FINITE-ELEMENT ANALYSIS OF TWO-PHASE FLOWS

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SUMMARY

Safety studies for liquid cooled reactors address the questions raised by postulated failures of the cooling system. It is of importance to predict the time history of the depressurization that follows a loss of coolant accident to guarantee the integrity of the overall system. In this report the sudden depressurization of a straight pipe filled with heated and compressed water is investigated. This condition is isolated from a complicated sequence of events triggered by a loss of coolant in a water cooled reactor.

The investigated water column contained in a straight section of pipe is released suddenly at one end. After a quick initial pressure drop, vaporization occurs. At this point, the rate of depressurization becomes much lower. The pressure inside the pipe is now controlled by the rate at which the steamwater mixture escapes through the open end.

Previous analytical investigations used the fluid dynamics equations for conservation of mass, momentum and equilibrium. The resulting differential equations in time were solved by means of finite-difference operators. This investigation represents the first attempt to employ a direct stiffness finite-element approach to the solution of the depressurization of a straight pipe. The MARC Nonlinear Finite Element Analysis program provided the necessary software capabilities.

The section of the pipe is represented by a number of one-dimensional finite-elements connected in series. Due to the large range covered by the full expansion of the compressed water to a steam-water mixture at ambient conditions, large displacement and large strain effects are considered in the problem formulation. Best suited for finite-element application of this scope is the Lagrangian approach where the variables of the current state are expressed in terms of the original reference state. The stress and strain measures are those of Piola-Kirchhoff and Green. A suitable constitutive law is generated from steam table data. The equations of motion are integrated by means of the Newmark-Beta method. The pronounced nonlinearities in geometry and material present no difficulties for this solution method.

Available experimental results indicated that the initial pressure drop from the start to a level where significant vaporization resulted in an almost constant pressure, is larger than expected. Neglecting the effects due to the transient nature of the depressurization and assuming thermodynamic equilibrium, vaporization was expected to set in at the saturation pressure corresponding to the initial enthalpy. Two analyses were performed. In the second, allowance was made for non-equilibrium flow. The pressure history results obtained with this approach confirm previously obtained analytical and experimental results.

1. Introduction

The Standard Problem No. 1 is the first of a series of test cases adopted by the USAEC in 1973 to validate existing computer codes for depressurization analyses. The problem is, in general terms, posed as follows. A column of compressed and heated water contained in a straight section of pipe is released suddenly at one end. After a quick initial pressure drop vaporization occurs. At this point the rate of depressurization becomes much lower. The pressure inside the pipe is now controlled by the rate at which the steam-water mixture escapes through the open end.

Edwards(1) first treated the problem both analytically and experimentally. The experimental results indicated that the initial pressure drop from the stability point to the level where significant vaporization resulted in an almost constant pressure is larger than expected. Neglecting the effects due to the transient nature of the depressurization and assuming thermodynamic equilibrium, vaporization was expected to set in at the saturation pressure corresponding to the given initial temperature. Edwards abandoned the concept of thermal equilibrium for one where heat conduction limited the rate of vaporization. Therefore, the pressure was allowed to drop below the saturization point indicated by thermal equilibrium. The participants in the "Comparative Analysis of Standard Problems - Standard Problem No. 1"(2) were allowed to assume an initial enthalpy distribution along the pipe. This was, however, not substantiated by the experimental conditions. It was designed to produce a pressure history to match the experiment as closely as possible. Another study by H. B. Gross and R. Hoffmann(3) generally corroborated the results found by the participants and the "Comparative Analyses".

These previous analytical investigations used the fluid dynamics equations for conservation of mass, momentum and equilibrium. The resulting partial differential equations were solved by means of finite difference operators. In this investigation the finite-element method is used to formulate and analyze the problem. The MARC Nonlinear Finite Element Analysis Problem(4) provided the necessary software capabilities.

The section of the pipe investigated is represented by a number of one-dimensional finite-elements connected in series. The formulation of an individual element is based on large strain and large displacements. Current states of the continuum are described from the Lagrangian viewpoint which references stresses and

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strains to the original state. Non-linearities present no difficulties due to the incremental solution procedure. The Lagrangian approach is favored for finite-element applications because the kinematic relations remain simple. However, the description of the constitutive relationship becomes more involved. The equations of motion of the system are integrated by means of the well-known Newmark-Beta method.

The geometry and the starting conditions of the problem investigated are described in the next section. The technical approach leading to the solution is outlined in Section 3, together with the theoretical basis and the underlying assumptions. The analysis results are presented in Section 4 which is followed by the discussion and conclusions.

2. Problem Definition

A straight pipe of circular cross-section is filled with water which is then heated and pressurized. One of the end diaphragms is removed suddenly. The initial pressure drop is propagated from the open end toward the closed end in the form of a wave. This drop takes place very rapidly. As the pressure reaches a certain value at the open end, the onset of nucleation is observed. The water begins to separate into a gaseous and a liquid component. This is combined with a large increase in volume at almost constant pressure. Thus, the generation of steam inside the tube and its discharge through the open end are characterized by a pressure equilibrium. This phase is followed by a relatively slow depressurization until the pipe is empty. The pressure history of the blowdown at selected locations along the pipe is of principal interest.

The original geometric configuration used by Edwards(1) and the participants in The Comparative Analyses of Standard Problems(2) was the following. The pipe had an inside diameter of 2.88 inches and a length of 13.44 feet. The SAI(3) report showed that the short term response near the open end of the pipe could be correctly reproduced by limiting the pipe model to a much shorter length of 2.92 feet containing only the first two of the seven original monitoring stations. For the present investigation the short pipe was used. The internal diameter remained unchanged. One end of the pipe is fixed and the other end is closed off by a removable diaphragm. In Edwards' experiment a glass disc that ruptured under the impact of a bullet was used.

The initial conditions are the following: the water is heated to a temperature of 467^{O}F and compressed to 1000 psi. The corresponding enthalpy is 449.4 Btu/lbm^{O}F.

The depressurization experiments were started by destroying the glass diaphragm. This event took place in less than 1 millisecond. Edwards observed that parts of the glass disc that remained in place reduced the cross-section of the opening by an average of 13%. The analytical investigation by Gross and Hoffmann(3) showed that this reduction of the opening cross-section had only little effect on the pressure history. For this study the diaphragm was considered fully removed at the elapsed time of 1 millisecond.

The primary results are the pressure histories of the water column at the two monitoring locations. The present analysis will allow comparison of pressure versus time plots between the experiment and the analytical results. The finite-element results will also yield the displacement history of all nodal points.

3. Technical Approach

The principal features of the current approach to the solution of the Standard Problem No. 1 are the following: a) The depressurization process is modeled by the finite-element method based on the direct stiffness approach. The calculations are carried out using the MARC Non-linear Finite Element Analysis Program. b) An analogy is developed in the constitutive domain replacing quantities of stress and strain with pressure and volume. The element used is a straight linear truss element formulated for large strain and large displacements. For the constitutive relationship an analogy is developed replacing quantities of stress and strain with pressure and volume. The necessary data for the description of the constitutive law is taken from the steam tables.

The following assumptions are also made. A one-dimensional model is sufficient to represent the blowdown in a straight pipe. The process takes place in an isentropic fashion. This assumption affects the constitutive law. At the open end the pipe is considered to extend continuously at the same diameter for a distance much longer than the observed section. For the first analysis only mechanical work terms are considered. After the initial pressure loss without phase changes, a period of rapid expansion started at a pressure level much higher than was predicted by Edwards' experiment. This phenomenon was common to all numerical investigations. For the second analysis a temperature drop between the liquid and the liquid-vapor mix was established by means of an empirical correlation. This non-equilibrium model furnished results showing much better agreement with the experiment.

The basis for the analysis is given by the finite-element formulation of the nonlinear dynamic problem

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K(u)]\{u\} = \{P(t)\} \qquad (1)$$

where		
	$[M]$	mass matrix
	$[C]$	damping matrix
	$[K(u)]$	non-linear stiffness matrix
	$\{P(t)\}$	time-dependent loading
	$\{\dot{u}\}$	nodal velocity
	$\{\ddot{u}\}$	nodal accelerations

For the mass matrix the "lumped" formulation was chosen. No damping was included. The stiffness matrix reflected the non-linearites introduced by the material and by the large deflections. For this reason, [K(u)] was recalculated for each analysis step. The resulting equations of motion are integrated with the aid of the Newmark-Beta method.

In structural mechanics the constitutive law relates increments of stress to increments of strain.

$$\{\Delta\sigma\} = [D]\ \{\Delta\varepsilon\} \qquad (2)$$

For a one-dimensional formulation the above formula is simplified to a single equation with D representing the slope of the uniaxial stress-strain curve. For the Standard Problem No. 1 the increment of stress $\Delta\sigma$ is replaced by Δp, the increment of pressure and the linear strain increment $\Delta\varepsilon$ is replaced by $\frac{\Delta V}{V_o}$ representing the normalized increment of volume. The constitutive coefficient D which depends on the current volume V is derived from steam table data.

The finite-element discretization of the one-dimensional continuum consisted of 15 elements of equal size. The constitutive law was input as a continuous piecewise linear function through specification of 35 pairs of data representing slopes and breakpoints.

For this study two analyses were completed. The first one assumed thermal equilibrium where nucleation would start at the saturation pressure corresponding to the uniform initial temperature. The participants in the "Comparative Analyses of Standard Problems - Standard Problem No. 1" assumed a uniform enthalpy distribution over the length of the tube. The results did not compare well with the experiment. For a second series of analyses an initial enthalpy distribution was determined according to the experimentally observed pressure plateaus reached after the steep initial pressure drop. This was effected without any rational explanation. Edwards who had recognized the need for including heat transfer considerations in his analysis disagreed with this expedient.

A second analysis incorporating an empirical formula for the temperature drop between the liquid and the liquid-vapor mix was carried out. In this non-equilibrium model the temperature difference is due to the heat flux between the two phases. The literature contains many empirical correlations yielding the temperature drop in non-equilibrium flow. In the current study we have adopted the relation by Jens and Lottes(5)

$$\Delta t_{sat} = \phi 8^{0.25} \ \exp \frac{p}{62} \qquad (3)$$

where ϕ = heat flux in
and p = pressure in bars

The above relation was given for boiling water at high pressure in low quality forced convection boiling. These conditions are similar to the ones encountered in this study. The sequence of calculations is as follows:

1. A pressure drop of Δp below the saturation pressure indicated by the initial enthalpy is postulated. From the steam tables the corresponding drop in enthalpy Δh is found.
2. The heat flux ϕ across a fixed observation plane at the tip of the water column is calculated by multiplying the mass flux found from the analysis by the enthalpy change Δh.
3. The value for heat flux ϕ and current pressure p is substituted into the empirical formula and the drop in the saturation temperature Δt_{sat} is evaluated.
4. The drop in saturation pressure associated with Δt_{sat} is found from the steam tables and compared with the initially assumed Δp.
5. If the correlation is unsatisfactory, a different Δp is used and steps 1 through 4 are repeated.

An accurate simulation of the system is expected for the following reasons:

1. In order to solve the equations of motion the Newmark-Beta operator is used. This method has proven its reliability in many previous applications. Complete flexibility exists in the choice of time steps to adapt to the current behavior of the system. Residual load correction is included to prevent the accumulation of non-equilibrium errors. Reassembly of the dynamic matrix is forced for each analysis step to adequately represent non-linearities.
2. A simple one-dimensional element is used. This keeps the analysis simple and transparent.
3. The overall behavior of the modeled system is well known. The current solution can be checked against existing solutions.

4. Results

Two different analyses have been performed. The consideration of interphase heat transfer yielded a significantly different constitutive relationship for the second analysis. Both were started from the same physical configuration.

The results presented here include the pressure versus time plots for two stationary locations referred to as GS-1 and GS-2. Nodal point pressures were averaged from the neighboring element pressures. In order to evaluate the pressure at GS-1 or GS-2 a third order polynominal defined by the pressure at four nodal points was used. Two nodes each were located on both sides of the monitoring station.

2) Analysis I

The temperature of the compressed water column is 467°F and the enthalpy of the corresponding saturation state is 449.4 btu/lbm°F. The pressure-time history in Fig's. 1 and 2 shows that the pressure plateau reached after the first millisecond is 500 psi, the saturation pressure corresponding to a temperature of 467°F.

The agreement between the present analysis and the Aerojet Nuclear RELAP3 results is good. The slower initial decompression is probably due to the linear release of the balancing force taking place during the first millisecond. After reaching the saturation point the present pressure history was very smooth and did not show the temporary pressure buildup indicated by the RELAP3 results.

3) Analysis II

According to the non-equilibrium model discussed in Chapter 3, a drop in the saturation pressure from 600 psi to 360 psi was assumed. The enthalpy loss was 36.7 Btu/lbm^{0}F and the saturation temperature dropped by 32.6^{0}F. The change in the saturation temperature predicted by the empirical formula (Eq. 3) is 30.7^{0}F. The discrepancy between the assumed and the calculated value of Δt_{sat} is 5% which is thought to be within the range of reliability for the empirical formula used. The results produced by this analysis compared much better with the experiment.

For both monitoring stations the observed pressures fell between the experimental results and the Aerojet Nuclear RELAP3 results. Again, the initial decompression was slower due to the linear release of the balancing force. Pressure oscillations predicted by the RELAP3 results were not confirmed by the present analysis.

5. Discussions and Conclusions

The numerical simulation of the Standard Problem No. 1 by means of finite-elements requires modeling of large geometry changes, non-linear strains, and transient response. The element formulation and the constitutive law reflected the large changes in geometry causing large strains. Short time-steps of .05 milliseconds were used to cover the steep drop in pressure at the start of the analysis. At later stages the step size was increased to 0.2 milliseconds.

The present approach did not require any assumptions with respect to wall friction, exit conditions, and other parameters pertinent to fluid dynamics solutions. Future analyses should cover the full length of the pipe and the full duration of the blowdown.

Conclusions drawn from the current investigation are the following:

1. The modeling capabilities of the MARC Non-Linear Finite Element Analysis Program permit numerical simulation of the blowdown in a straight pipe.

2. The present structural dynamics solution using finite elements leads to results of the same quality level as obtained by fluid dynamics solutions.

3. The assumption of thermal equilibrium between the liquid and the mixed liquid-vapor phase leads to pressures that are too high. Edwards' contention that interphase heat transfer had to be considered is confirmed by this investigation.

4. An empirical formula was used to predict the drop in the the saturation temperature due to heat transfer considerations. The agreement with the experimental results was much proved.

ACKNOWLEDGEMENT

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References

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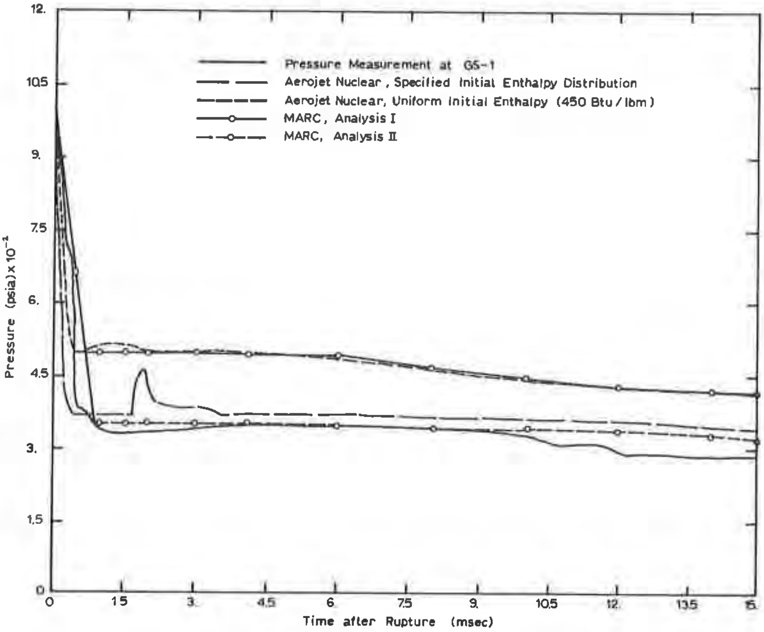

Figure 1 -- Pressure-Time Diagram at GS-1 for Analyses I and II

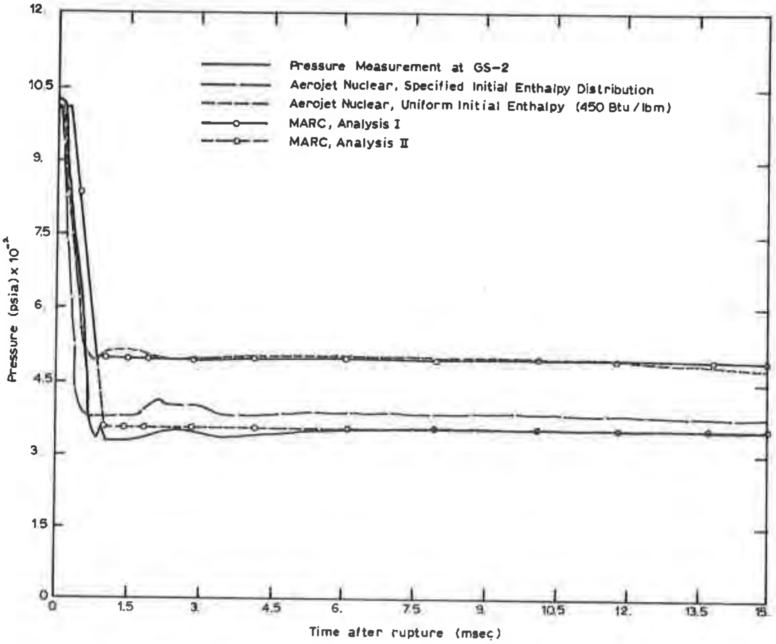

Figure 2 -- Pressure-Time Diagram at GS-2 for Analyses I and II