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ENHANCING COMPUTATIONAL EFFICIENCY OF BAYESIAN INFERENCE BY IDENTIFYING THE INTENSITY MEASURE RANGE TO UPDATE SEISMIC FRAGILITY CURVES

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ABSTRACT

The United States Nuclear Regulatory Commission (US NRC) has established stringent criteria for the acceptance of new Standard Plant designs. These criteria require that the fragility curves have a high degree of confidence, especially in the low-probability regions. However, conventional methods of developing seismic fragility curves require a substantial number of dynamic analyses, which can be computationally intensive. To address this challenge, the Bayesian framework offers a more efficient and effective solution. Bayesian Inference enables the integration of prior knowledge with newly acquired data to refine the data-generating process. This paper presents a systematic Bayesian framework to update the seismic fragility curve of structures, systems, and components (SSCs). The efficiency of the framework is illustrated using an application to the seismic fragility of a concrete shear wall. The seismic fragility curve developed with our proposed approach aligns closely with those generated through conventional nonlinear simulations while significantly reducing the computational cost.

INTRODUCTION

In recent years, the United States nuclear industry has progressed towards the use of performance-based design (PBD) and risk assessment methodologies for structures, systems, and components (SSCs), with a particular emphasis on addressing seismic loads. The performance-based design is preferred over the deterministic design because of its ability to capture the inherent uncertainty in seismic performance caused by unpredictability in seismic loads. Moreover, the PBD also helps in risk-informed decision-making, as it provides insight into the potential consequences of seismic failures, allowing for proactive measures to be taken to reduce the risk of accidents. In the context of PBD, the term "failure" refers to the occurrence when the structures, systems, and components (SSCs) do not meet a specified performance level, such as serviceability, life safety, collapse prevention, or when they reach a prescribed limit state (ASCE 43-19, 2021). The PBD approach evaluates the seismic risk of SSC failures by utilizing the seismic fragility curve to predict the conditional probability of SSC failure when subjected to a suite of ground motions. Moreover, seismic fragility assessment also provides insights into developing more robust designs, incorporating necessary upgrades, and implementing effective mitigation strategies for increased resiliency against earthquakes.

In assessing seismic risks for nuclear power plants, it is important to determine the seismic vulnerability of SSCs. Evaluating seismic risk at the system level (nuclear power plant) necessitates a detailed sequence analysis of potential failures at the component levels. The commonly used "safety factor" method by EPRI (2018) offers a simplified procedure to determine the seismic fragility of SSCs without conducting complex numerical simulations. Researchers have performed experimental studies to determine the seismic fragility of various SSCs such as shear walls, piping components, and systems. Nevertheless, the process of conducting multiple experiments to develop the entire seismic fragility curve is cost-intensive and impractical. As a result, the numerical simulation approach using experimentally validated high-fidelity finite element (FE) models (Kitada et al., 2004; Syed, 2013) is a commonly adopted alternative.

Multiple Stripe analysis (MSA) is one of the widely used conventional methods for developing fragility curves. It relies on numerical simulation and maximum likelihood estimation methods in seismic fragility curve development (Zentner et al., 2017). However, MSA requires a substantial number of dynamic analyses, which can be computationally intensive and time-consuming. Many researchers (Sandhu et al., 2019; Syed, 2013; Tadinada and Gupta, 2020) have developed frameworks based on Bayesian Inference to reduce the number of time history analyses at varying intensity measure levels. Bayesian Inference allows for combining the “prior” fragility curve with newly acquired data to obtain the “posterior” fragility curve. Although prior studies have used Bayesian approaches to update fragility curves, applying such an approach to actual large-scale structural systems remains impractical. The primary reason for this is the lack of guidance on the range of intensity measures within which one must conduct time-consuming nonlinear time history analyses to determine the likelihood function. The lack of such guidance leads to a large number of nonlinear analyses that often fall outside the range of intensity measures corresponding to the true posterior fragility curve.

The proposed approach recommends developing an equivalent SDOF system corresponding to a critical element. The aim, however, is not to substitute the full model with the SDOF system. Instead, the SDOF system is utilized to determine the initial range of the intensity measure and select a few intensity measure levels as a preliminary step for conducting nonlinear time history analyses of the full model. Finally, the fragility parameters are updated through an iterative process using Bayesian inference. The proposed approach significantly reduces the computational resources needed to determine the appropriate posterior curve.

SEISMIC FRAGILITY ANALYSIS

The seismic fragility, denoted as $P_{f|IM}(im)$, or simply $P_f(im)$, gives the conditional probability of reaching to or exceeding a specific limit state under a given earthquake intensity measure IM, such as PGA or spectral acceleration at a specified period. In most practical applications, a fragility function is usually expressed using the cumulative lognormal distribution function given by Eq. (1), which represents the family of fragility functions for various confidence levels (Q) and incorporates the treatment of uncertainties related to nuclear applications (EPRI, 2018; Tadinada and Gupta, 2020; Zentner et al., 2017).

$$P_f(im) = \Phi\left(\frac{\log\left(\frac{im}{a_m}\right) + \beta_U \Phi^{-1}(Q)}{\beta_R}\right) \tag{1}$$

The Safety factor method (EPRI, 2018) employs a simplified approach to estimate lognormal fragility parameters a_m, β_R, β_U. In this approach, the acceleration median capacity (a_m) is expressed as the product of a given seismic design level im_{design} (typically PGA) and various safety margin factors (F_i), which account for conservatism and uncertainty in the design parameters.

$$a_m = (\Pi F_i) \cdot im_{\text{design}} = F_C F_{RS} \cdot im_{\text{design}} \tag{2}$$

Broadly, F_i is a function of the capacity factor (F_C) and the structural response factor (F_{RS}). These two factors can be further divided into many sub-factors to incorporate the effects of strength, inelastic absorption, model response, soil-structure interaction, model combination, earthquake component combination, etc. Each F_i, where the subscript i represents the subfactors mentioned above, is characterized as a lognormal variable with a median value of 1, and the logarithmic standard deviations β_R^i and β_U^i represent aleatory uncertainty and epistemic uncertainty, respectively.

Bayesian framework to update seismic fragility curve

The seismic fragility of SSCs for a given earthquake intensity, expressed as $P_f(im) = f(im; \boldsymbol{\Theta})$, is typically influenced by a collection of random variables $\boldsymbol{\Theta} = [\theta_1, \ldots, \theta_p]^\top \in \Omega$ living in some parameter space Ω. For example, if the fragility curve is represented using a lognormal model as shown in Eq. (1), then $\boldsymbol{\Theta} = [a_m, \beta_R, \beta_U]^\top$. A joint probability density function, $f_{\boldsymbol{\Theta}}(\theta_1, \ldots, \theta_p)$, can describe the current knowledge of these parameters $\boldsymbol{\Theta}$, obtained through a combination of expert judgment and pre-existing data. This joint density function is referred to as a "prior". If new observations $\boldsymbol{y} = [y_1, \ldots, y_m]^\top$ become available from experimental or high-fidelity simulation studies, Bayes' theorem can be utilized to update the understanding of the parameters using the new data $\boldsymbol{y}$. This updated information is known as the "posterior" and is represented by $f_{\boldsymbol{\Theta}|\boldsymbol{y}}(\theta_1, \ldots, \theta_p|\boldsymbol{y})$ (Bodda et al., 2022).

$$f_{\boldsymbol{\Theta}|\boldsymbol{y}}(\theta_1, \ldots, \theta_p|\boldsymbol{y}) = \frac{\mathcal{L}(\theta_1, \ldots, \theta_p|\boldsymbol{y}) f_{\boldsymbol{\Theta}}(\theta_1, \ldots, \theta_p)}{f(y)} \tag{3}$$

where, $\mathcal{L}(\theta_1, \ldots, \theta_p|\boldsymbol{y})$ is the "likelihood function," representing how likely it is to observe the data, assuming the data has been generated using parameters $\boldsymbol{\Theta}$; $f(\boldsymbol{y})$ is the marginal likelihood of data, and it does not depend on $\boldsymbol{\Theta}$ and evaluates to a constant. In this study, a two-parameter lognormal model is considered for seismic fragility, as given by Eq. (4). The prior distribution for a_m and β_R can be defined using Eq. (5). If $y_i = \{im_i, n_i \text{ analysis}, r_i \text{ failures}\}$ denotes the fragility data at $IM = im_i$, the likelihood function $\mathcal{L}(a_m, \beta_R \mid \boldsymbol{y})$ can be represented by a binomial distribution using Eq. (6).

$$P_f(im; a_m, \beta_R) = \Phi\left[\frac{\log im - \log a_m}{\beta_R}\right] \tag{4}$$

$$\begin{aligned} a_m &\sim \mathcal{LN}(\hat{a}_m, \beta_U) \\ \beta_R &\sim \text{Unif}\left(\hat{\beta}_R(1-\delta_R), \hat{\beta}_R(1+\delta_R)\right) \end{aligned} \tag{5}$$

$$\begin{aligned} \mathcal{L}(a_m, \beta_R \mid \boldsymbol{y}) &= \prod_{i=1}^{k} P(y_i \mid a_m, \beta_R, r_i, n_i) \\ &\propto \prod_{i=1}^{k} \left(P_f(im_i \mid a_m, \beta_R)\right)^{r_i} \left(1 - \left(P_f(im_i \mid a_m, \beta_R)\right)\right)^{n_i - r_i} \end{aligned} \tag{6}$$

where, $\hat{a}_m$ is the median capacity obtained from the safety factor method based on Eq. (2); β_U is the combined epistemic uncertainty, reflecting the uncertainty in the estimate of $\hat{a}_m$; $\hat{\beta}_R$ is the combined aleatory uncertainty. The parameter δ_R represents the variation around the mean value.

The final step of the Bayesian inference process is to estimate the posterior distribution of parameters by combining the likelihood function with the prior density function, as shown in Eq. (7). In this study, Markov chain Monte Carlo (MCMC) algorithms are employed using the Nimble package (de Valpine et al., 2023) in R to determine the posterior distribution of parameters.

$$f_{\boldsymbol{\Theta}|\boldsymbol{y}}(a_m, \beta_R|\boldsymbol{y}) \propto f_{\boldsymbol{\Theta}}(a_m, \beta_R) \times \mathcal{L}(a_m, \beta_R \mid \boldsymbol{y}) \tag{7}$$

ILLUSTRATION OF THE PROPOSED FRAMEWORK FOR A 3-STORY SHEAR WALL STRUCTURE

The flowchart proposed methodology to develop the seismic fragility curve for SSCs using Bayesian Inference shown in Fig. 1. The methodology is demonstrated through an experimentally validated high-fidelity FE model (Lee, 2021) of a 3-story concrete shear wall structure shown in Fig. 2. The experimental data used in this study is obtained from the SMART-2013 benchmark program (Benjamin et al., 2016), organized by the French Sustainable Energies, Atomic Energy Commission (CEA), and Electricité De France (EDF).

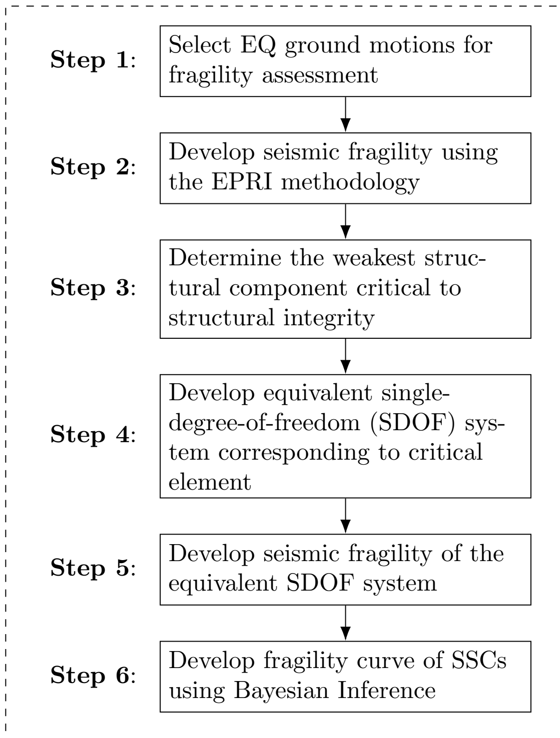
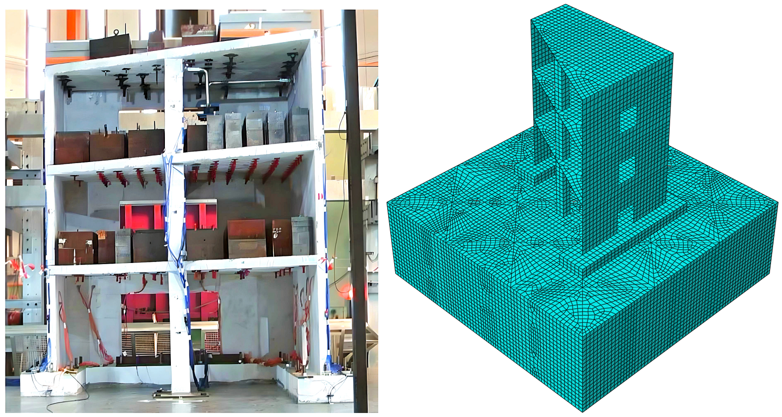


Figure 1. Flowchart of the proposed methodology

The resulting seismic fragility curve corresponds to the limit state C (ASCE 43-19, 2021). The following steps are executed sequentially to develop the seismic fragility using the proposed Bayesian framework.

Step #1: ***Select earthquake ground motions for fragility assessment***

The ground motions used for the seismic analyses should be representative of the site of interest. In this study, uniform hazard spectra (UHS) is considered as target spectra in accordance ASCE standards (ASCE 43-19, 2021) and the EPRI guidelines (EPRI, 2018), which represents the state-of-practice in nuclear industry for assessing the seismic fragility of SSCs. In this research, Seabrook, New Hampshire is chosen as a site and 11 pairs of ground motions are selected for seismic analysis through deaggregation analysis. The spectral matching of selected earthquake ground motions with the target spectra is performed using the commercial software program Seismosoft (Seismosoft, 2019), as illustrated in Fig. 3.



(a) Experimental set up (b) Finite element model

Figure 2. SMART 2013 shear wall structure

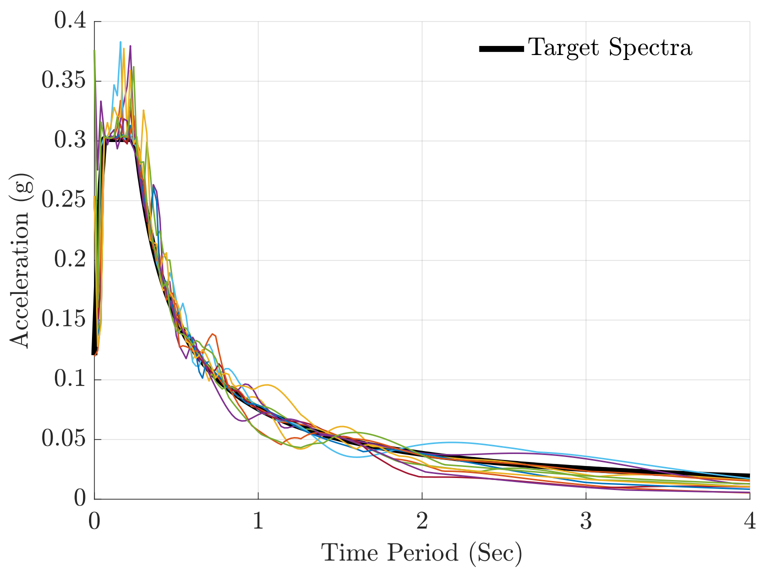


Figure 3. Spectral matching of ground motion spectra with target spectra

Step #2: ***Develop seismic fragility curve using EPRI methodology***

The seismic fragility curve for the 3-story shear wall, based on EPRI methodology, is developed using safety factors corresponding to the capacity and structural responses, as shown in Table 1. Comprehensive details on the derivation of these safety factors can be found in Zentner et al. (2017). The median capacity, β_R, β_U, and High Confidence of Low Probability of Failure (HCLPF) value of the seismic fragility curve developed using EPRI methodology are $0.92\,\mathrm{g}$, $0.28\,\mathrm{g}$, $0.45\,\mathrm{g}$, and $0.27\,\mathrm{g}$, respectively.

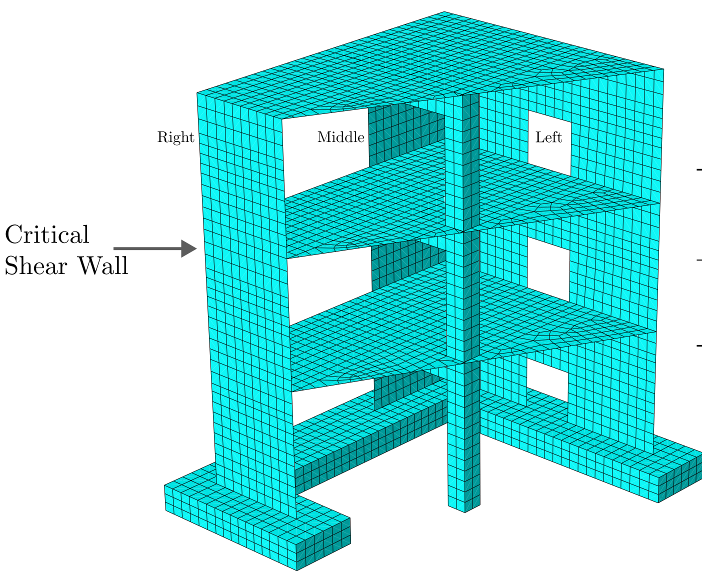
Step #3: ***Determine weakest structural component critical to overall structural integrity***

The failure sequence within a structure begins with the weakest (or the most critical) load-bearing element. In this study, linear time-history analysis of the 3-story shear wall structure is conducted to determine the most critical shear wall using all the 11 pairs of earthquake ground motions selected in Step #1. Each pair of earthquake ground motions are applied in the orthogonal direction. The engineering demand parameter

(EDP) considered for each analysis is the inter-story drift of each shear wall. The shear wall exhibiting the maximum inter-story drift is identified as the most critical shear wall. As illustrated in Fig. 4, the right shear wall exhibits the maximum values of inter-story drift for each analysis. The critical shear wall identified through time history analysis aligns with the critical shear wall identified based on the EPRI methodology, as established by Zentner et al. (2017).

Table 1: Factor of safety (EPRI methodology)

Factors	Median Value	β_R	β_U
F_s	2.74	0.04	0.22
F_μ	2.41	0.04	0.14
F_{RS}	1.00	0.27	0.36
F_{SS}	1.00	0.20	0.24
F_{MC}	1.00	0.15	0.00
F_{EC}	1.00	0.11	0.11
F_M	1.00	0.00	0.25



Story	Maximum Interstory Drift (%) of Shear Walls		
	Left	Right	Middle
First	0.53	0.58	0.12
Second	0.14	0.14	0.12
Third	0.14	0.13	0.11

Figure 4. Critical shear wall of the 3-story shear wall structure

Step #4: ***Develop equivalent single-degree-of-freedom system corresponding to critical element***

The SDOF system is developed by tuning it to resonate with the fundamental frequency of the critical shear wall identified in step #3, which is $20.65\,\mathrm{Hz}$. The stiffness of the SDOF system ($k = 14\,492.75\,\mathrm{N/mm}$) is computed through a static analysis of the critical shear wall, where a unit force is applied. While all the degrees of freedom are restrained at the base of the SDOF system, only one translational degree of freedom is released at the top of the system.

Step #5: ***Develop seismic fragility curve of equivalent SDOF system***

In this step, perform nonlinear time history analyses (NLTHA) of the SDOF system for all ground motions corresponding to the HCLPF value obtained from the EPRI fragility curve. However, no failure is observed when the NLTHA of the SDOF system is carried out by scaling all 11 ground motions to $0.27\,\mathrm{g}$ PGA. Therefore, all 11 ground motions are scaled to $0.5\,\mathrm{g}$ PGA, a value greater than the HCLPF value, to perform NLTHA of the SDOF system. Then, use the principle of standard normal (Z-score) and logarithmic standard deviation of aleatory uncertainty (β_R) of the EPRI fragility curve as the standard deviation to develop the fragility curve of the SDOF system.

Step #6: ***Develop fragility curve of SSCs using Bayesian Inference***

The final step of the proposed framework involves an iterative process, updating the prior fragility curve with the NLTHA of the 3-story shear wall structure. The NLTHA of the complete structure is conducted at three IM levels corresponding to 5%, 50%, and 95% on the prior fragility curve, considering two randomly selected ground motions from the pool of 11 ground motions. The prior fragility curve is combined with the likelihood function to derive the posterior fragility curve. Iterate through this process until convergence is achieved. The convergence criterion is established based on the median capacity, and the convergence is considered to be achieved when the median capacity of the successive fragility curve falls within the $\pm 3\%$ range. Table 2 shows the estimated median capacity values and the convergence status for each iteration.

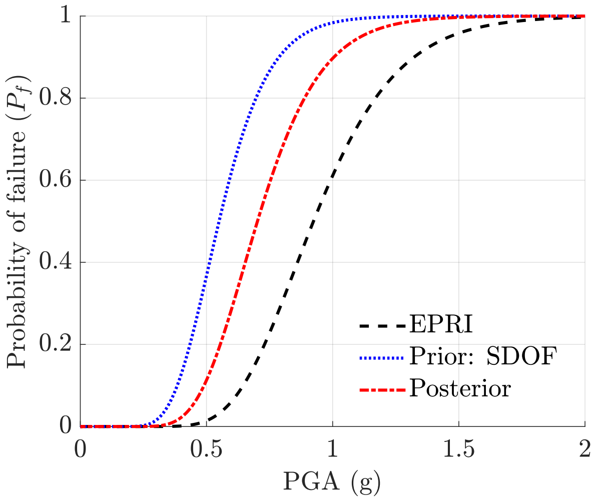
Comparison of proposed approach with conventional MSA

The posterior fragility curve derived from the proposed framework is compared with the fragility curve obtained through the conventional NLTHA of the 3-story shear wall structure for all 11 ground motions across five IM levels: $0.2\,\mathrm{g}$, $0.5\,\mathrm{g}$, $1\,\mathrm{g}$, $1.5\,\mathrm{g}$, and $2\,\mathrm{g}$. Upon extracting the NLTHA results, the failure probability for each IM level is computed. The lognormal fragility parameters are then estimated through maximum likelihood estimation (MLE)-based regression (Baker, 2015). As depicted in Fig. 5(b) and Fig. 5(c), the fragility curve based on the proposed method closely matches the conventional fragility curve developed using the MSA and MLE approach. The results presented in Table 2 demonstrate that the proposed method required only 12 NLTHAs, whereas the conventional approach necessitated 55 NLTHAs, indicating a significant reduction in computational effort.

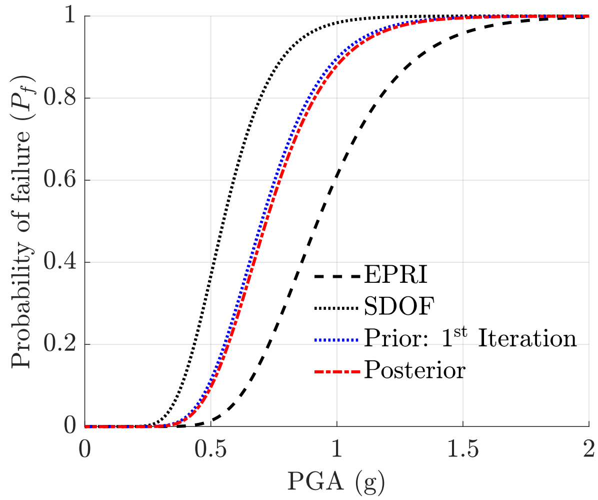
Table 2: Median capacity from all iterations

Iteration	**Median Capacity**				**NLTHA Runs***		**Error**	**Convergence Remarks**
	Prior	**Posterior**	**EPRI**	**MSA**	**Proposed**	**MSA**		
1st	$0.58\,\mathrm{g}$	$0.702\,\mathrm{g}$	$0.92\,\mathrm{g}$	$0.75\,\mathrm{g}$	6	55	–	–
2nd	$0.702\,\mathrm{g}$	$0.72\,\mathrm{g}$	$0.92\,\mathrm{g}$	$0.75\,\mathrm{g}$	12	55	2.56%	Achieved
3rd	$0.72\,\mathrm{g}$	$0.73\,\mathrm{g}$	$0.92\,\mathrm{g}$	$0.75\,\mathrm{g}$	18	55	1.39%	Confirmed

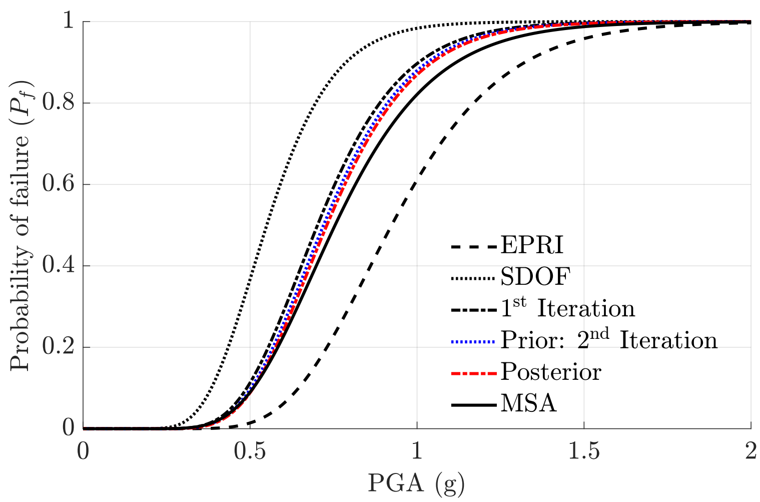
*total number of cumulative runs at the end of each iteration.



(a) After 1st Bayesian iteration



(b) After 2nd Bayesian iteration



(c) After 3rd Bayesian iteration

Figure 5. Seismic fragility curves at the end of each iteration

CONCLUSIONS

This paper presents a methodological approach for developing the seismic fragility of SSCs in nuclear facilities. The proposed methodology combines performance-based design principles with Bayesian statistics and is demonstrated through the application of a case study to a 3-story shear wall structure, which was used in the experimental study for the SMART-2013 research initiative. The key conclusions of this research are:

- The seismic fragility curve developed through the proposed methodology aligns well with the conventional analysis method.
- The EPRI methodology, often used for developing seismic fragility curves, does not always provide conservative results.
- The proposed methodology for developing an equivalent SDOF system to determine a preliminary estimate of the range for earthquake intensity measure is highly effective in reducing the number of

computationally intensive nonlinear time history analyses.

- The proposed methodology for developing a seismic fragility curve further reduces the computational effort by employing the Bayesian inference method.
- The proposed methodology requires significantly fewer NLTHA to develop the seismic fragility curve than the conventional simulation-based methods.

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