.847	9.303	8.804	5.306	7.937	0.04	0.054
111.8									
	9.252	7.916	9.632	9.627	5.722	11.74	8.819	0.04	0.054
111.8									
	8.898	9.614	8.037	10.07	12.88	12.90	9.260	0.04	0.054
Natural Period (s)	0.315	0.321	0.327	0.316	0.317	0.306	0.316		
Damping Ratio	0.05								

3 NUMERICAL EXAMPLE AND DISCUSSION

3.1 Model structures and Monte Carlo simulation

Six-story shear wall building models are used for constructing the relationship between the bilinear and linear responses. Seven buildings with different shear stiffness distributions are modeled into lumped mass and shear spring systems as shown in Table 1.

The relationship between the bilinear response and linear response is constructed with the aid of the Monte Carlo simulation technique. The phase angles in the input motion are considered as the random parameters. 45 artificial earthquakes (each of three sets of 15 earthquakes have the same S_0 value as: 1.0, 2.5 and 6.25 ft^2/sec^3) are applied to each structural model in order to evaluate the effect of randomness in the wave shape on the bilinear response. All the parameters of the structural models are assumed to be deterministic at this time.

The Zion auxiliary building (7-story) model [6] is also analyzed as a typical existing structure. In this case, a similar analysis is carried out by generating 75 artificial earthquakes.

3.2 μ-m relationship for system

The distribution of story ductility factors obtained by the bilinear response analysis for the model structures #1, #4 and #7 are shown in Fig.3. A strong damage concentration is observed at the fifth-story of the model #4. The same tendency is also seen in the diagrams for the models #5 and #6, although they are not shown here. The model #7 is a well-balanced structure, thus its damage concentration is not so large. The models #1, #2 and #3 are in between these two kinds of the ductility

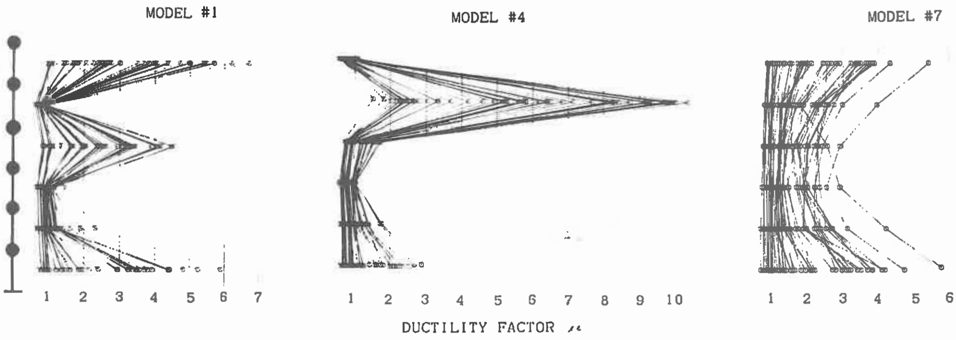

Figure 3. Distribution of story ductility factors

distribution. Anyhow, each model exhibits the damage concentration to some extent.

It is obviously difficult to construct the general μ-m relationship valid for all the stories. However, the largest ductility factor among all the story ductilities may be most significant when evaluating the safety of structures. Therefore, in this study, this largest story ductility factor is defined as the system ductility factor. The system linear response factor is also determined as the corresponding story's m and it is, in most cases (more than 99% in our examples), the largest linear response factor among all the stories. This fact implies that the distribution of the story linear response factor give us very useful information about the damage concentration.

The system μ-m relationships for all these seven model structures are summarized in Fig.4. No clear difference is observed among the data points of these different structures although the degree of the damage concentration are quite different as shown in Fig.3. Thus, the system μ-m relationship may be used as a unique tool for the safety evaluation of a certain class of structures.

3.3 Statistical modeling of μ-m relationship

The system μ-m relationship is constructed with the aid of the least square method using the simulated data points of the ductility factor between 1 and 10. The assumed form of the median relationship, $\breve{m}=(p\mu-p+1)^r$, appears quite reasonable as shown by the solid line in Fig.4. The assumption that m can be represented by the log-normal distribution for each value of μ also looks appropriate. It is confirmed that the range of the deviation Δd from the median relationship should be treated as a function of μ.

The results of Monte Carlo simulation for the Zion auxiliary building model are plotted in Fig.5 along with the currently used conversion factor $\breve{m}=\sqrt{2\mu-1}$ in the dashed line and the median relationship for the above 6-DOF models in the straight line. This median relationship fits the Zion model's results better than $\sqrt{2\mu-1}$. Thus, $\sqrt{2\mu-1}$ is found to be not always a conservative estimation when evaluating the acceleration capacity of structures. Monte Carlo technique appear to provide more reasonable approximation for the relationship. The values of p, r, s and t are indicated in Fig.4.

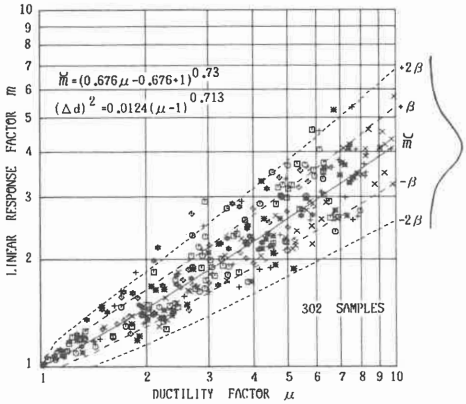

Figure 4. System ductility factor for 6-DOF models

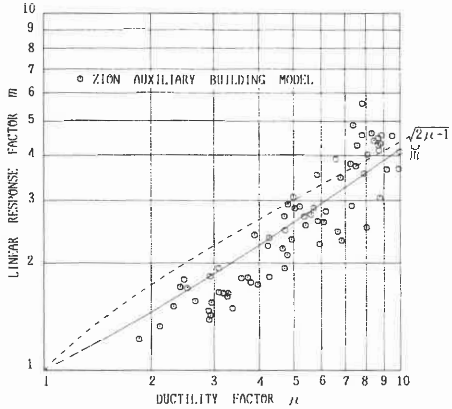

Figure 5. System ductility factor for Zion model

4 CONCLUSION AND ACKNOWLEDGMENT

A Monte Carlo simulation study is carried out for 6-DOF shear wall structures as typical examples of multi-DOF systems in order to furnish the data base for constructing a relationship between the elasto-plastic and linear responses. The results are summarized as follows:

1) The maximum story displacements of elasto-plastic shear building systems may be estimated on the basis of the linear responses and ductility factors. Also, their variations can be taken into consideration properly.
2) The unique relationship between the system ductility factor and linear response is derived for seven different shear building models when the wave shape of the input motion is a random parameter.
3) The above relationship fits the simulated results of another shear wall type model better than the energy absorption factor, $\sqrt{2\mu-1}$, currently used in the PRA procedure. However, further study is suggested for more general conclusions.

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