An Efficient Axisymmetric Shell Finite Element for the Computation of the Vibrations of Fluid-Structure Systems

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Summary

It is proposed to investigate the finite element analysis of the harmonic vibrations of coupled fluid-structure axisymmetric systems by using a new efficient and economical shell element. The main advantages are the following : the problem of spurious displacement field on the axis of revolution is no more encountered, the added-mass matrix is of lower dimension than usual because it does not involve rotation degrees of freedom. The methodology utilizes a local mixed variational formulation. Numerical and experimental results are presented. The results show the accuracy of the method.

1. Introduction

It is proposed to investigate the finite element analysis of the harmonic vibrations of coupled fluid-structure systems of revolution by using a new efficient and economical shell element.

Using mixed variational formulations through Lagrange multipliers concepts, see R. Valid [1,2], various symmetric variational formulations for bounded coupled fluid-structure systems have been established by R. Valid, R. Ohayon, H. Morand, H. Berger [3,4,5,6], leading to generalized added mass concepts.

In the case of thin shells of revolution, use has been made of a standard conical element based on classical conical shell theory [1]. The element has two nodes whith four degrees of freedom at each node (three displacements and one rotation).

In the hydroelastic finite element analysis, the fluid-structure interaction involves a coupling term between the displacement potential of the fluid φ and the normal displacement w of the shell. Therefore, in order to remain consistent with the conical shell stiffness matrix, a cubic interpolation is used for w and implies rotation type degrees of freedom. The use of added mass matrices (which corresponds to an elimination of the degrees of freedom of the displacement potential φ) leads to full matrices.

In order to increase the sparse properties of the fluid-structure matrix system, especially concerning the wetted surface coupling term, or, which is equivalent, in order to reduce the size of the full added mass matrix, a variational formulation which involves independant rotation and displacement fields will be prefered for the shell. Therefore, w can be linearly interpolated (instead of cubically). The added mass hydroelastic matrix is of lower dimension than usual because it does not involve any degree of freedom of rotation type.

The idea of introducing independent rotation and displacement field for thin beams, plates and shells (under Kirchhoff-Love assumptions) has already been used in static cases. Reduced integration techniques appear in references [7,8,9] by O.C. Zienkiewiez, T.J.R. Hughes, R.L. Taylor, W. Kanoknukulchai, J. Bauer, K. Morgan and E. Onate. For axisymmetric and non axisymmetric vibrations of thin shells of revolution, it appears that the reduced integration technique cannot be justified and can lead to poor approximations if this technique is carried out for all terms, instead of a selective one.

Even in that last case and for vibration problems, poor results were found. Instead of using reduced integration technics, mixed finite element procedures have been tentatively used.

In some cases, there is an equivalence between mixed methods and reduced integration technics ; let us mention the works of T.J.R. Hughes, D.S. Malkus [10] and M. Bercovier [11].

The method introduces a dual variable, namely the shear stress in the meridian plane, through a regularization technique. This variable is never assembled from one finite element to the next, following general mathematical concepts introduced by P.A. Raviart [12] and used in [10,11] and by P. Gibert in [13]. The elimination of the dual variable leads to an indirect displacement - rotation variational formulation through a penalty factor ε .

For axisymmetric vibrations, the physical meaning of the procedure corresponds to the introduction of a shear effect. Such effects are not introduced for non axisymmetric motions in directions orthogonal to the meridian plane as we shall see later. Let us mention a promising idea presented by P. Destuynder [14], starting from the same concepts, but leading to a formulation which does not contain the penalty factor ε , but which assembles the dual variables.

In a first section, we shall recall briefly the classical direct displacement procedure. In a second section, we present the method leading to a so-called indirect displacement-rotation formulation.

In a third section, numerical results are presented, compared with the results provided by usual methods previously employed. We shall see that the problem of spurious displacement fields on the axis of revolution is no more encountered. Let us recall that this problem occurs in the analysis of free vibrations of liquid filled tanks for the computation of the hydroelastic modes of liquid propelled launchers.

Let us also mention that the rotation degrees of freedom can rigorously be eliminated from the final matrix system. Therefore, when comparing numerical results and experimental data, the identification procedure which nowadays worked out to improve computanal models through experimental results is much more efficient since the rotations are usually not measured.

2. Direct displacement shell formulation

Let us first briefly recall the primal displacement principle for the vibrations of linear hyperelastic thin shells under Kirchhoff-Love hypothesis [1,2]. Let us call by Σ the middle surface of the shell of tangent plane $\vec{E}_{\varphi_0}$ in the undeformed state, by α the strain energy surface density function of the displacement field V through the surface membrane and flexural deformations Γ and K, by A the constitutive anisotropic elastic operator, and Tr the trace symbol operator.

The primal principle can be written :

Find $\omega \in \mathbb{R}^+$ (ω, circular frequency) and $V \in \mathcal{C}$, $\mathcal{C}$ being the admissible kinematic displacement field space, such that :

$$\delta\left[\frac{1}{2}\int_\Sigma \widetilde{D(V)}\, A\, D(V) - \omega^2\,\frac{1}{2}\int_\Sigma \rho\,|V|^2\right] = 0 \quad , \forall V \in \mathcal{C} \tag{1}$$

where δ denotes the virtual variation symbol, and where the scalar product symbol $\sim$ is defined by :

$$\widetilde{\Gamma_1}\,\Gamma_2 = Tr\left[\Gamma_1 \Gamma_2\right] \quad , \quad \forall \Gamma_1, \Gamma_2 \in \mathcal{L}\left(\vec{E}_{\varphi_0}, \vec{E}_{\varphi_0}\right)$$

$$D = \begin{bmatrix} \Gamma \\ K \end{bmatrix} \quad , \quad A = \begin{bmatrix} {}^1A_1 & {}^1A_2 \\ {}^1\widetilde{A_2} & {}^2A_2 \end{bmatrix} \tag{2}$$

In the case of conical axisymmetric shell element (fig. 1) , if n denotes the classical circumferential Fourier number ($\cos n\theta$ for u and w, $\sin n\theta$ for v normal to the meridian $(z - r)$ plane) , Γ and K are written as follows [1,2] :

$$\Gamma \begin{cases} \gamma_s = \dfrac{\partial u}{\partial s}\cos n\theta \\ \gamma_\theta = -\dfrac{v\,n\cos n\theta}{r} + \dfrac{\cos n\theta}{r}\left(w\cos\varphi + u\sin\varphi\right) \\ \gamma_{s\theta} = \sin n\theta\left(-\dfrac{nu}{r} + \dfrac{\partial v}{\partial s} - \dfrac{v\sin\varphi}{r}\right) \end{cases} \tag{3}$$

$$K \begin{cases} K_s = -\dfrac{\partial^2 w}{\partial s^2}\cos n\theta \\ K_\theta = \cos n\theta\left(\dfrac{n^2}{r^2}w + \dfrac{n\cos\varphi}{r^2}v - \dfrac{\sin\varphi}{r}\dfrac{\partial w}{\partial s}\right) \\ K_{s\theta} = 2\sin n\theta\left(\dfrac{n}{r}\dfrac{\partial w}{\partial s} - \dfrac{n\sin\varphi}{r^2}w + \dfrac{\cos\varphi}{r}\dfrac{\partial v}{\partial s} - \dfrac{\sin\varphi\cos\varphi}{r^2}v\right) \end{cases}$$

Remark

It should be stressed that, due to the Kirchhoff-Love assumptions, K involves second order partial derivatives of w , usually : $\dfrac{\partial^2 w}{\partial s^2}$, $\dfrac{\partial^2 w}{\partial \theta^2}$, $\dfrac{\partial^2 w}{\partial s \partial \theta}$; but for axisymmetric shells, only $\dfrac{\partial^2 w}{\partial s^2}$ remains. This remark is of primary important for the presentation of the method.

3. Indirect displacement - rotation variational formulation

The displacement formulation (eq. 1) together with eq. (3) leads to the standard two nodes finite element with a linear interpolation for u and v , and cubic interpolation for w , as mentioned in the introduction.

In order to introduce independant displacement and rotation fields, a penalty function procedure can be applied to the Kirchhoff-Love equation :

$$\beta - \frac{dw}{ds} = 0 \tag{4}$$

Let us recall that the physical meaning of eq. (4) is one of the Kirchhoff-Love hypothesis by which no distinction is made between the geometric and material normals.

For static cases and for axisymmetric motions of the shell, the penalization of eq. (4) together with reduced integration technique have been already used

We shall introduce a so-called indirect displacement - rotation variational formulation by using a regularization procedure which introduces the shear stress - shear strain constitutive equation as follows :

$$\begin{cases} \beta - \dfrac{dw}{ds} = \varepsilon \lambda \\ \varepsilon > 0 \qquad \dfrac{1}{\varepsilon} \quad \left(\dfrac{1}{\varepsilon} \text{ will be a penalty factor}\right) \end{cases} \tag{5}$$

Using Lagrangian multiplier techniques [1,2], one can obtain a mixed variational principle if, starting from eq. (1) expressed in terms of (u,v,w, β), one adds to the result the following energy contributions :

$$\int_\Sigma \left(\beta - \frac{dw}{ds}\right) \lambda \, d\Sigma - \frac{\varepsilon}{2} \int_\Sigma \lambda^2 d\Sigma \tag{6}$$

The multiplier λ does not need to be assembled (as $\lambda \in L^2(\Sigma)$) : we choose λ = constant per element.

The fact that λ is not an "assembled" type variable leads to a mixed variational formulation of local type, i.e. the dual field can be eliminated, after discretization, as a function of β and w inside each element.

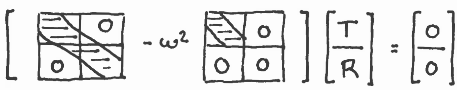
We can show that, if λ is chosen constant, the result corresponds to the following penalization term :

$$\frac{1}{\varepsilon} \sum_{\text{elements}} \frac{1}{\text{meas}(\Sigma)} \left[\int_{\Sigma_{\text{element}}} \left(\beta - \frac{dw}{ds}\right) d\Sigma \right]^2 \tag{7}$$

Of course, one can obtain more complicated formula by chosing higher order interpolation for λ (non assembled) and by applying a Gauss elimination inside each element.

In any case, the discretized matrix system is only expressed in terms of displacement and rotation as it is the case for the elemen- matrices.

As β is not involved in the expression of the kinetic energy of eq. (1), the matrix eigenvalue can be written as follows (the stiffness matrix depending upon ε) :



(8)

T denotes the nodal displacement degrees of freedom, and R denotes rotation type degrees of freedom. Eq. (8) shows that R can be also rigorously eliminated as expected.

4. Numerical results

4.1 Vibrations of an elastic sphere

When calculating the vibrations modes of a sphere represented by cylindrical coordinates, some group theory considerations lead to the following result regarding the eigenfrequencies of the system : The frequencies corresponding to a circumferencial number n are contained into the corresponding $(n-1)$ circumferencial number. Table 1 shows the accuracy of the element.

Figure 2 shows the mode shape corresponding to the new element and to the classicle one : the spurious deformation at the axis of revolution has disappeared.

4.2 Composite material cylinder immersed in a liquid filled tank with a free surface (fig. 3)

Hydroelastic computations have been carried by using the methods developped in references [3,4,5,6] and the research code [15] developped from the algorithms of a modular efficient code [16]. Table 3 shows the comparison between experimental and computed results for the frequencies corresponding to the value n = 4,5,6,7.

4.3 Third stage of the liquid propelled launch vehicle Ariane - (fig. 4)

5 Conclusion

We have presented a new finite element for shell of revolution which is particularly efficient for the study of fluid structure vibration analysis. One of the main feature of the methodology is that, through the concept of locally mixed type variational formulation, the elementary matrix (of displacement - rotation type) has the same usual size as the classical one, with a high degree of accuracy. An extension of these concepts to transient analysis of fluid - structure interaction is an open path under development.

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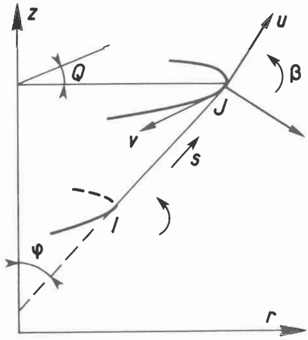

Fig. 1 - Axisymmetric shell definition .

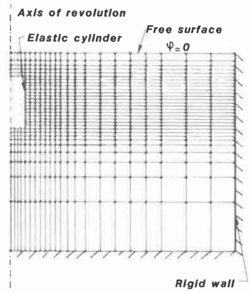

Fig. 3 - Fluid - structure mesh.

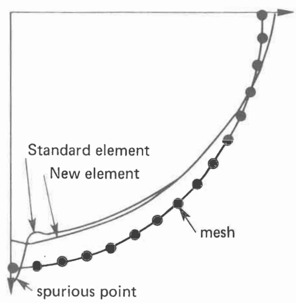

Fig. 2 - Mode shape of a spherical shell.

n = 0 f_0 Hz	n = 1 f_1 Hz	$\frac{f_0 - f_1}{f_0}$	n = 2 f_2 Hz	$\frac{f_2 - f_1}{f_1}$	n = 3 f_3 Hz	$\frac{f_3 - f_2}{f_1}$	n = 4 f_4 Hz	$\frac{f_4 - f_3}{f_3}$	$\frac{f_4 - f_0}{f_0}$
46.86 s	46.83 a	0.6 ‰	46.79 s	0.8 ‰					
62.67 a	62.55 s	1.9 ‰	62.42 a	2.1 ‰	62.24 s	2.9 ‰			
75.26 s	75.00 a	3.4 ‰	74.76 s	3.2 ‰	74.61 a	2.0 ‰	74.10 s	6.5 ‰	15.4 ‰
86.61 a	86.22 s	4.5 ‰	85.84 a	4.4 ‰	85.42 s	4.9 ‰	84.94 a	5.6 ‰	19.3 ‰
97.42 s	96.89 s	5.4 ‰	96.39 a	5.2 ‰	95.83 a	5.8 ‰	95 23 s	6.3 ‰	22.5 ‰
107.98 a	107.34 s	5.9 ‰	106.72 a	5.8 ‰	106.03 s	6.5 ‰	105.30 a	6.9 ‰	24.8 ‰
118.46 s	117.70 a	6.4 ‰	116.99 s	6.0 ‰	116.18 a	6.9 ‰	115.34 s	7.2 ‰	26.3 ‰
128.92 a	128.08 s	6.5 ‰	127.28 a	6.2 ‰	126.37 s	7.1 ‰	125.42 a	7.5 ‰	27.1 ‰
139.41 s	138.64 s	6.2 ‰	137.64 s	6.2 ‰	136.64 a	7.3 ‰	135.60 s	7.6 ‰	27.3 ‰
149.96 a	149.00 s	6.4 ‰	148.90 a	61 ‰	147.01 s	7.3 ‰	145.90 a	7.5 ‰	27.1 ‰

Table 1. Frequencies of a spherical elastic shell (n : circumferencial number) -

s : symmetric modes

a : antisymmetric modes

COMPARAISON CALCUL–EXPERIENCE

Module en composite dans bassin d'essai

n	f_{exp} Hz	f_{comp} Hz	ϵ %
4	42,4	43,7	3 %
5	59,3	60,57	2 %
6	133,5	140,73	5 %
7	128,0	130,49	1,9 %
8	177,9	168,72	3,2 %
8	206,7	212,51	2,8 %

Table 2.- Comparison between theoretical and experimental frequencies -

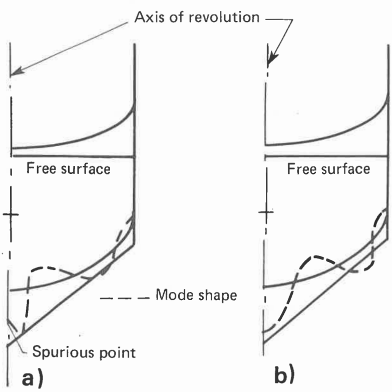

Fig. 4 - 6th. eigenvector of the lower tank of third stage of liquid-propelled launcher.

4a - Standard element

4b - New element