SURVEY AND EVALUATION OF THE FINITE ELEMENT METHOD IN LINEAR FRACTURE MECHANICS ANALYSIS

R.H. GALLAGHER,

Department of Structural Engineering
School of Civil and Environmental Engineering,
Cornell University, Ithaca, New York, U.S.A.

ABSTRACT

Approaches to finite element fracture mechanics analysis are classified and reviewed. Four categories are defined for linear fracture mechanics: direct, energy-based, superposition, and singularity function methods. Advantages and limitations of the respective methods are discussed and avenues of approach to more efficient and accurate analyses are identified. A separate section is devoted to inelastic fracture mechanics analysis.

1. INTRODUCTION

The role of fracture mechanics analysis in the design of nuclear structures is well established. Various sessions of this conference are devoted to this topic, its design aspects have been surveyed by Mager [1], and an international conference [2] recently focused upon its implications in pressure vessel technology.

The finite element method is also well established as an independent field of activity. Only recently, however, has attention been drawn to the value of finite element analysis in fracture mechanics. Classical approaches to the necessary determination of the stress intensity factor for linear fracture mechanics (Irwin [3], Sneddon and Lowengrub [4], and Paris and Sih [5]) have been and continue to be appropriate for this purpose in the presence of simple geometries and load conditions. Nevertheless, for complicated situations, numerical methods come to the fore and the most generally applicable of these is the finite element method.

Despite the relative novelty of finite element analysis as a tool for fracture mechanics studies -- the earliest papers appeared approximately three years ago -- a bewildering array of separate studies have gained publication. Equally remarkable is the diversity of approaches represented by these papers. The objective of this paper, therefore, is a classification and review of the various approaches to finite element fracture mechanics analysis and, to a limited extent, an assessment of their advantages and limitations.

The field of analysis for fracture phenomena is conveniently divided into two classifications: "linear", or elastic, fracture mechanics, and inelastic analysis. The attention of design practitioners is exclusively directed towards the former and this situation is reflected in the predominance of finite element publications for linear fracture mechanics. Four classifications of these efforts are identified and discussed under as many separate headings in the sections to follow. These headings are: (a) direct, (b) energy-based, (c) superposition, and (d) singularity function methods. A single section is devoted to inelastic fracture mechanics studies. The advantages and limitations of the respective approaches are then described.

Anticipating a more thorough coverage of the direct and energy solutions in the paper at this conference by Rowe [6], the coverage of these aspects in the following is somewhat limited in comparison with the extensive amount of published data. Also, in the discussion of linear fracture mechanics, the specific treatments are limited to consideration of the K_I class of stress intensity factor. As will be apparent, however, the general methodologies of the various schemes are in no way limited in applicability to this mode of crack extension.

2. DIRECT METHODS

"Direct" methods involve the performance of a straightforward finite element analysis with a high degree of refinement in the region of the crack tip and seek determination of the stress intensity factor by interpretation of the calculated stresses or displacements. Interpretation of some form is needed since, due to its "finite" nature, the conventional finite element representation (in contrast with classes of representation discussed in Sections 4 and 5) is incapable of producing a direct evaluation of the crack tip stress intensity factor.

We consider first the direct methods which operate upon the calculated joint displacements. One approach, the crack-opening-displacement method (Kobayashi, et al [7]), is based upon an examination of the opening displacement of the crack at a point near the crack tip (see Fig. 1) using the equation for displacement (Irwin [3])

$$v = \frac{K_I}{G} \sqrt{\frac{r}{2\pi}}\, f_1(\theta,\mu) \qquad (1)$$

where G is the shear modulus, μ is Poisson's ratio, and θ and r are polar coordinates. Solving for K_I

$$K_I = \frac{Gv}{\sqrt{\frac{r}{2\pi}} \cdot f_1(\theta,\mu)} \qquad (2)$$

where, for example

$$f_1(\theta,\mu) = \sin\frac{\theta}{2}\ \frac{2}{(1+\mu)} - \cos\frac{\theta}{2} \qquad (3)$$

Miyamoto [8], at this conference, employs the same approach in three-dimensional linear fracture mechanics analysis. The analytical model is defined with use of tetrahedronal elements and a 2-step process, in which a coarse grid of the complete structure is analyzed, followed by the analysis of the region of the crack with a refined grid, is introduced.

In an attempt to supplant still further the requirements of grid idealization through use of interpretive procedures, Chan, et al [9] apply an extrapolation scheme. See Figure 2. A radial line at a fixed value of θ is first established and by finite element analysis and use of Equation 2 the stress intensity factor (K_I) is calculated at points along the line. Then, by extrapolation of a line through the so-determined points on a K_I vs. radial distance plot, the value of K_I at zero radius is established.

The method of extrapolation has been studied by Anderson, et al [10] and by Watwood [11]. The work of Reference 10 includes comparisons with alternative methods; the conclusions reached will be cited in Section 7.

Alternative to the interpretation of displacement values to obtain K_I is the interpretation of stresses for this purpose. The interpretation of stress is effected in the manner of the above-described procedures. The stress field in the vicinity of the crack tip is described by

$$\sigma_r = \frac{K_I\ f_2(\theta)}{\sqrt{2\pi r}} \tag{4}$$

and solving for K_I:

$$K_I = \frac{\sigma_r \sqrt{2\pi r}}{f_2(\theta)} \tag{5}$$

This expression is evaluated at various locations along a given radius and the results extrapolated in order to define K_I as r approaches zero. This procedure is due to Chan, et al [9] and has also been applied by Watwood [11]. The commonly-employed constant-stress elements give an erratic basis for interpretation. If linearly varying stress elements were used the problem would be diminished; Figure 3 shows how the stresses might be interpreted in evaluation of K_I.

3. ENERGY METHODS

A variety of choices is available under this general heading, as follows: (a) total energy method, (b) local energy method, and (c) line integral method.

The total energy approach is based upon the following relationship between the rate of change of strain energy with respect to crack length ($\frac{\partial U}{\partial a}$) and the stress intensity factor (Irwin [3])

$$\frac{\partial U}{\partial a} = \frac{K_I^{\,2} t}{E} \tag{6}$$

where t is the plate thickness.

Expressing $\frac{\partial U}{\partial a}$ in terms of finite differentials and solving for K_I, we have

$$K_I = \sqrt{\frac{E\Delta U}{t\Delta a}} \tag{7}$$

Hence, in this procedure (see Figure 2) one performs an analysis for a stipulated crack length (a) and computes the strain energy U. The crack is then opened a distance Δa, a new strain energy (U + ΔU) is computed.

The procedure, then, is to perform two analyses of a crack situation which differ only in the increment of crack size. The resulting difference in strain energy is not as sensitive to grid refinement in the vicinity of the crack as is the determination of stress intensity factors on the basis of the "direct" methods and a relatively coarse mesh can be used in each analysis.

Finite element applications of this scheme are described by many authors, including Anderson, et al [10], Watwood [11], Dixon and Pook [12], Deverall and Lindsey [13], and Mowbray [14]. The survey by Rowe [6] at this conference comprises a more detailed and comprehensive examination of the total energy approach.

It is of interest to note, in passing, Watwood's [11] demonstration that the value of stress intensity factor calculated in a valid minimum potential energy solution is a lower bound on the actual value when K_I increases with the crack length. It is well known, of course, that the predicted strain energy is a lower bound for a given solution, but this gives no measure per se of the difference in strain energy in successive solutions.

An expression relating the stress intensity factor to the strain energy contained within a region surrounding the crack tip, originally presented by Irwin [3], forms the basis for the local energy method (Deverall and Lindsey [13]). The subject expression is of the form

$$U = \frac{(5-3\mu)}{8E} r_1 K_1^{\ 2} \tag{8}$$

or, as an equation for the stress intensity factor

$$K_I = \sqrt{\frac{8EU}{(5-3\mu)r_1}} \tag{9}$$

Thus, a circle of radius r_1 is inscribed with origin at the crack tip (see Figure 4a), the strain energy of all elements within the circle is computed, and K_I is calculated from Equation (8).

Only a single computation is needed to determine K_I for a given crack length. On the other hand, difficulties are introduced in the isolation of the data of interest in a large scale analysis, considerable refinement is required in the vicinity of the crack, and the circular boundary must be correlated with elements whose boundaries are generally straight lines. No substantive studies of the accuracy of the method have achieved formal publication as of this date.

The line integral, or path-independent integral method was introduced into linear fracture mechanics by Rice [15]. Similar to the local energy method, but with no restriction on the shape of the region studied, a region containing the crack tip is delineated as in Figure 4b. Rice [15] shows that the following integral, taken over the boundary (Γ) of the region is equal to a constant J:

$$J = \int_{\Gamma} (U\,dy - T \cdot \frac{\partial u}{\partial s}\,ds) \tag{10}$$

where $\bar{U}$ is the strain energy density, T is the traction vector defined according to the outward normal along Γ, and u is the displacement vector. ds is the element of arc along Γ. The stress intensity factor for plane stress is related to J by the expression

$$K_I = \left[\frac{JE}{(1-\mu^2)}\right]^{1/2} \tag{11}$$

Delineation of the isolated region around the crack and the determination of the line integral requires significant modification of existing finite element programs. A finite element application of this concept is described by Chan, et al [9]. It has also been applied in the evaluative study of Anderson, et al [10].

4. SUPERPOSITION METHODS

The superposition of classical and finite element solutions, as advocated by Yamamoto [16][17] and by Morley [18] seeks to compute the stress field with the coarsest finite element idealization.

The concept is to define the classical solution for a problem as close to the problem of interest as possible. For example, in the present case, the solution for a crack in an infinite plate subjected to biaxial stress is first obtained. This solution will not satisfy all conditions for the actual problem and disparities can be interpreted as body forces and edge loadings. The body forces and edge loadings are applied to the finite element analysis in reversed direction and the results from the finite element and classical solutions are superimposed.

The general scheme is sketched in Figure 5. The classical solution is denoted by σ_a, while finite element solutions are given by σ_r and σ_e. The appropriate superposition of solutions yields the "exact" answer σ^*.

Papers published to date (Yamamoto [16], Morley [18]) do not deal with sharp cracks. The more recent, as-yet unpublished work of Yamamoto and Tokuda [1], however, solves the problem of edge-cracked plates and achieves agreement with alternative solutions (Isida, et al [19]) to within 1%.

5. SINGULARITY FUNCTION FORMULATIONS

One of the most appealing approaches, from both a theoretical and computational view, is to formulate a special element containing a singularity, to be employed in the region of the crack tip. Regions beyond the crack tip are represented by conventional elements. Many attempts to exploit this concept have appeared, including the papers at this conference by Carmignani and Cella [20], and by Rao, et al [21].

A sophisticated singularity function element formulation is due to Byskov [22], who constructs the basic element relationships with use of the complex stress functions of Muskhelishvili [23]. The basic stress field is transformed into relationships in terms of element joint displacement parameters and a stiffness matrix representation is established. Because of the origin of the formulation in an assumed stress field the conditions of interelement continuity of displacement are not met when the so-derived element is joined to a conventional finite elements. Although the approach is described in general terms and draws upon the generality of numerical integration in numerical evaluation of the element stiffness coefficients only triangular elements of the type shown in Figure 6 are used by the author in illustrative numerical solutions.

The singularity element described by Rao, et al in Reference [24] is less restrictive in geometric representation. Using the "hybrid" approach to element formulation (Pian and Tong [25]), which permits basic representation in terms of complementary energy but effects transformation to stiffness format, the authors employ a series representation of the stress field. The series accounts for the state of stress at the crack tip; the geometric representation includes a circular element. The paper in these conference proceedings (Rao, et al [21]) describes this work and extensions beyond the developments of Reference [24].

The representation described by Levy, et al [26], formulated for elastoplastic fracture mechanics analysis, represents an alternative singularity formulation. The "basic" element, shown in Figure 7, consists of a transformation into polar coordinates of the familiar rectangle based upon a linear edge-displacement field (see Raju and Rao [27] for stiffness coefficient details). The singularity is achieved by allowing the joints a and b to coalesce into joint 1 as the element is used as a "near tip element" with joints 1-2-3. This scheme yields a $\frac{1}{r}$ singularity of the type associated with inelastic crack tip phenomena, in contrast to the $1/\sqrt{r}$ singularity of linear fracture mechanics.

It is of interest to observe that many possible displacement "shape functions" can be devised to reflect a singularity at a joint. For a triangular element, at the crack tip for example (Figure 7), the following polar coordinate function for the radial displacement u meets this condition

$$u = (1 - \frac{r}{r_o})\, u_1 + (\frac{r}{r_o})(1 - \frac{\phi}{\phi_o})\, u_2 + (\frac{r}{r_o})(\frac{\phi}{\phi_o})\, u_3 \qquad (12)$$

This singularity is also of the 1/r type, however, and clearly there is a difficulty in effecting transition of geometric description at the outer (circular) boundary.

6. INELASTIC ANALYSIS

Finite element analysis possesses special advantages in elastoplastic analysis where opportunities for classical solutions are very limited and some form of numerical analysis is required for virtually all circumstances. A description of progress in the development of this aspect of the finite element method is beyond the scope of this paper; the interested reader is advised to consult the excellent survey by Marcal [28]. The significance of elastoplastic studies in the prediction of fracture is reviewed by Rice and Drucker [43]. Our purpose here is simply to cite efforts towards the analysis of inelastic behavior in the region of the crack tip.

One of the most thorough treatments of the subject problem is due to Levy, et al [26]. As noted in the previous section, the 1/r singularity hypothesized for stresses in the presence of plastic deformation at the crack tip is represented in this work.

Another attempt at the representation of the crack tip singularity is due to Hilton and Hutchinson [29], who employ an analytical model of the singularity in a circular region surrounding the crack tip and represent the structure outside this region with conventional finite elements. Swedlow [30] and Tuba [31] perform finite element elastoplastic analyses for cracked and notched plates, basing their representations entirely upon conventional elements.

The nature of elastoplastic deformation phenomena at the tip of cracks leads naturally to the desirability of three-dimensional stress analysis. Two-dimensional analytical models continue to predominate in linear fracture mechanics; 3-D analyses are described by Miyamoto [8] at this conference and by Johnson [32]. A significant share of the few elastoplastic finite element analyses have been of 3-D form, however.

Miyamoto, et al [33] have examined plastic flow in the zone of a crack, using tetrahedronal elements. Levy [34], and Visser, et al [35] have undertaken studies which attempt to include considerations of stress components in three directions.

7. ASSESSMENT OF METHODS

The following comments, which apply only to the linear fracture mechanics methods described in Sections 2-5, are intended as a summary of the more apparent advantages and limitations of the respective methods and give reference to results of studies which have compared these methods. The measures of value of the respective methods must principally pertain to computational efficiency, rather than accuracy, since improved accuracy is always possible at increased cost. Furthermore, account must be taken of the possibility of required additional research or major modifications of existing computer programs to establish working capabilities for practical analysis.

The direct methods are appealing in that any available computer program can be employed "as is". Available evidence reflects unfavorably on the economics of such methods, however. The use of 200 to 1000 elements has been found necessary if an accuracy within 5% is to be realized in a plane stress problem. This degree of refinement is economically unacceptable for a simple geometry and load condition and is unquestionably so for more complex geometries and three-dimensional stress situations. This disadvantage is reduced if an analysis is first performed for a coarse grid of the complete structure and then the region of the crack is studied in detail, using the internal stresses calculated from the coarse grid for applied loads in the latter (Miyamoto [8]). The data processing requirements are amplified by this scheme, however.

Among the direct methods the extrapolation procedure on displacements is clearly preferable when the element formulations are based upon assumed displacement fields. Considerable reliability is introduced through the device of extrapolation; many more elements must be employed in the crack-opening-displacement method to obtain as accurate a result. Extrapolation upon stresses may prove more profitable if the finite element model is an "equilibrium field" model (see paper by Gallagher and Dhalla [36], this conference).

Advantages of the direct approaches include the possibility for improved efficiency through use of "higher order" finite elements and automatic mesh-generation schemes. The former is discussed in these proceedings by Rowe [6]; available programs may require significant modification to include such elements. The automatic mesh generation scheme enables considerable refinement in the vicinity of the crack tip, as in the papers by Chan, et al [9] and Anderson, et al [10]. This consideration would appear to deserve closer examination, however. A rigorous mathematical proof of convergence generally depends upon successive refinements throughout the complete domain, in which case interpretation of successive solutions yields only a general indication of rate of convergence. Presuming that the element and system representation does not violate any of the conditions on a valid finite element analysis, however, convergence in the limit is assured.

Among the energy methods, the total energy scheme is the simplest to apply. The line integral and local energy schemes require substructuring capabilities in the finite element analysis system of the user and have not been particularly accurate for refined grids.

Various studies ([9], [10], [13]) have effected numerical comparisons of procedures in the direct and energy-based categories. It is of interest to note that Anderson, et al [10] found that the order of merit of methods in the two categories could be interchanged by appropriate definition of the finite element grid (node spacing). Virtually all studies discussed herein rely upon the simplest type of element (triangle with linear edge displacements). Thus, conclusions reached in comparison studies may be altered by use of higher-order elements.

The superposition method is highly efficient from the view of computer costs, but requires formidable expense in data preparation. It does not appear that the process can be automated for the general situation.

Singularity function elements represent the most promising of all approaches. In general, these require that the program into which they are to be incorporated be able to accommodate procedures for numerical integration of the individual element stiffness coefficients.

8. CONCLUDING REMARKS

Cumulative evidence supports the view that either the energy-based or singularity function procedures are preferable for stress intensity factor determination in linear fracture mechanics, the choice being dependent upon the form of available software. For users of widely-available computer programs, (Gallagher [37]) the total energy method is immediately applicable. Computational efficiency can be greatly enhanced through incorporation of new and more sophisticated elements in the program. In the limit of element sophistication one arrives at the singularity function elements.

Few of the widely-available computer programs possess "substructuring" capabilities to enable isolation of the region around the crack for detailed treatment. A capability of this type yields major efficiencies, as demonstrated by users of special purpose programs which incorporate it. Indeed, the essential theory of linear fracture mechanics and such related theoretical developments as Rice's [16] path-independent integral formulation make clear the appropriateness of study of the region of the crack; the analysis of the larger structure serves only to define loads upon the region of the crack.

The present survey is far from complete in identification of candidate methods of numerical analysis of linear fracture mechanics. Contributions presented here (e.g., Giencke [44], Bazant [38]), and at recent conferences (e.g., Hartz and Chopra [39]) are not yet available in the printed form for study. Other contributions will shortly appear (e.g., Pian, et al [40]). Additionally, contributions have been made to this topic via special forms of discrete element analysis (Mohraz, et al [41]) and through studies which deal with the cracking of concrete (Nilson [42]). A broader survey of the topic, including extensive numerical comparisons, is highly desirable. It should be emphasized, nevertheless that acceptable methods are already at the disposal of the practitioner and that improvements upon these can be accomplished as outlined above.

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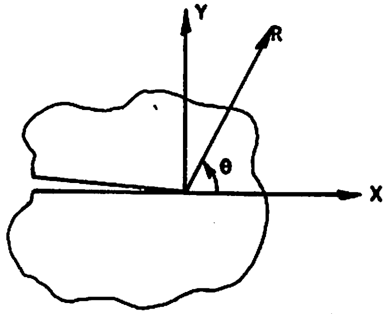

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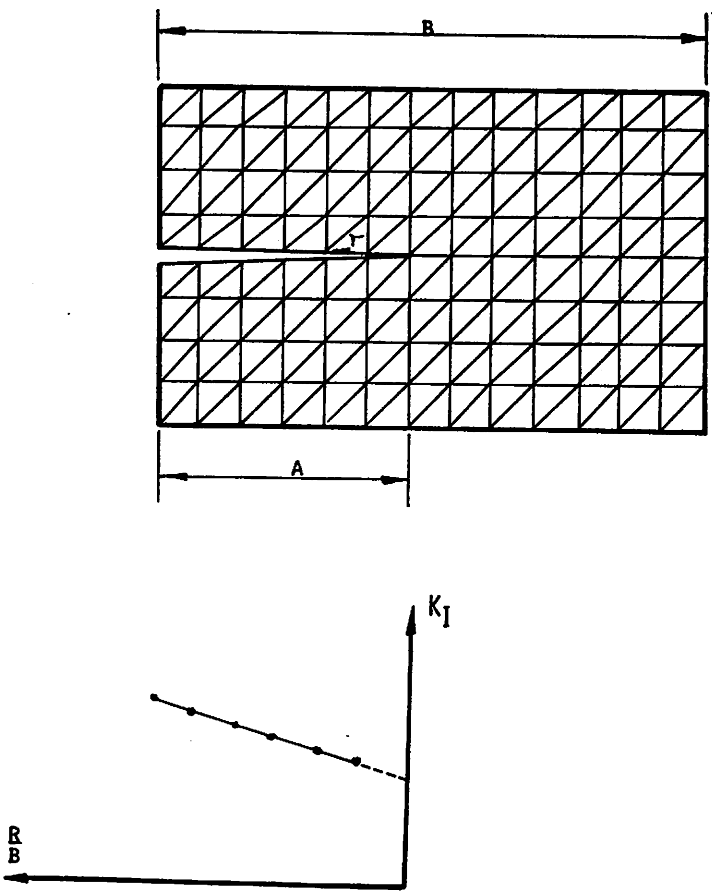

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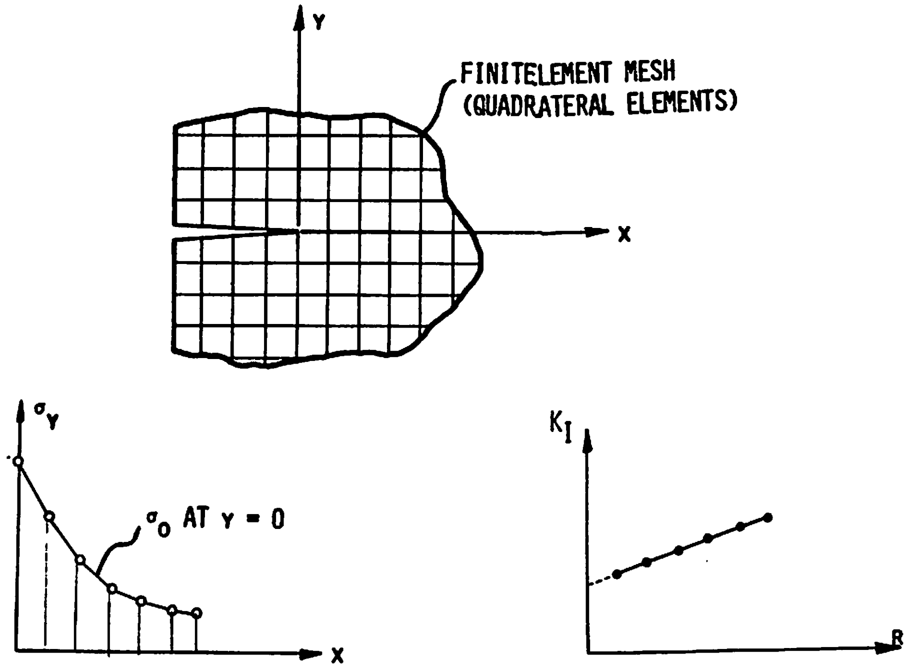

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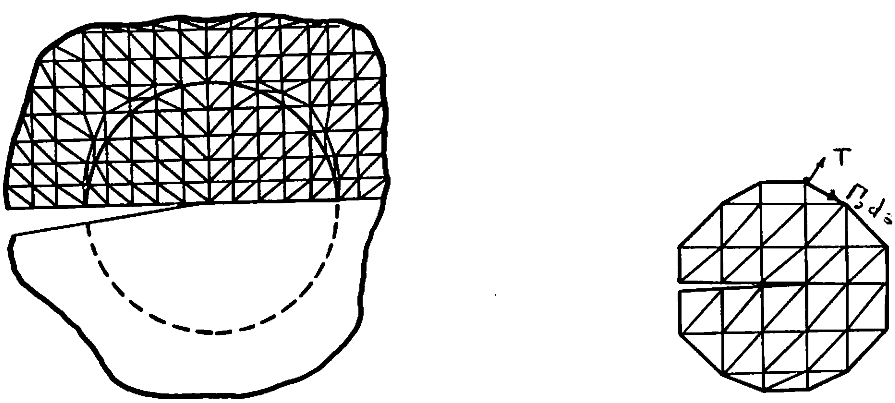
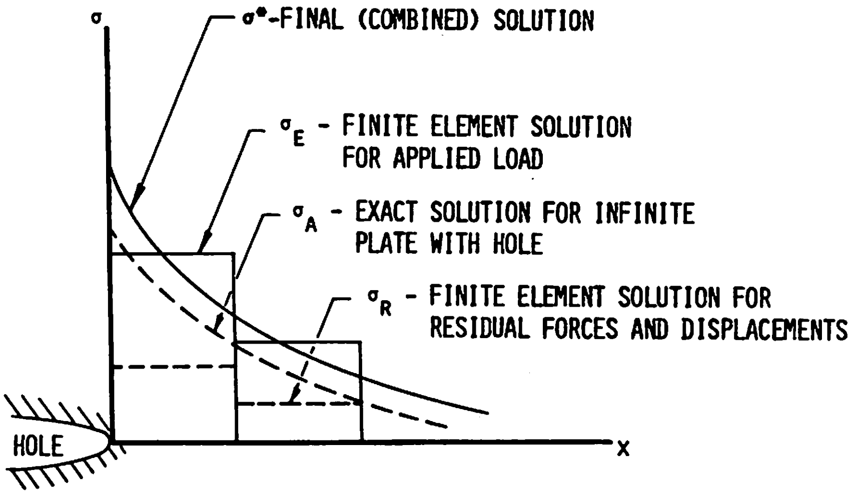
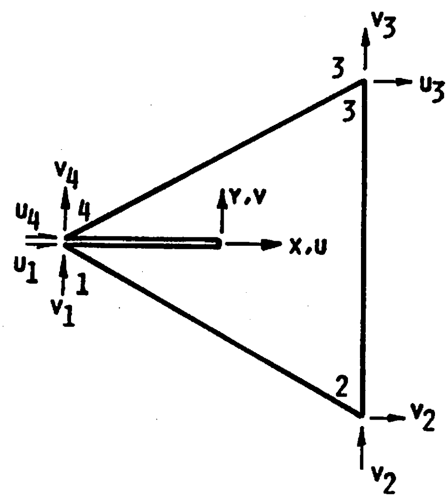


A. LOCAL ENERGY METHOD B. LINE INTEGRAL METHOD

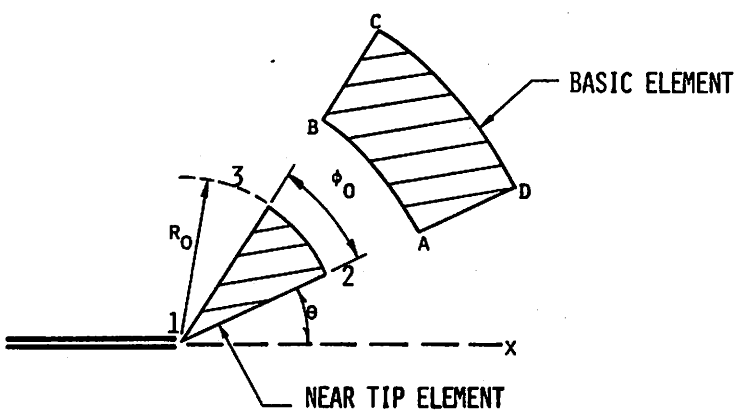
4. ENERGY METHODS BASED ON LOCAL REGIONS.



5. SUPERPOSITION METHOD (REFERENCE 16).



6. TRIANGULAR SINGULARITY ELEMENT (REFERENCE 22).



7. SINGULARITY ELEMENTS IN POLAR COORDINATES.

DISCUSSION

Q

D. COSTES, France

May I know if three-dimensional codes with plasticity have allowed to verify the linear elastic fracture mechanics (L. E. F. M.) assessments ?

A

R. H. GALLAGHER, U. S. A.

Three-dimensional analyses of crack tip strain distributions, including plasticity, have been published by Miyamoto (8, 33) and by Levy (26, 34). I would say that the work reported to date is not of sufficient scope and depth to assert that a verification of linear fracture mechanics has been achieved. On the other hand, a limited verification of prior hypotheses related to inelastic fracture mechanics has been accomplished.

Q

P. O. SCHILDKNECHT, Germany

How do you determine whether the crack is a tension or a shear crack, in other words how do they differ in affecting the stiffness matrix modification ?

A

R. H. GALLAGHER, U. S. A.

A discussion of procedures for distinguishing between K_I and K_{II} stress intensity factors is given by Kobayashi et al. (7). In general, other papers cited in this survey give no explicit treatment of this question.