A PROBABILISTIC APPROACH TO DETERMINE THE INTEGRITY OF MULTI-COMPONENT REACTOR STRUCTURES USING LIMITED TEST DATA

T. Y. STOKOE

Central Electricity Generating Board (SER), Scientific Services Department, Gravesend Laboratory, Suffolk Road, Gravesend, Kent DA12 2SW, United Kingdom

SUMMARY

A system of high reliability, such as an internal reactor structure of high redundancy which is in a corrosive environment, may contain some failed components. This leads to uncertainties in the integrity of such a structure when subjected to fault loading conditions. Since the steels from which in-reactor structures are fabricated are defined by their mechanical properties, the failure probability of components with the same nominal specification and under the same conditions can vary appreciably. Also corrosion rates can be very sensitive to small differences in temperature which may not be known precisely. Therefore unless all the components of a structure are inspected some uncertainty in the component failure probability will exist. It is likely however that a thorough inspection procedure is hindered by the inaccessibility of components and also the large number of components involved. A general method for determining the overall failure probability of such a structure, when limited test data is available, will be presented.

Many factors affect the failure probability of a multi-component structure which contains some failed components. Initially the stresses which arise in a structure under fault loading conditions must be determined. The magnitude of these stresses will indicate the degree of redundancy in the system and thereby its likely ability to withstand the loading. A consideration which could have some influence is whether the structure is subject to continuous loading or the sudden application of a load. Typically the core restraint structure of a gas cooled graphite moderated reactor contains a few thousand bolts. It is likely that any sample population will be a large fraction of the total bolt population. Therefore any predicted bolt failure probability distribution must take into account the finite size of the bolt population. Using the theorem of Inverse Probability the overall failure probability for such a structure will be determined, where all the above factors are taken into account.

Data may be available on the failure probability of bolts of the same specification which have experienced similar corrosion conditions. This information could be obtained either from experiment or in-reactor inspection. It will be shown that this information can be combined with information from a test result, thereby reducing the size of the test sample required to demonstrate the desired degree of structural integrity.

The preceding theory will be applied to a typical example which could arise in estimating the integrity of the core restraint structure of a gas cooled reactor.

1. Introduction

Reactor Systems are often required to have high reliability and this can be achieved by designing the system with adequate redundancy. Such a system in a corrosive environment could contain some failed components, but as it has high redundancy certain numbers and configurations of failures would be permissible before the system as a whole could be said to have failed. Information regarding the reliability of the system can be obtained by a process of examination and testing, but it may be that a test result is only available for some of the constituent components, possibly due to the large numbers involved, or even the inaccessibility of some components. Information may also be available from other sources, such as test data derived from similar components, although difficulties can often arise in the interpretation of the data. This report will describe a method for determining the overall failure probability of a high reliability system by the application of sampling theory to a limited amount of test data obtained directly from the structure. A method for including additional data from similar components in other structures will also be given.

2. Estimation of Failure Probability from Direct Inspection

The two factors which determine the overall failure probability of a system are, firstly the chance of a particular number of failed components occurring, and secondly the chance that such a number of failed components can result in failure of the whole system. Direct inspection of some of the components of a system gives information about the chance of a particular number of failed components occurring. Consider a total population of N components, of which r may be failed, from which a sample ℓ, containing m failed and n unfailed components, is selected. It is straightforward to show that the likelihood of there being r failures in the total population, assuming the sample is random, can be expressed in terms of binomial coefficients by the hypergeometric distribution, when it is assumed that all arrangements are equally likely, so that

$$L_r = \binom{r}{m}\binom{N-r}{n} \Bigg/ \sum_{r=m}^{N-n} \binom{r}{m}\binom{N-r}{n} \quad . \tag{1}$$

The failure probability of a system with a particular number of failed components will depend on the manner in which failed components can be arranged to produce failure of the system. Knowledge concerning the specific location of failed and unfailed components derived from a test result will have some influence on the overall failure probability. Every system will in general have a different function describing its failure probability but initially a typical problem will be considered.

For example if a structure consists of N individual components which are grouped into g assemblies, each with b components, the failure of this system could be defined such that when all b components of a single assembly fail, the structure as a whole would fail. Consider the case of r randomly distributed failed components. The structure cannot fail if $r<b$ and will certainly fail if $r>(b-1)g$. The r failed components can be arranged amongst the N locations in $N!/(r!(N-r)!)$ ways. If one of the g assemblies fails the number of different ways of arranging the remaining components is $(N-b)!/((r-b)!\,(N-r)!)$.

For all the different arrangements the total number of failed assemblies which can occur is

$$g \binom{N-b}{r-b} \qquad (2)$$

Interpretation of this expression in terms of arrangements involves counting multiple failures more than once. Including correction terms and dividing by the total number of arrangements, the fraction of arrangements for which failure of the system occurs is

$$f_r = \frac{r!}{N!} \sum_{i=1}^{i \leq r/b} \frac{(-1)^{i-1} \, g!}{i! \, (g-i)!} \, \frac{(N-ib)!}{(r-ib)!} \; . \qquad (3)$$

This expression would be applicable to a system which may fail under the sudden application of a load. For any system as more failures occur the likelihood of failure of the system as a whole increases. But for a system which undergoes continuous loading certain arrangements are precluded from producing failure because the system would have failed previously. In the present example failure under continuous loading could only occur through the failure of one assembly. It can be deduced that the fraction of arrangements in which only one assembly fails is

$$f_r = \frac{r!}{N!} \sum_{i=1}^{i \leq r/b} \frac{(-1)^{i-1} \, g!}{(i-1)!\,(g-i)!} \, \frac{(N-ib)!}{(r-ib)!} \; . \qquad (4)$$

where the number of multiple failure arrangements have not been subtracted from the total number of possible arrangements since there are so few.

If a system is now considered for which failure can occur when the number of failed components in an assembly is c (<b) then it can be shown that

$$f_r = g \frac{r!}{N!} \sum_{x=c}^{b} \frac{b! \, (N-b)! \, (N-r)!}{x! \, (b-x)!\,(r-x)! \, (N-r-b+x)!} \; . \qquad (5)$$

This equation only includes the leading terms because for a system of high reliability the terms arising from multiple failure are likely to be small.

In general, for a system of high reliability, the fraction of possible arrangements of failures, which could cause failure of the system can be written as

$$f_r = \sum_{j,k} C_{jk}(N) \binom{N-j-k}{r-j} \Big/ \binom{N}{r} \; . \qquad (6)$$

j and k are the number of failed and unfailed components respectively for a particular mode of failure and C_{jk} (N) is the number of ways the mode of failure can occur noting that double counting must be avoided.

Equation (1) gives the likelihood of r failures occurring amongst the total population N, assuming ℓ components are sampled of which m are failed and n unfailed. Consequently r can take values from m to N-n, and f_r gives the chance of failure of the system should r failures occur. Therefore, using Bayes Theorem of Inverse Probability the overall failure probability for the structure is

$$P = \sum_{r=m}^{N-n} f_r' \; L_r \tag{7}$$

where f_r' is the fraction of arrangements in the uninspected sample for which failure of the system can occur and can be obtained from equation (6) by changing $r \rightarrow r-m$, $N \rightarrow N-\ell$ and $C_{jk}(N)$ accordingly. For a system of high reliability where r/N is expected to be small only the early terms in the summations of eq.(7) will be significant, so any assumed prior distribution is taken to be slowly varying in this region. Making use of the equation

$$\sum_{r=m}^{N-n} \binom{r}{m} \binom{N-r}{n} = \binom{N+1}{\ell+1} \tag{8}$$

the proof of which is given in ref. [1] equation (7) can be expressed as

$$P = \sum_{j,k} C_{jk}(N,\ell) \; T_{jk}^{mn} \tag{9}$$

where the matrix

$$T_{jk}^{mn} = \frac{(m+n+1)!}{(m+n+j+k+1)!} \cdot \frac{(m+j)!}{m!} \cdot \frac{(n+k)!}{n!}$$

, and $C_{jk}(N,\ell)$ is the number of ways in which the mode of failure, containing j failed and k unfailed components, can occur giving due regard to the location of components in the test result and avoiding double counting.

3. Estimation of Failure Probability from Similar Test Data

If no direct test data is available for a particular structure it may be possible to make use of information based on similar data from other sources. For a particular structure whose material properties and environmental conditions are known accurately it is straightforward to use other data derived from a structure with exactly the same conditions. If for example data is available for identical components, which have experienced identical conditions, of which a are failed and b unfailed the failure probability of the original structure is given by equation (9) where a = m, b = n, and $N \rightarrow N + a + b$.

It is not necessarily the case that inspection data from similar structures can be used in the way just described for a large complex structure. For a structure where failures are due to the effects of corrosion, the failure probability of components fabricated from materials of the same nominal specification, and under nominally the same environmental conditions, can vary appreciably. This is because within a particular material specification there will be large variations of the relevant mechanical properties. Also corrosion rates can be sensitive to small changes in environmental conditions, which may not be known precisely. Therefore care must be taken in assessing the usefulness of similar test data. A method of using test data will now be described.

The material properties and the environmental conditions for the components of a particular structure will be known within certain limits. Obviously test data derived from the components of a similar structure, which experience conditions outside these limits, will be less useful than data from similar structures within these bounds. There can be many different groups of tested components which are within the limits of the known material specification and environmental conditions of the components in question and each group may have a different fraction of failed components. If the relevant test data contains a representative sample of the different groups of components which can occur, then a failure probability distribution for the components of the structures of interest can be obtained.

A failure probability distribution $G_{r,N}$ will be defined as the probability of selecting components from groups such that out of N components r have failed. The probability of a structure failing if r components are failed can be obtained from equation (6), and the overall failure probability of the structure, assuming a probability distribution $G_{r,N}$, is then given by

$$P = \sum_{r} f_r \, G_{r,N} \qquad (10)$$

A convenient way of evaluating the above expression will now be described. Consider the equation

$$G_{r,N} = \sum_{a,b} D_{ab} \binom{r+a}{a} \binom{N+b-r}{b} \bigg/ \binom{N+a+b+1}{a+b+1} \qquad (11)$$

where $\sum_{a,b} D_{ab} = 1$.

It is straightforward to show that any function can be expressed by the RHS of this equation. Substituting equations (6) and (11) into (10) and using the summation given by equation (8) it can be shown that

$$P = \sum_{j,k} \sum_{a,b} C_{jk}(N)\, D_{ab} \frac{(a+b+1)!}{(a+b+j+k+1)!} \frac{(a+j)!}{a!} \frac{(b+k)!}{b!} \qquad (12)$$

where it should be noted that a and b do not need to be positive integers.

The distribution described by equation (11) can be interpreted in the following manner. Consider t groups of components where the number of failed and unfailed components is $a_1, b_1; a_2, b_2; \ldots\ldots\ldots\ a_t, b_t$ for each group. If the relative proportion of a group a, b occurring is D_{ab}, then the probability of there being r failures out of a total population N is given by equation (11).

To simplify the application of equation (12) an approximation will now be introduced. If the ith term in equation (11) has a separate average value $\bar{p}_i$ and a variance σ_i^2, then it is straightforward to show that the average value and variance of the total distribution is given by

$$\bar{p}_T = \frac{1}{t} \sum_{i=1}^{t} \bar{p}_i, \quad \text{and} \quad \sigma_T^2 = \frac{1}{t} \left\{ \sum_{i=1}^{t} \sigma_i^2 + \bar{p}_i^2 \right\} - \bar{p}_T^2 \qquad (13)$$

where it is assumed, in the absence of other evidence, that all groups are equally representative and therefore all $D_{ab} = 1/t$. A hypergeometric distribution can be defined by parameters $\bar{a}$ and $\bar{b}$ such that

$$G_{r,N} \simeq \binom{r+\bar{a}}{\bar{a}} \binom{N+\bar{b}-r}{\bar{b}} \Big/ \binom{N+\bar{a}+\bar{b}+1}{\bar{a}+\bar{b}+1} \qquad (14)$$

It can be shown that such a distribution can be described, which has a mean fraction of failed components $\bar{p}_T$ with variance σ_T^2, where $\bar{a}$ and $\bar{b}$ are obtained from the equations

$$\bar{a} = (1-\bar{p}_T)\, Q - 1 \quad \text{and} \quad \bar{b} = \bar{p}_T Q - 1$$

with

$$Q = \frac{\bar{p}_T (1-\bar{p}_T)}{\sigma_T^2} \; \frac{N - 1}{N - \dfrac{\bar{p}_T (1-\bar{p}_T)}{\sigma_T^2}} - 1 . \qquad (15)$$

Therefore the overall failure probability is given by

$$P = \sum_{j,k} C_{jk}(N) \frac{(\bar{a}+\bar{b}+1)!}{(\bar{a}+\bar{b}+j+k+1)!} \frac{(\bar{a}+j)!}{\bar{a}!} \frac{(\bar{b}+k)!}{\bar{b}!} . \qquad (16)$$

There are two reasons for justifying the approximation given by equation (14). Firstly it has the correct functional form and will represent a distribution with any mean value and variance. Secondly it is not generally necessary to determine overall failure probabilities to great accuracy since they are only used to indicate a level of integrity, and a distribution defined by only a mean and variance is usually adequate.

If information is also available from the direct inspection of the components of a system the overall failure probability will be given by

$$P = \sum_{j,k} \sum_{r} f'_r \; L_r \; G_{r,N} \Big/ \sum_{r} L_r \; G_{r,N} \quad .$$

Making use of the approximation given by equation (14) and summing over r using equation (8) leads to

$$P = \sum_{j,k} C_{jk} \; (N,\ell) \quad T_{j,k}^{m+\bar{a},n+\bar{b}} \tag{17}$$

where $$T_{j,k}^{m+\bar{a},n+\bar{b}} = \frac{(m+n+\bar{a}+\bar{b}+1)!}{(m+n+\bar{a}+\bar{b}+j+k+1)!} \quad \frac{(m+\bar{a}+j)!}{(m+\bar{a})!} \quad \frac{(n+\bar{b}+k)!}{(n+\bar{b})!}$$

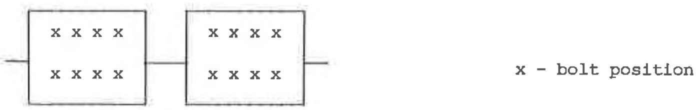
and $C_{jk}(N,\ell)$ is the number of ways in which the mode of failure containing j failed and k unfailed components can occur giving due regard to the location of components from direct testing and avoiding double counting.

4. Potential Application of the Theory

In gas cooled Magnox reactors the components of the core restraint structure are bolted together to produce designs with a high level of redundancy and hence, high reliability. However, the use of carbon dioxide as coolant gives rise to corrosion of mild steel components, and in bolted assemblies the growth of oxide at interfaces can strain the bolts and could ultimately lead to their failure [2,3]. It is important in safety studies to assess the effects of oxidation and possible failure of bolts and to relate this to the observed incidence of failure so that the continued integrity of the structure can be demonstrated with the requisite level of confidence.

The theory presented here can be applied to such a problem where only a small proportion of the total number of bolts is amenable or accessible to inspection to determine their status. The principles are illustrated in the following example:

Assume a core restraint structure is constructed from sub-assemblies bolted together by, say, groups of eight bolts as indicated diagramatically below. Furthermore assume that



stress analysis of this highly redundant structure under a maximum fault loading condition shows that at least 10 bolts out of 16 in 2 adjacent groups need to fail before the integrity of the structure would be imperilled . Since 10 failed bolts can be distributed amongst the 16 locations in several thousand ways (some configuration probably would not lead to failure) and there are likely to be a few hundred 8 bolt groupings in the reactor, there will typically be about 10^6 ways in which the most likely mode of failure could occur. For a system which is expected to have a small fraction of failures other modes of system failure i.e. modes involving more than 10 bolt failures are significantly less likely, therefore the overall failure probability of the system, given by equation (17) is

$$P \simeq 10^6 \; T_{10,6}^{m+\bar{a},n+\bar{b}}$$

where it is assumed that the inspected bolt population is only a small fraction of the total bolt population. m and n are the failed and unfailed bolts from direct inspection and $\bar{a}$ and $\bar{b}$ are determined by indirect test data.

If only direct test data is available $\bar{a} = \bar{b} = 0$ and plot 1 Fig.1 shows the sample size, with a given number of failures, required to demonstrate the integrity of the structure to a probability of less than 10^{-5}. For example, assume that it is required to demonstrate the integrity of the system to 10^{-5}. Then plot 1 Fig.1 shows that the minimum number of bolts to be tested is 45 and providing no failures are found amongst the sample this would be a sufficient test programme. However if one failure is found it would be necessary to test an additional 14 bolts without finding further failures in order to prove the system to the required level of integrity. If further failures were found the size of the total bolt sample which would be required can be obtained from Figure 1.

The size of the sample required from direct inspection can be reduced by the use of indirect test data as indicated in Section 3. Indirect test data based on bolts which have experienced similar conditions to the bolts of the structure in question may be employed. This indirect data may predict low mean failure probabilities but the variance about the mean may be large and account must be taken of this. For example, if the mean $\bar{p}_T = 0.04$ and the standard deviation $\sigma_T = 0.05$ $\bar{a}$ and $\bar{b}$ can be determined from equations (15) where N = 2000. It is found that $\bar{b} = 12.8$ and $\bar{a} = -0.43$ which leads to an overall failure probability for the structure.

$$P = 10^6 \; T_{10,6}^{m-0.43,\, n + 12.8}$$

In Fig.1, plot 1 indicates the sample size required to justify the integrity of the structure to a failure probability of 10^{-5} where information from direct inspection only is used, and plot 2 shows the sample size when a prior distribution with $\bar{p}_T = 0.04$ and $\sigma_T = 0.05$ is assumed as given by equation (14). It can be seen that the prior information reduces the sample size from direct inspection by about 20 bolts for sample sizes up to 100.

If it were assumed that $\bar{p}_T$ = 0.04 and σ_T = 0.04 the prior distribution would be equivalent to a direct inspection of about 24 bolts. Similarly if σ_T = 0.06 the direct inspection equivalent is 17 bolts. No direct inspection is needed for $\bar{p}_T$ = 0.04 if σ_T = 0.25 or less.

5. Conclusions

The integrity of a multi-component system, designed to have high reliability but which may now contain a number of failed components, can be assessed by combining information from the direct inspection of a small component sample, and the results of indirect inspection data, using the theory presented here provided that the failed components are distributed in a random uncorrelated manner.

6. Acknowledgment

This paper is presented with the permission of the Director General, South Eastern Region, Central Electricity Generating Board.

References

[1] JEFFREYS H J, Theory of Probability, Clarendon Press Oxford (1961)

[2] PAYER J M and STAELE R W, Localised Attack on Metal Surfaces, CORROSION FATIGUE, NACE - 2, 211 (1972)

[3] BROWN J, Design Engineering Implications and Assessment, B.N.E.S. International Conference on Corrosion of Steels in CO_2. (1974).

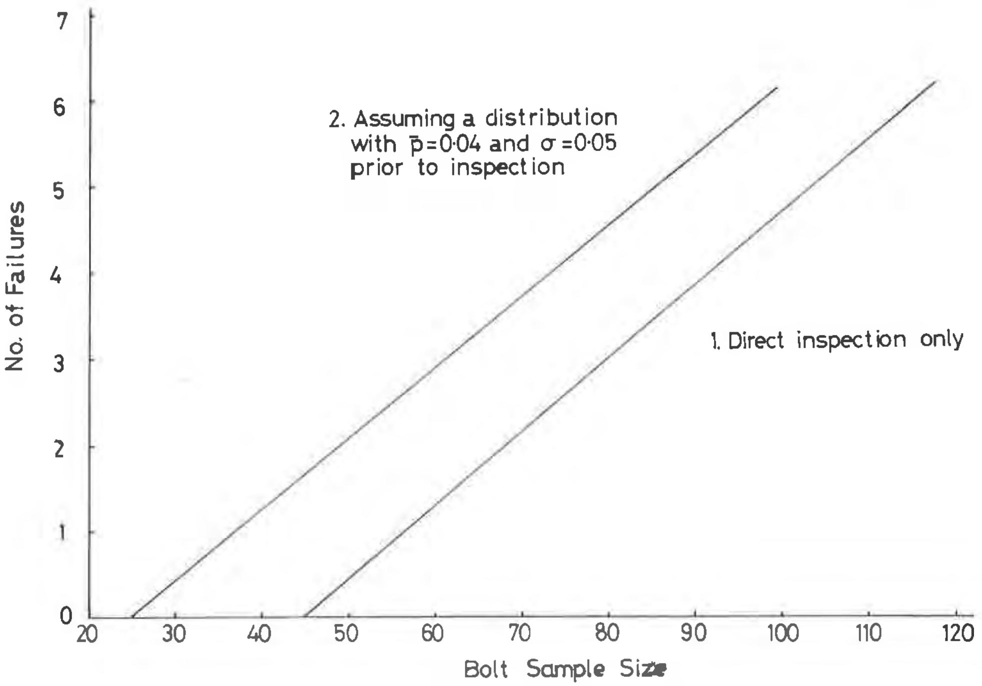


Fig. 1 Bolt sample size required to ensure structural integrity to a failure probability < 10^{-5}