On Stress Waves Produced by a Variable Magnetic Field in an Elastic-Plastic Conductor

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Abstract

The purpose of this study is to provide some information on the initial stage of the process of interaction between the impulsive magnetic field and the conducting body accompanied by elastic and plastic deformations. Assuming the conductor immobility, a form of the penetrating magnetic field is, first, determined. Then, this field is used to express the body force which causes the deformation of the conductor.

The study is limited to motions which can be described with a single spatial coordinate. The problem of the plane waves propagation, describing such a behaviour is solved by the method of characteristics. The configuration of the elastic and plastic deformation regions in the phase plane is determined.

1. Introduction

The interaction between an electromagnetic field and conductors belongs to problems yet not well recognized like those related to interactions between other fields of a different physical nature. The modern science and technology require, from the other side, some problems of this kind to be solved. These arise, for instance, from a technique of generation of very strong magnetic fields / see Kolm and Freeman [1] /. The impulsive magnetic fields the most frequently used there often effect a destruction of current carrying windings of electromagnets coils.

Similarly, an experimental device designed to produce a controlled fusion reaction may be considered as an example of the modern structure operating under extremely complex conditions. Such conditions result from both very high and very low values of temperature, very strong variable magnetic fields which generate variable mechanical forces acting on structural elements.

The above two examples present undesired effects of forces produced within conductors by variable magnetic fields. There are some cases where the action of these forces is necessary. Here, the industrial process of the metal forming utilizing the impulsive magnetic field should be mentioned / see Waniek [2] /.
It is well known that in the case of variable magnetic field / especially,

impulsive one / the higher values of intensity, compared to those from the static case, can be reached. Such fields always cause a dynamic motion of the conducting body. For this reason the dynamic behaviour of deformable conductors under the impulsive magnetic field is studied.

The first paper / by Knopoff [3] / devoted to the interaction between magnetic field and the elastic conducting body has appeared about thirty years ago. Fields considered in subsequent papers were usually stationary and conductors / often taken as ideal / were assumed to be elastic. A considerably less work has been done to investigate the plastic deformations of conductors. The detailed study on the propagation of various deformation regions into the conducting body has not been undertaken even in papers devoted to electromagnetic forming processes. It has been assumed there that the forces produced by the variable magnetic field have taken a form of a pressure acting on the surface of the deformed element.

It is observed that some parameters describing the elastic properties of materials placed in magnetic field change their values. Such effects are well known from the literature concerned with problems of electromagnetism.

Some additional features in a plastic behaviour of conducting materials were recently discovered by Hayashi [4] and Troitskii [5]. These were named "magnetoplastic effect" and "electroplastic effect" respectively. Hayashi and Troitskii observed a considerable reduction in the yield limit at a presence of magnetic or electric fields, in independent experiments carried out by themselves. The described behaviour of materials up to now has no clear and unique interpretation.

It follows from the presented above remarks that further studies on interactions between the electromagnetic field and conductors are needed. The results obtained from those considerations will be applied to explain some ob-served features of phenomena under study as well as to predict a behaviour of various elements in structures and devices operating under complex conditions.

The present paper should be considered as an introductory one to further studies on similar problems taking into account more-dimensional stress or strain states which reflect actual situations in a better way.

Notation

$x_1, x_2, x_3 = x$ - coordinates of a spatial reference system,

$\bar{x}$ - dimensionless space coordinate,

t - time coordinate,

$\bar{t}$ - time dimensionless coordinate,

$H(x,t)$ - function of variables x,t describing a magnetic field intensity,

H_0 - amplitude of an external magnetic field intensity,

h - coefficient representing damping of an external magnetic field,

ω - frequency of an external magnetic field,

c - speed of the light in vacuum,

μ - permeability coefficient of a conducting body,

λ_e - coefficient of Ohmic conductivity,
$E(x,t)$ - function of variables x,t describing an electric field intensity,
ρ - coefficient of the conductor mass density,
$F_e(x,t) = F_e$ - body force produced by an electromagnetic field,
$\sigma(x,t) = \sigma$ - normal component of stress along x_3 coordinate,
$v(x,t) = v$ - displacement velocity in x_3 direction,
G - shear modulus,
G_p - modulus of hardening,
K - bulk modulus,
σ_s - yield limit,
σ_i - stress intensity,
ε_i -strain intensity,
a_o -velocity of longitudinal elastic waves.

2. Problem Formulation

Consider a motion of the conducting body taking a form of a semi-space, caused by an external impulsive magnetic field acting on a surface of the semi-space / see Fig.1 /. The study is restricted to the process of the propagation of plane stress waves. This means that every field under study can be described with a single spatial coordinate $x_3 = x$ in a (x_1,x_2,x_3) reference system shown in Fig.1. Assume the changes in both electric and magnetic fields intensities caused by the conductor motion to be negligible compared to intensities of the respective fields which are induced in the conducting body by the external magnetic field. By this means a magnetic field penetration can be described independently on the solution of the problem of the conductor motion.

A parabolic equation describing the magnetic field in the conductor is obtained from the set of Maxwell's equations under following assumptions:
(a) the conductor immobility,
(b) the material isotropy with regard to electromagnetic properties,
(c) the displacement current being negligible compared to conduction current,
(d) there are no free electric charges in conductor.

The initial boundary-value problem for the mentioned above parabolic equation is solved by the method of Laplace's transforms, under the following conditions:

$$H(x,t)\Big|_{t=0} = 0 \tag{1}$$

$$H(x,t)\Big|_{x=0} = H_o \exp(-ht) \sin\omega t \tag{2}$$

$$\lim_{x\to\infty} H(x,t) < \infty \tag{3}$$

The solution of the above problem representing the magnetic field in the conducting body takes the following form:

$$H(x,t) = H_o \exp\left[-(th + \frac{x}{\sqrt{\kappa}} A)\right] \sin(\omega t - \frac{x}{\sqrt{\kappa}} B) - - \frac{H_o\omega}{\pi} \int_0^{\infty} \exp(-tr) \frac{\sin(\frac{x}{\sqrt{\kappa}}\sqrt{r})}{\omega^2 + (h-r)^2} dr, \tag{4}$$

where

$$A = \sqrt{\frac{1}{2}\left(\sqrt{h^2 + \omega^2} - h\right)},$$

$$B = \sqrt{\frac{1}{2}\left(\sqrt{h^2 + \omega^2} - h\right)},$$

$$\kappa = \frac{c^2}{4\pi\mu\lambda_e}.$$

The electric field intensity is described by the following relation:

$$E(x,t) = -\frac{H_o\, c}{4\pi\lambda_e\sqrt{\kappa}}\left\{A \exp\left[-(th + \frac{x}{\sqrt{\kappa}}A)\right]\frac{\sin(\omega t - \frac{x}{\sqrt{\kappa}}B + \gamma)}{\cos\gamma} + \right.$$

$$\left. + \frac{\omega}{\pi}\int_0^{\infty}\exp(-tr)\frac{\sqrt{r}\ \cos\frac{x}{\sqrt{\kappa}}r}{\omega^2 + (h - r)^2}\, dr\right\}, \qquad (5)$$

where

$$\gamma = \operatorname{arc\,tg}\frac{A}{B}.$$

The relations (4) and (5) are used to determine the body force which effects the conductor motion:

$$\rho F_e = -\frac{\lambda_e \mu}{c} E(x,t)\, H(x,t). \qquad (6)$$

The function expressed by eq. (6) is then used to formulate the essential part of the problem under study, i.e., an initial boundary-value problem for a set of hyperbolic equations describing the conductor motion. The formulation of this problem requires some conditions representing mechanical properties of the conducting body to be taken into account. It is assumed that the material is homogeneous and isotropic. The behaviour of the conductor is described within the infinitesimal strains framework by the model of an elastic-plastic body with hardening, as it is shown in Fig.2. A material compressibility under plastic deformations is admitted. The conducting body is free of external loads of a mechanical origin. It is assumed that under above specified conditions in the conducting semi-space the plane stress waves propagate.

The analytical formulation of the initial boundary-value problem describing the conductor motion takes the form:

(a) equations of motion

$$\frac{\partial \sigma}{\partial x} - \rho\frac{\partial v}{\partial t} = - F_e, \qquad (7)$$

$$\frac{\partial \sigma}{\partial t} = E_\alpha \frac{\partial v}{\partial x}, \qquad (8)$$

where

$$E_\alpha = \begin{cases} \frac{1}{2}(4G + 3K) & \text{- for elastic loading and unloading} \\ \frac{1}{2}(4G_p + 3K) & \text{- for plastic loading;} \end{cases}$$

(b) initial and boundary conditions

$$\sigma(x,t)\Big|_{t=0} = v(x,t)\Big|_{t=0} = 0 \qquad (9)$$

$$\sigma(x,t)\Big|_{x=0} - \frac{1}{8\pi}\Big\{(\epsilon - 1)\, E(x,t)\Big|_{x=0} + (\mu - 1)\, H(x,t)\Big|_{x=0}\Big\} = 0 \qquad (10)$$

3. Method of Solution

A method of characteristics is used to solve the initial boundary-value problem described by eqs. (7) ÷ (10). The equations of characteristics and the corresponding relations take the form:

$$dx - \sqrt{E_\alpha/\rho}\, dt = 0, \qquad (11)$$

$$dv - \frac{1}{\rho\sqrt{E_\alpha/\rho}}\, d\sigma - \frac{F_e}{\sqrt{E_\alpha/\rho}}\, dx = 0, \qquad (12)$$

$$dx + \sqrt{E_\alpha/\rho}\, dt = 0, \qquad (13)$$

$$dv + \frac{1}{\rho\sqrt{E_\alpha/\rho}}\, d\sigma + \frac{F_e}{\sqrt{E_\alpha/\rho}}\, dx = 0. \qquad (14)$$

To integrate the expressions (12) and (14) some simplifying assumptions should be introduced into eq. (4). The obtained relations are then applied to determine functions describing stress and displacement velocity fields. The image of the solution in the phase plane (x,t) shown in Fig. 3 is used to facilitate the determination procedure and the interpretation of the required fields. The analysis of the configuration of strain regions in the phase plane leads to some conclusions related to the deformation process of the conducting body in the initial stage of its motion.

To illustrate characteristics features of the calculated fields some diagrams are presented. Fig.4 shows a typical variability of the relative stress σ/σ_s and the displacement velocity v/a_0 in a plane determined by a fixed space coordinate $\bar{x}$ = const / cf Fig.3 /. In Fig.5 changes in these quantities are plotted against space coordinate $\bar{x}$, for a fixed time coordinate $\bar{t}$ = const / cf Fig.3 /. Some results arising from this study are compared with those obtained from the similar problem considered in the earlier author's paper [6].

4. References

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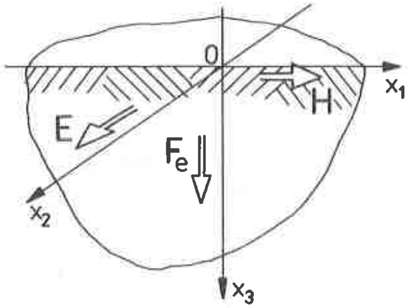

Fig. 1

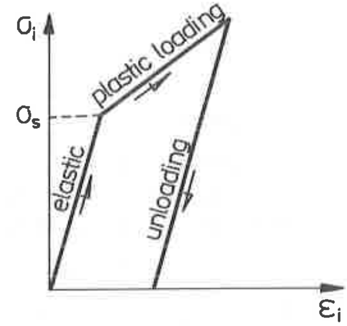

Fig. 2

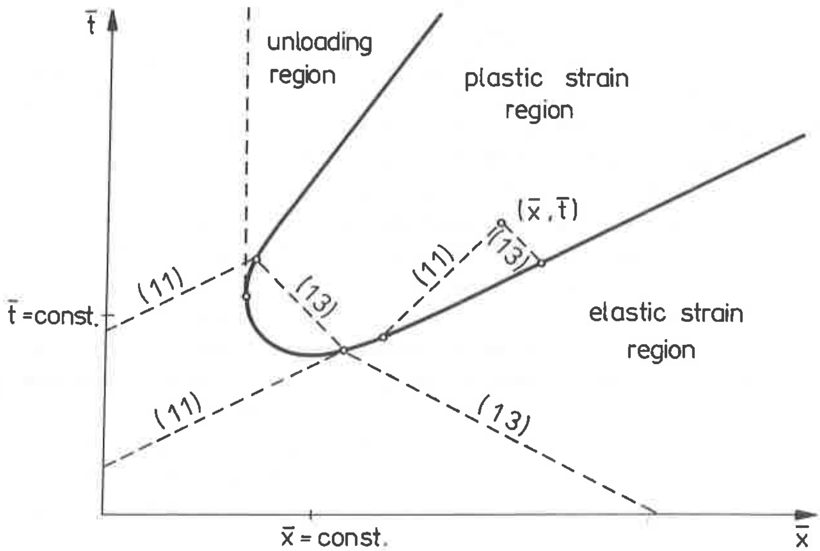

Fig. 3

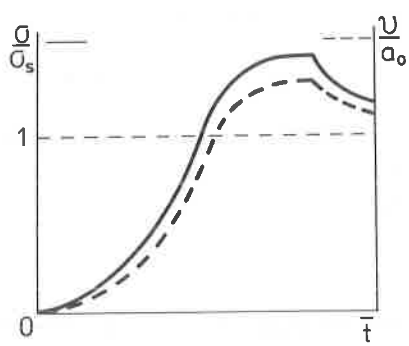

Fig. 4

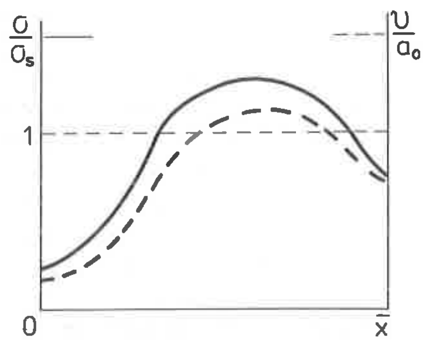

Fig. 5