ABSTRACT

HSU, DE FEN. Essays on Mechanism Design and Social Network Dynamics in Health and Education. (Under the direction of Thayer Morrill and Melinda Morrill).

This dissertation consists of two essays analyzing how individuals and institutions interact with mechanisms and interventions, using both theoretical and empirical approaches. These essays showcase two lines of my research: the first focuses on health and social networks, while the second addresses challenges in school choice mechanisms.

Chapter 1 investigates individual decisions to participate in programs aimed at improving adolescent well-being, motivated by observed disparities in participation by socioeconomic status. These interventions not only influence behaviors but also alter participants' social network dynamics. I examine ways to increase participation rates among currently underrepresented groups – particularly those from low socioeconomic backgrounds – who are more at risk for mental health disorders or engaging in risky behaviors. Research on adolescent health and well-being emphasizes that successful behavioral health interventions, such as Positive Youth Development (PYD) programs, hinge on fostering connections among participants and creating environments that influence network dynamics. This implies that policy approaches leveraging social networks to enhance intervention uptake must also account for how networks evolve endogenously in response to participation. Yet, existing literature primarily assumes exogenous and fixed social networks. Based on a structural action-network model and restricted Add Health data, I capture the quantitative significance of interdependence between intervention participation and social network dynamics through a Bayesian Markov Chain Monte Carlo technique. My counterfactual analysis identifies the role of social network dynamics in enhancing intervention efficiency and shows that incorporating network dynamics into target strategies can nearly double uptake and improve efficiency by better reaching those who benefit most in some contexts.

This work provides insight into policy approaches that leverage social networks, depending on how networks evolve in response to treatment participation; it also has important implications for social determinants of health and social capital formation. Focused on PYD programs, my work contributes to the PYD literature by examining how endogenous participation decisions influence sample generation, thereby aiding in the accurate interpretation of existing results on the effectiveness of PYD programs.

The second chapter, coauthored with Dr. Thayer Morrill, addresses potential endogenous capacity problems within the Wake County school system. Unlike most existing literature that assumes fixed and exogenous capacities for each school, our study considers the dynamic

nature of capacities. In Wake County, students are initially assigned to a base school based on geographic location but are allowed to apply for reassignment through magnet program application process. When students are successfully reassigned, they free up seats at their base school, making capacity inherently endogenous to the assignment process itself. This dynamic can lead to unfair outcomes and incentivizes students to strategically misreport their preferences. This issue is particularly exacerbated by Wake County's design of a 10% lottery policy, wherein 10% of the seats at each magnet school are allocated through a lottery.

Chapter 2 characterizes and provides the conditions under which endogenous capacities by a mechanism must meet to ensure the system is strategy-proof and fair. These conditions build upon and expand those established in prior studies, offering a more nuanced understanding that specifically focuses on the capacity aspect of school assignments. Our findings have significant theoretical implications, suggesting the possibility of multiple strategy-proof and fair assignment mechanisms.

This dissertation explores two important mechanisms influencing young individuals – social networks and school choice. The findings contribute to our understanding of how peer dynamics shape behavioral health intervention participation and how the design of school assignment mechanisms affects fairness and access to educational opportunities. These insights can inform public policy decisions aimed at improving adolescent well-being, guiding the design of programs that leverage social networks and ensure equitable access through transparent, strategy-proof assignment systems.

Essays on Mechanism Design and Social Network Dynamics in Health and Education

by
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BIOGRAPHY

De Fen Hsu, who goes by Ariel, was born in Yuli, Taiwan. She earned a Bachelor of Arts in Economics from National Chung Cheng University. She entered the Ph.D. program in Economics at North Carolina State University, where she also earned a Master of Arts in Economics en route to the doctorate. Her research interests include mechanism design, labor economics, and health economics. She aims to inform and improve public policy and interventions through evidence-based analysis and innovative mechanisms.

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This dissertation marks a deeply meaningful milestone in my life, both academically and personally. The journey of earning a Ph.D. has been full of challenges, growth, and joy–and I am immensely grateful for everyone who has supported me along the way.

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To my family, your support has been my foundation. I thank my dad for his steady encouragement and belief in me, and my siblings for their companionship along the way. Above all, I am deeply grateful to my grandmother, whose unconditional love and support gave me strength through every difficult moment.

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INTERVENTION IN NETWORKS: EVALUATING POSITIVE YOUTH DEVELOPMENT PROGRAM PARTICIPATION IN THE U.S.

1.1 Introduction

Recent studies have increasingly focused on the role of social influences in shaping health behaviors (Rutter et al. 2017). This has given rise to research on network-based targeting strategies, which aim to identify key individuals within networks who can drive widespread behavioral change and enhance information diffusion (Kim et al. 2015; Badham et al. 2018).[1] These strategies provide valuable insights into selecting targets for health interventions and exploiting social network, ultimately improving the efficiency of health policies. However, interventions or treatment designed to change behaviors may, in turn, alter the networks themselves, as intervention programs can create environments that influence the overall dynamics of partici-

[1]These strategies are also frequently studied in the fields of finance (Banerjee et al. 2013), agriculture (Cai et al. 2015), and communication (DeGroot 1974).

pants' networks. Given the interdependence between treatment participation and network dynamics, how should a policy be designed to optimally leverage social networks to encourage participation among the target population? Additionally, how can we evaluate the efficacy of the treatment – the potential gains for participants from the treatment – in this context?

This paper investigates individual decisions to participate in interventions and offers insights into which policy approach to take, depending on how social networks evolve in response to treatment participation. Friends typically connect due to shared commonalities, which can be exogenous characteristics (e.g., race, gender identity) or shared behaviors (e.g., engaging in the same activities). If individuals prefer to form friendships based on similar exogenous characteristics, then these connections may endure even when behaviors change. In such cases, policymakers can target individuals with more connections to encourage overall participation. If individuals are more connected due to shared behaviors, then interventions aimed at altering those behaviors may not enhance take-up efficiency through social networks. This is because changes in behavior might cause individuals to lose connections, as links formed through shared activities may dissolve when participation behaviors change. In such scenarios, any effort to leverage social networks must also focus on strengthening these friendship links first.

Network formation perspectives in the context of behavioral health interventions are critical not only for deriving important policy implications but also for addressing potential threats to the identification of peer effects. Essentially, peer groups are not formed randomly, and including peer effects in the model makes my approach subject to the *correlated effect* and *reflection problem* (Manski 1993). Introducing the network formation perspective helps distinguish between impacts driven by social influence and those resulting from friendship selection based on an individual's intrinsic characteristics. I closely follow the structural action-network model developed by Badev (2021) to incorporate network formation and a game-theoretic framework into the individual treatment decision equation.

Applying this model, I study the participation decision of Positive Youth Development (PYD) – an evidence-based intervention widely adopted in various settings, including after-school programs, mentoring initiatives, and national youth organizations – using a restricted dataset from the National Longitudinal Study of Adolescent to Adult Health (Add Health). In this paper, PYD program participants are identified as individuals who engage in Scouting or the Big Brother Big Sister program. Add Health is the largest, most comprehensive longitudinal survey of adolescents ever undertaken, featuring some of the most extensive and complete social network information in the U.S. The Add Health database has been widely used in empirical models of social influences due to its detailed social network information (Goldsmith-Pinkham and Imbens 2013; Hsieh and Lee 2016; Badev 2021).

I quantify the impact of various factors influencing treatment participation, such as peer

effects and network dynamics, with friendship value predicted by the degree of similarity between individuals. This allows me to distinguish between those who participate due to intrinsic preference and those who are influenced by their social networks. The predicted social influence, derived from network dynamics, provides the necessary variation to identify the treatment effect for marginal participants—those who are brought into the program by social influence but would not have participated otherwise. This enables policy evaluators to investigate how potential treatment gains vary across individuals with different levels of participation preference driven by social influences, and whether those who join interventions under peer pressure benefit more or less. The extent to which social networks enhance treatment efficiency depends on how treatment preferences correlate with potential gains. Specifically, if individuals whose participation is driven by higher social influence do not gain more from the treatment than others, relying on social influence to attract participants may inadvertently bring in those who benefit less from the intervention.

My empirical findings suggest that for every additional friend opting into PYD programs, an individual's participation probability increases by a statistically significant 4.8 percentage points.[2] This effect is comparable to the impact of having one grade point higher in math GPA (5.1 percentage points, significant) and having a parent with at least a high school or college degree (4.9 percentage points, not significant). Additionally, my counterfactual analysis results suggest that incentivizing the participation of a subset of individuals can generate a social multiplier effect ranging from 1.18 to 1.77, depending on the size of the targeted population. Furthermore, shifting from random assignment to a more refined targeted strategy aimed at marginal participants could increase the social multiplier effect to between 2.01 and 2.51. This means that for every individual incentivized by policy to join, the social influence and network changes induced by their participation will attract at least one additional participant. This simulation also highlights that network formation plays a crucial role in PYD program participation choices. Ignoring this aspect could lead to a 25% to 47% downward bias in predicting the social multiplier effect.

While Scouting and the Big Brother Big Sister program are not narrowly defined behavioral health interventions like life skills training (Botvin and Griffin 2004) or school-based social and emotional learning programs (Jones and Doolittle 2017; Schonert-Reichl 2017), they have been proven effective in enhancing adolescents' well-being, future mental health, and overall success. As general behavioral health interventions, they provide opportunities for children to connect with others and form supportive networks. Participants in these programs represent a broader population compared to those in targeted health interventions, reflecting policymakers' focus on improving overall well-being and personal development. By examining a general

[2]The sample population's overall participation rate is approximately 12%.

intervention, this paper aims to offer insights into how encouraging participation in self-improvement activities can lead to a broader enhancement of population mental health, rather than focusing solely on interventions targeting specific issues.

This issue is particularly relevant now, as the rising prevalence of mental health disorders among children and adolescents, coupled with the unmet need for treatment, has gained increased attention (US Public Health Service 2000; Kase et al. 2017). In addition, behavioral health interventions can be challenging for individuals from low-income families to access, even though they may be at greater risk than other demographic groups. Financial incentives are a common method to encourage participation in behavioral intervention programs among these demographics, nevertheless, such monetary incentives have been criticized for potentially leading to unsustainable participation, as they rely solely on extrinsic motivation (Gneezy et al. 2011). Participants attracted by financial rewards may not remain engaged in the program once the incentives are removed. Using social influence as a complementary strategy to engage individuals who may benefit more from the intervention could offer a more sustainable approach. Furthermore, for policymakers with limited resources or capacity to provide financial incentives to everyone, leveraging social networks to amplify the effects of these limited incentives is crucial. This phenomenon, known as the social multiplier effect, occurs when the actions of a few individuals influence others within their network, creating a larger overall impact.

This model is estimated using an approximate exchange algorithm (Murray et al. 2012; Mele 2017; Badev 2021). The likelihood employed is based on the stationary equilibrium distribution of a stochastic best-response process among myopic individuals. Specifically, individuals meet randomly in sequence and myopically optimize their treatment participation utility based on the people they encounter, making decisions on whether to update their actions and links according to these meetings. Because meetings occur randomly and a stochastic term is included in the utility of individual treatment participation, there is uncertainty regarding which states will evolve from the current states. This uncertainty results in a probability distribution over the transitions between potential states (i.e., network and action configurations), forming a Markov chain. Over time, this Markov chain is expected to converge to a stationary distribution, characterizing the probability of each potential state occurring in the long run. A stationary distribution captures the core of individual incentives in treatment participation and network formation, describing how the action-network will evolve and encapsulating all factors driving treatment participation and network formation.

This paper primarily relates to three strands of literature. First, by drawing on the social interaction literature that provides methods for identifying peer effects in the presence of endogenous network formation, I characterize how an individual's participation in a treat-

ment program is influenced by their friends' choices, while also accounting for the potential endogenous changes in the social network that may result from these individual treatment decisions. I also capture quantitative importance of social influence in individual behavioral health intervention participation. Typical health intervention studies focus on the relationship between health behavior and peer pressure, this paper shifts the focus to participation in behavioral interventions. Additionally, network dynamics perspectives are rarely discussed or incorporated in the existing literature of this field. To my knowledge, only two other papers attempt to incorporate network features in the individual intervention participation decisions: Lin and Vella (2021) exploit a Bayesian game to model endogenous treatment decisions and estimate with a nested fixed point algorithm, while Hersey (2018) exploit the SAR model and estimate using a Bayesian estimation method. The main difference between my approach and theirs is that I capture the two-way interdependence between treatment participation and friendship links – incorporating network formation dynamics in the individual treatment participation model while also allowing the network to respond to changes in treatment decisions. This two-way interdependence is crucial especially in behavioral health interventions, where programs often foster connections among participants.

Second, in the empirical application, I contribute to the policy and intervention evaluation literature, especially in development contexts (Attanasio et al. 2011; Todd and Wolpin 2006), by providing insight into how peer-influenced treatment participation correlates with Positive Youth Development (PYD) program outcomes and how general behavioral health intervention take-up can be improved using social networks. Understanding individual treatment decisions is crucial for intervention efficacy evaluation, as non-random participation can lead to nonequivalent comparison designs. Extensive work in this field focuses on statistical models for selection bias driven by preference for treatment, particularly methods to control for essential differences between the treated and untreated. Although social interaction is not a new concept in policy evaluation literature(Weiss 1972; Latkin et al. 1996; Prochaska et al. 1997), most studies focus on the spillover effects of treatment on outcomes. I turn my attention to the role of social interactions in treatment participation decisions. Additionally, the integration of network frameworks into policy evaluation literature, especially concerning selection bias, remains underdeveloped. This is due to the relatively recent emergence and ongoing evolution of relevant theories and methodologies (Lin and Vella 2021; Abadie and Cattaneo 2018). In fact, a common underlying assumption in these works is stable unit treatment value assumption (SUTVA), which posits that an individual's treatment decision should not be affected by the choices of others.

Finally, my paper also belongs to the broader literature on the effects of Positive Youth Development (PYD) programs on adolescents' future outcomes. Many studies on the effec-

tiveness of PYD programs, such as the 4-H Study of Positive Youth Development (Lerner et al. 2005; Tirrell et al. 2019; Lerner et al. 2017, 2018), draw their samples from participants who voluntarily join these programs. However, if there exists *selection on gains* – where individuals who expect to benefit the most from the program are more likely to participate – then the estimated PYD effect could be overstated, as the participants may not be representative of the broader population, skewing the results toward those who are more likely to experience positive outcomes. My research contributes to the PYD literature by examining how these samples are generated through endogenous participation decisions, helping to accurately interpret the results of existing studies.[3] In addition, since PYD program participation is linked to efforts to enhance individual health, overall lifestyle, and social connections, my findings also help explain why some groups may face greater challenges in improving their well-being. These challenges may stem from the influence of social dynamics, which can lead to a series of decisions that ultimately impact their overall well-being.[4]

The remainder of this paper is organized as follows: Section 2 provides a detailed description of the Positive Youth Development (PYD), introduces the Add Health dataset, and reviews related literature on estimating social influences and networks. Section 3 describes the model and outline the game-theoretic approach employed. Section 4 introduces the estimation framework. Section 5 presents an empirical investigation of the impact of social networks on the PYD program. Section 6 discusses simulation results that explore the potential magnitude of the social multiplier effect under various policy scenarios. The paper concludes in Section 7. Details of the estimation process, sample construction, and robustness checks are provided in the Appendix.

1.2 Data and Background

1.2.1 National Longitudinal Study of Adolescent to Adult Health (Add Health)

This research uses survey data from the National Longitudinal Study of Adolescent to Adult Health (Add Health), a comprehensive dataset tracking over 20,000 nationally representative adolescents from 144 schools since 1994-95, when they were in grades 7-12. The latest data wave was collected in 2018, when participants were aged 33-43. Add Health was initiated following a directive from the U.S. Congress to support a thorough study of adolescent health. Add Health has conducted five survey waves in total: wave 1 (1994-1995), wave 2 (follow-up in 1996),

[3] In Section 1.2.2, I will provide a detailed review of the PYD literature.

[4] Exploring the correlation between participation preferences in behavioral health activities and the potential gains from them may offer insights into social reproduction – the perpetuation of social inequalities – as certain social groups are more likely to engage in behaviors or access resources that reinforce their existing social status.

wave 3 (follow-up in 2001–02), wave 4 (follow-up in 2007–08), and wave 5 (follow-up in 2016-18). Add Health is the most extensive and comprehensive longitudinal survey of adolescent populations conducted to date in the U.S.. This dataset includes detailed information on participants' demographics, health, social networks, behavioral aspects, socioeconomic status, family backgrounds, and life experiences. This rich dataset allows us to explore the relationship between PYD program participation and peer dynamics in adolescence.[5]

Sample Construction and Descriptive Summary My sample is drawn from a subset of schools where all students have been interviewed, and their friendship information has been collected (i.e., a saturated sample). This ensures that the social network constructed using the friendship information of students from these schools will not suffer from nonrandom missing nodes or links due to incomplete data collection within the same school. By collecting comprehensive information on all students' connections within a school, this dataset provides unbiased and complete coverage of the social networks.

The Add Health survey asks each student to list up to five best female and five best male friends. I construct the friendship network using these responses, and the social network is restricted to reciprocal links (following the assumption that friendships are formed with mutual consent). Sixteen networks are constructed using the Add Health saturated sample, but the largest network and any network with fewer than 100 students are dropped to ensure comparable size for each sample network. Eventually, the initial sample size of 20,745 are reduced to a final analytic sample size of 1,353 (8 networks), as shown in A.1. Table 1.1 presents the summary statistics for all individuals in my sample, and Table 1.2 presents the summary statistics at network level.

My goal is to understand the relationship between PYD program participation and social networks, and how participation choices are driven by social influences. Therefore, the dependent variable in my analysis is an indicator for PYD program participation. I define PYD program participation as a binary indicator (i.e., =1 if participated; =0 otherwise) based on a single item: "*Which of the following types of organizations have you been involved with in your volunteer or community service work in the last 12 months?*". An individual is identified as a PYD program participant if they marked "*youth organizations, such as Little League or scouts*" or "*service organizations, such as Big Brother or Big Sister*". These were chosen because they are national youth organizations with a consistent framework across the country, ensuring generalizability in program structure and goals.[6] As shown in Table 1.1, overall, 12.6% (138 individuals)

[5]See `https://addhealth.cpc.unc.edu/` and Harris (2013) for more detailed information on the Add Health.

[6]An alternative definition could exploit the item "*At any time during your adolescence, when you were between 12 to 18 years old, did you regularly participate in volunteer or community service work? Don't count things like*

of participants were identified as having participated in some PYD during adolescence.

Key explanatory variables include the number of friends participating in the PYD program (as a discrete variable) and the number of friends who did not participate in the PYD program. The corresponding parameter for the former term captures the peer effect for people who participate in the PYD program, while the latter term captures the peer effect for people who did not participate in the PYD program. Other covariates include age (as a continuous variable), gender, race (recoded as five dummy variables: White, Black, Hispanic, Asian, Others), English GPA (as a discrete variable), Math GPA (as a discrete variable), standardized self-efficacy (as a continuous variable),[7] standardized risk aversion (as a continuous variable), an indicator for whether the individual's school has more than 10% of students participating in the PYD program, parent education (=0 if less than high school; =1 if high school/B.A), and household income (as a continuous variable). The explanatory variables for network formation include a set of indicators for each individual and each of their friends to indicate whether they have the same gender, are in the same grade, belong to the same race, and have similar self-efficacy levels and similar risk aversion levels.[8]

Appendix Table A.1 provides detailed summary statistics comparing PYD program participants (Column 6) and non-participants (Column 7); Appendix Table A.2 summarizes the characteristics of peers for both participants and non-participants. In general, PYD program participants are, on average, more likely to be white, younger, and male. They also participate more in other extracurricular activities (ECA), have higher levels of self-efficacy, higher parental education, and higher household income. Additionally, they have more friends participating in PYD programs and school clubs.

1.2.2 Positive Youth Development (PYD)

While Positive Youth Development (PYD) has not been extensively studied in the field of economics, there is a substantial body of literature discussing its principles and impacts within psychology, education, and public health. PYD is a strengths-based framework that focuses on developing important skills necessary for adulthood, rather than addressing the problems youth may face. It aims to provide young people with diverse experiences that foster intentional

washing cars or selling candy to raise money." This item is not explored in the current paper.

[7] Self-efficacy refers to an individual's belief in his or her capacity to execute behaviors necessary to produce specific performance attainments. Self-efficacy reflects confidence in the ability to exert control over one's own motivation, behavior, and social environment (Bandura 2006). Details on how these self-efficacy measures are conducted can be found in the appendix.

[8] Risk aversion refers to the preference for certainty over uncertainty and the tendency to avoid risks when possible. According to Jackson et al. (2023), malleable characteristics such as risk aversion serve as good explanatory variables for network formation: people with similar risk-aversion preferences tend to be friends. Therefore, I include risk aversion in my analysis to explain network formation.

self-regulation and cultivate multiple positive assets such as self-esteem and an optimistic mindset. These improvements in well-being, in turn, enhance educational outcomes and buffer against risky behaviors. PYD programs and activities encompass any extracurricular activities aligned with this framework. In the 1970s, the Youth Development and Delinquency Prevention Administration, part of the Department of Health, Education, and Welfare, adopted PYD as a tool to support youth. This marked a significant conceptual shift in adolescent behavior intervention, emphasizing what supports kids in staying on track, as opposed to the prevailing focus of the time on understanding why kids get into trouble (Lerner et al. 2006).

Common examples of PYD programs and activities include service learning, community service, national youth organizations like Scouting, Big Brothers Big Sisters (BBBS) programs, and 4-H programs. Scouting, organized by the World Organization of the Scout Movement, helps young people develop physically, intellectually, socially, and spiritually, and has proven effective in promoting youth character development (Wang et al. 2021). BBBS programs are designed to enhance self-competence, boost academic performance, and strengthen relationships with family, peers, and other adults through one-on-one mentoring (Park et al. 2017). Empirical evidence supports their positive impact on the social-emotional, behavioral, and academic outcomes of participants (Herrera et al. 2023; Park et al. 2017). The 4-H programs, offered by the United States' largest youth development organization, focus on mentorship and educational activities to help young people develop skills for positive personal and community change.

Other unconventional examples that align with the PYD framework, by fostering the acquisition of essential developmental assets and positive outcomes for adolescents, include church attendance, performance activities, sport teams, school involvement, and academic clubs. While these activities vary in their structure, content, and focus, they all demonstrate a positive impact on youth development assets (Eccles et al. 2003). In the Appendix, I further explore the relationship between school club involvement and social networks, providing a comparison to my main analysis, which focuses on the relationship between social networks and national youth organizations as PYD programs.

Over the past two decades, researchers have increasingly demonstrated that PYD can effectively reduce risky behaviors and this framework has been prevalently used in all kinds of prevention programs. For example, Schwartz et al. (2010) found a significant negative association between PYD and the initiation of tobacco use, marijuana use, and early sexual activity among girls.[9] Lerner and Lerner (2013) reported that 4-H participants are nearly twice as likely to make healthier choices. Bonell et al. (2015) explored how PYD interventions might reduce

[9]Although the study was funded by the National 4-H Council, the sample included adolescents from various after-school activities, with fewer than half involved in 4-H.

substance use. Specifically, PYD programs/activities provide youth with effective relationships and diverse experiences, fostering the development of multiple positive assets that compensate for involvement in substance use and violence. These findings align with Bleck and DeBate (2016), who emphasized the importance of improving developmental assets like self-control, self-esteem, and a positive mindset. Greater developmental assets were linked to reduced cigarette smoking, substance use, fast food consumption, and increased physical activity. Moreover, meta-analyses conducted by Durlak et al. (2011), Ciocanel et al. (2017), and Taylor et al. (2017) have shown that various school-based Social Emotional Learning (SEL) interventions and after-school programs are associated with reduced emotional distress and improved academic performance.[10] These findings underscore the potential of a diverse range of positive development interventions as effective approaches for improving educational outcomes and reducing risky behaviors.

1.2.3 Related Literature on Estimation of Social Influences and Networks

Typically, empirical studies on peer effects take the friendship network as given (Manski 1993; Bramoullé et al. 2009; Lee 2007; Goldsmith-Pinkham and Imbens 2013; Brock and Durlauf 2001) and predominantly exploit the linear-in-means model (i.e., the linear social interaction model). However, Manski (1993) highlights that this model is subject to the *reflection problem*, making it challenging for researchers to distinguish between *endogenous peer effects* and *contextual effects*.[11] Various solutions have been proposed to address this issue. For instance, Aguirregabiria and Mira (2007) and Lin and Xu (2017) incorporate a Bayesian game that resolves the simultaneity of peers' choices and estimates the endogenous peer effect through a sequential nested fixed-point algorithm. Alternatively, Bramoullé et al. (2009) extend the linear-in-means model to structure the social interaction through networks, identifying endogenous peer effects using exogenous characteristics of excluded peers (i.e., friends' friends that are not your friends) as instrumental variables. Their method involves the spatial autoregressive model (SAR) and is related to Lin (2010) and Lee (2007). Since individual peer groups are unlikely to completely overlap, the differences among their friend compositions introduce crucial nonlinearity for identifying the endogenous peer effect with a spatial weight matrix. This approach is further refined by Goldsmith-Pinkham and Imbens (2013) and Hsieh and Lee (2016) to address the endogeneity of friendship formation arising from potential correlations between friendship decisions and economic outcomes. Their model identifies unobserved idiosyncratic character-

[10] SEL, as defined by Elias (1997), refers to the process that enables people to learn the "ability to understand, manage, and express the social and emotional aspects of one's life."

[11] An endogenous peer effect refers to the impact on an individual's behavior that arises directly from their peers' behaviors, whereas a contextual peer effect refers to the impact stemming from the characteristics or attributes of their peers.

istics that influence both the friendship formation process and the determination of observed outcomes. This approach accounts for network endogeneity in individual action decisions and relaxes the assumption that networks are given and fixed.

While the SAR approach provides an improvement on existing measures and a solid base for econometric models of networks and actions, it does not account for potential changes in the network caused by changes in individual actions. In other words, they account for factors that may influence network formation and individual actions and how the decisions to form friendships influence actions, but not how actions may influence the decisions to form friendships. In the context of PYD programs, participants are likely to form connections with each other because adolescents tend to befriend those who share similar experiences, and these activities facilitate friendship formation (Dworkin et al. 2003). On the other hand, Badev (2021) incorporates Mele (2017)'s network formation model and extends the focus to examine how the social network responds to changes in individuals' behaviors (e.g., smoking) while still accounting for how the network facilitate these changes in actions. He models the network formation process as a potential game that in the long run converges to an exponential random graph model (ERGMs), which can be estimated using a Bayesian MCMC method. Of these studies, this paper is most closely related to the work of Badev (2021) and Mele (2017).

1.3 Model

In this section, I detail the specification of the latent treatment utility I_D that determine participation decision. To account for peer effects on individual treatment participation and the interplay between network formation and actions (i.e., treatment participation), the framework and parameter assumptions for I_D closely follow the network-action game model developed by Badev (2021).

The treatment considered in this paper is PYD program. As noted earlier, PYD programs are fundamentally extracurricular activities. Students weigh multiple factors when deciding on extracurricular activity participation. Key considerations include social influences such as peer effects, family support, the availability of time and financial resources, along with personal interests and motivations (Lane 2023; Mittermeyer 2011; Mohamad Sari and Esa 2017). Additionally, decisions regarding PYD program participation and forming friendships are considered to occur within the same period, reflecting their intertwined nature and the simultaneous consideration by individuals.[12]

[12]Students discussed their friends' involvement in activities as a reason to initiate or continue their own participation (Fredricks et al. 2002; Juvonen et al. 2012; Bramoullé et al. 2009). Extracurricular activity settings facilitate the formation and maintenance of friendships through common experiences and the opportunity to

Suppose there's a finite population $I = \{1,2,3,\ldots,n\}$, where $n \in \mathbb{Z}^+$. D represents treatment participation, i.e., participation in a PYD program. If $D_i = 1$, student i participates in some PYD programs; otherwise $D_i = 0$. Let G represent a network matrix where each element g_{ij} describes whether student i is a friend with j ($g_{ij} = 1$) or not ($g_{ij} = 0$), where $j \neq i$, and G_i indicates the friendship links of individual i.

Following Badev (2021), each individual $i \in I$ simultaneously chooses whether to participate in PYD programs $D_i \in \{0,1\}$ and a set of friendship links $g_{ij} \in \{0,1\}$ for every $j \neq i$, to maximize her treatment utility I_{D_i}:

$$I_{D_i} = D_i * \left(W_i' \beta_w\right) + D_i \phi \sum_{j \in I} D_j + \underbrace{\phi_{(D=1)} * \sum_{j \neq i} g_{ij} D_i D_j + \phi_{(D=0)} * \sum_{j \neq i} g_{ij} (1 - D_i)(1 - D_j)}_{\text{endogenous peer effect}} - V_i \quad (1.1)$$

$$+ \sum_{j \neq i} g_{ij} * \underbrace{\omega(W_i, W_j; \beta_{homo})}_{\text{homophily}} + q \underbrace{\sum_{j,k \neq i} g_{ij} * g_{ik} * g_{jk}}_{\text{clustering}} - \underbrace{\psi * \left(\frac{1}{2}(d_i^2 + d_i) + \sum_{j \neq i} g_{ij} d_j \right)}_{\text{network formation cost}} \quad (1.2)$$

where $d_i = \sum_{j \neq i} g_{ij}$ is the total number of links for i, which is also referred to as degree. W includes pretreatment observed characteristics (e.g., demographic and family socioeconomic background).

Let $W_i = (w_i^1, w_i^2 \ldots w_i^K)$ be a vector of exogenous pretreatment characteristics of i, and $W_j = (w_j^1, w_j^2 \ldots w_j^K)$ be a vector of exogenous pretreatment characteristics of j, where $K \in \mathbb{Z}^+$. $\omega(.)$ is a function of homophily that captures the utility an individual derives from forming a friendship with another individual j based on their exogenous characteristics:[13]

$$\omega(W_i, W_j; \beta_{homo}) = ([\mathbb{1}(w_i^k = w_j^k)])_{k=1}^K * \beta_{homo} + \omega_0 \quad (1.3)$$

Specifically, it implies that for each individual i, the benefit from forming a link with j is influenced by the degree of similarity between the individuals across various characteristics (e.g., demographics, malleable characteristics). In other words, $\omega(.)$ encapsulates how much an individual values a friendship with j based on how similar they are in multiple aspects.

The first part of I_D, component (1.1), implies that, conditional on the network G, individual i's utility from her action ($D_i = 1$ or 0) depends linearly on i's exogenous pretreatment characteristics (W_i), the number of friends playing the same action as her (i.e., $\sum_{j \neq i} g_{ij} D_i D_j$ if

develop skills important for positive peer interactions (McPherson et al. 2001; Dworkin et al. 2003). Moreover, participating in the same activities is closely linked to forming friendships during middle and high school, even more so than the usual preference for selecting friends from the same ethnic background during adolescence (Schaefer et al. 2011, 2018; Alwin et al. 2018).

[13] ω_0 is the constant in the network formation model.

$D_i = 1$; $\sum_{j \neq i} g_{ij}(1-D_i)(1-D_j)$ if $D_i = 0$), the overall participation of all individuals within the same network as i ($\sum_{j \in I} D_j$), and on unobserved personal taste V_i that determine valuations of PYD programs. (1.1) captures the effects of all the important factors on participation choice: β_w captures the effect of individual attributes W_i on intrinsic preferences for participation; ϕ captures a environmental effect that depends on entire surrounding population with a specific network, while $\phi_{(D=1)}$ and $\phi_{(D=0)}$ capture local endogenous peer effects that depend on the individual's neighbors in the network and their own actions.

The second part of I_D, component (1.2), characterizes the network formation process. Conditional on individual action D_i and the actions of others (D_{-i}), individual i's utility to befriend another individual j depends on j's exogenous pretreatment characteristics (how similar they are, i.e., degree of *homophily* $\omega(W_i, W_j; \beta_{homo})$),[14] the number of mutual friends with j ($\sum_{j,k \neq i} g_{ij} g_{ik} g_{jk}$), and the total number of friends i and j have respectively (implied by the third term of (1.2), a differential of the convex friendship formation cost). Additionally, she also considers *action homophily* (i.e., the number of friends playing the same action as her).

Fundamentally, there are three main factors driving network formation: homophily (or, reciprocity), clustering, and the cost of forming/maintaining friendships. The utility of the network is the sum of the net benefits received from each factor. In social interaction and network formation literature, the importance of homophily has been highlighted in multiple papers (McPherson et al. 2001; Jackson et al. 2023). It has been shown to be a reliable predictor for the existence of friendships. More importantly, the similarity of exogenous characteristics is not the only factor; homophily in terms of actions and behaviors also plays a significant role. Individuals who engage in the same activities are more likely to form connections due to the human tendency to seek companionship with those who share common interests. In the context of PYD, shared experiences can provide more topics for conversation, fostering the initiation or maintenance of friendships. Additionally, participating in the same program increases opportunities to meet and spend time together, further facilitating the formation and strengthening of friendships.

On the other hand, clustering, i.e., a triad, is often observed in a social network. Mechanically, an individual is more likely to form friendships with people who have a mutual friend, either because the frequency of interaction is higher due to the mutual friend or because of the tendency to be with someone who shares something in common, including a friend.[15] Additionally, if a particular individual has mutual friends with a large number of other people, this may imply that the individual is "popular" (i.e., having many friendship links), so this

[14] In the network formation literature, the degree of similarity between two individuals is referred to as the degree of homophily, and it is sometimes also referred to as the degree of reciprocity.

[15] However, the clustering effect (i.e., q) will be negative if an individual prefers her friends to be exclusive to herself.

clustering effect could also be driven by the desire of individuals to associate with a popular peer.

Lastly, as all economic questions inevitably face cost constraints; here, the constraint in network formation is time and energy available for maintaining friendships. The costs of establishing a friendship between two individuals are directly reflected by how many friends they already have, respectively. Therefore, the more friends an individual has, the more costly it is for her to establish an additional friendship. These costs are shared, so it is also more costly for her to maintain friendships with someone who has many links to maintain.

The unknown parameters to be estimated in (1.1)-(1.2) are β_w (effects of exogenous pretreatment characteristics), ϕ (global endogenous peer effect), $\phi_{(D=0)}$ (local endogenous peer effect for non-participants), $\phi_{(D=1)}$ (local endogenous peer effect for participants), β_{homo} (effects of homophily in terms of exogenous pretreatment characteristics), q (clustering effect), and ψ (effect of friendship formation costs). Let these parameters be denoted as θ. The estimation of θ will be detailed in section 1.4.

1.3.1 How Social Influence Drives Treatment Participation

One of the purposes of this paper is to identify *marginal participants* by social influences – individuals who, by intrinsic preference, would not participate in treatment without any incentive but will do so if provided with an incentive boost resulting from participating friends. Here, the incentive considered is the peer effect. Specifically, I aim to identify individuals who participate because their friends' participation provides enough incentive to increase their utility from participating in the treatment. Without this peer influence, their utilities would fall below the minimum threshold for participation.

Understanding who the marginal participants are is beneficial, as it provides insight into whom we should target and offer extra incentives to achieve a social multiplier effect. Specifically, if we have limited resources to offer financial incentives to a select group of people to encourage participation in a specific treatment, targeting the friends of marginal participants is advantageous. Once their friends participate, the peer effect will bring in the marginal participants, thereby increasing overall participation more efficiently than if financial incentives were directly given to all potential participants.

The parameters θ defined in my model summarize the determinants affecting treatment participation decisions, including peer effects and network formation based on homophily, which influences the composition of peer groups and the number of peers engaging in participation. Below, I demonstrate how the participation utility constructed using θ can help us identify the marginal participants.

Let's consider a simple environment with three individuals $I = \{i_1, i_2, i_3\}$, each differing in their exogenous individual characteristics. These differences in characteristics imply that their intrinsic preferences for participation will also vary. Let $v_i^0 = W_i' \beta_w$ represent the intrinsic preference for individual i. In this scenario, assume $v_{i_3}^0 > v_{i_1}^0 > v_{i_2}^0$.[16] Here, i_3 has a high enough intrinsic utility to participate in the treatment regardless of her friends' participation, as demonstrated in Figure 1.1.

Network formation is an ongoing process where individuals continuously meet different groups of people and make decisions to change or maintain their current links and actions. The realized state (i.e., network and action configuration) we observe depends on time and the individuals involved in these meetings. Let's consider a moment when i_1 meets i_2 and i_3, and she is deciding whether to change her action or form a link with either i_2 or i_3, as demonstrated in Figure 1.2.

Both individual characteristics and the degree of characteristic homophily between individuals are exogenous and random, determining the intrinsic preference for participation and forming friendships with similar individuals. These factors characterize the potential tendency for participation and network formation before any social influences. This implies that whenever there is a discrepancy between the potential tendency and the realized states (i.e., if individuals take actions that differ from their intrinsic preferences or form friendships not predicted by their homophily), it is likely that social influence (i.e., peers' actions), change in peer group compositions, or external shock are driving the difference. The dynamics of network formation and action determination are intricate, and intertwined. Therefore, without accounting for network dynamics in participation decisions, we may misclassify individuals as marginal participants, resulting in less efficient targeting strategies.

In Figure 1.3, I list all the possible states that may occur upon i_1's decisions to update her actions/links, some of them potentially are driven by a different set of θ (i.e., different combinations of effect values for the factors driving network/actions). Each case represents a different network dynamic scenario, with case (a) potentially reflecting real-world dynamics. I will explain, for each case, the underlying θ implied and in what scenarios we might encounter misidentification issues if we do not account for network formation.

Case (a) Based on intrinsic preferences, i_1 would not participate in the treatment, but the realized action indicates her participation despite her intrinsic utility being lower than the

[16] From my estimates, individuals identified as male or possessing higher self-efficacy tend to have a stronger preference for participation, with males generally exhibiting higher utility than those with self-efficacy alone. In the example considered here, where intrinsic preferences follow the order $v_{i_3}^0 > v_{i_1}^0 > v_{i_2}^0$, i_3 could represent a male with high self-efficacy, i_1 a male with lower self-efficacy, and i_2 a female with lower self-efficacy.

threshold.[17] This suggests the presence of an external shock or social influence. Social influence appears to be the more likely explanation, given that i_1 has a friend participating in the treatment. The pressure exerted by this friendship likely influenced her to follow i_3's action. If the peer effect $\phi_{(D=1)}$ were zero or negative, she would not have participated, hence $\phi_{(D=1)} > 0$. Furthermore, the realized state in case (a) implies that the benefit derived from her friendship with i_3 must be significant compared to not participating in the treatment. Otherwise, she would have preferred to forego this relationship rather than engage in a treatment she initially did not favor (i.e., $\omega(W_{i_1}, W_{i_j}; \beta_{homo}) + \phi_{(D=1)} > -\nu^0_{i_1}$).

Case (b) The friendship link between i_1 and i_3 was not realized despite the high degree of characteristic homophily between them and the fact that they are engaging in the same action. This suggests that i_1 does not form friendships based on characteristic homophily (i.e., $\beta_{homo} < 0$) and action homophily (i.e., $\phi_{(D=1)} < 0$). Additionally, i_1 participates in the treatment despite having an intrinsic utility lower than the threshold, while her realized friend i_2 is performing the opposite action. This implies that the peer effect from nonparticipants is likely zero or even negative (i.e., $\phi_{(D=0)} \leq 0$), and her action is most likely driven by an external shock at that time.

Case (c) Despite the high degree of homophily and the fact that both i_1 and i_3 are engaging in the same action, the friendship link between them did not form. This indicates that i_1 does not form friendships based on homophily of characteristics (i.e., $\beta_{homo} < 0$) or shared actions (i.e., $\phi_{(D=1)} < 0$). Moreover, i_1 participates in the treatment despite having an intrinsic utility lower than the threshold, which suggests her action is most likely driven by an external shock at that time.

Case (d) The realization of the friendship link between i_1 and i_3 suggests that characteristic homophily positively affects friendship formation (i.e., $\beta_{homo} > 0$). However, i_1 did not participate in the treatment despite her linked friend i_3 doing so. This indicates that her action is either influenced by an external shock at that time, or the peer effect is zero or negative (i.e., $\phi_{(D=1)} \leq 0$).

Case (e) The friendship link between i_1 and i_3 was not realized despite their high degree of characteristic homophily, suggesting that i_1 does not form friendships based on characteristic homophily (i.e., $\beta_{homo} < 0$). On the other hand, i_1 befriending i_2 despite the low degree of

[17] In this model, an individual will only participate if her utility, including the stochastic term V_i, exceeds 0; thus, the threshold is 0.

characteristics homophily between them suggests that their link is potentially formed due to action homophily (i.e., $\phi_{(D=0)} > 0$), if not due to an external shock. This case implies that when two individuals form a link despite a low degree of characteristics homophily, their actions are unlikely to be influenced by each other's actions through this kind of "weak links". Conversely, it is their action similarity that lead to their connection.

Therefore, when determining targets for intervention, links like these should not be exploited: if i_2 is given an incentive and her action changes, it is less likely to influence i_1's action and bring i_1 into the treatment through their link. Instead, i_1 may drop the link as they will no longer be performing the same actions. This case also highlights the potential risk of treating the network as fixed when studying treatment participation. Misidentifying the reason for i_1's non-participation as a lack of social influence from participating friends could lead to biased conclusions. Further details on this issue will be discussed below.

Case (f) i_1 did not form any links or change her action, implying a zero or negative effect of homophily on network formation (i.e., $\beta_{homo} < 0$). Additionally, action homophily (i.e., $\phi_{(D=0)}$) is likely to be close to zero as well.

Marginal participants due to social influences exist only in cases where $\phi_{(D=1)} > 0$. Therefore, case (a) meets this requirement. The magnitude of peer effect $\phi_{(D=1)}$ is ambiguous in cases (e) and (f), and the remaining cases are ruled out. In case (a), individual i_1 is considered a marginal participant. She will not participate based on her intrinsic preference for participation but will do so upon observing her friend's participation, as demonstrated in Figure 1.4.

When evaluating the influence of peers' actions on individuals, it is crucial to condition on the degree of characteristics of homophily between the individual and her peers. Links between individuals with a low degree of characteristic homophily must have other commonalities to establish a connection. These commonalities could include unobserved characteristics or action similarity. Links based on action homophily are susceptible to changes in actions, which should be not considered when designing targeted strategies.

For example, consider an environment where individual actions are predominantly determined by intrinsic preferences based on exogenous characteristics, and links frequently form between people who engage in the same actions (i.e., the benefit from actions are higher than links). Without considering how networks are formed and how actions may influence the network, peer groups are assumed to be exogenous, and individual actions are assumed to influence each other through given fixed links. Consequently, a high correlation of participation choices among linked individuals would be identified as the results of high social influence. However, by incorporating a network formation perspective and considering the

role of homophily, we can distinguish whether similarity in action results from social influence or whether it is the reason for the connection.

Specifically, if network formation estimation shows that existing links cannot be explained by characteristic homophily but always vary with action status, then similarity in action must be what connects people, as there must exist other commonalities for individuals to establish a connection. In this case, social influence is low, contrary to what we might conclude without considering network formation. Failing to include the network formation perspective may lead to overestimating social influence.

Note that friendships are also choices, and social influence occurs only if the benefits of maintaining these connections are significant enough to resist change. In such scenarios, individuals tend to alter their actions – the relatively less costly option – when their peer environment changes, making their behaviors subject to social influences. The interaction between actions and connections determines which aspect is more prone to change, based on which one provides less utility to remain unchanged.

Moreover, network dynamics are essential when designing targeted strategies. For example, consider an environment with three individuals $I = \{i_1, i_2, i_3\}$, as demonstrated in Figure 1.5. Here, i_1 and i_3 are linked due to characteristic homophily, and individual i_2 is the only one participating in the treatment. If i_1 is given an incentive and subsequently participates, this will exert peer pressure on i_3 to follow i_1's action, as illustrated in Figure 1.5, panels (a) and (c). If the network structure is relatively fixed (case 1 in Figure 1.5), then the peer effect on i_3 from i_1's action will be the only new social influence induced by getting i_1 to participate, and estimation of policy effectiveness would not be greatly impacted if we do not account for network formation. However, the social influence may be greater if network structure is relatively dynamic (case 2 in Figure 1.5), as i_1 may form a link with i_2 due to action homophily. Specifically, having i_2 as a mutual friend may drive i_3 to form a friendship with i_1 (i.e., clustering effect). This means that eventually, the peer effect on i_3 could originate not only from i_1 but also from i_2 if we get i_1 to participate. In this scenario, comparing the models that consider network formation and those that do not, the magnitude of social influence on i_3 induced by getting i_1 to participate could be underestimated in the model that does not consider network formation due to the missing links not predicted by the model. Now let's consider another example where we have four individuals $I = \{i_1, i_2, i_3, i_4\}$. Individuals i_1, i_2, and i_3 are linked, while individual i_4 is the only one participating in the treatment. As demonstrated in Figure 1.6.

If we assign some incentive to i_1, and i_1 participates in the treatment as a result, the states may evolve in three different directions, depending on θ, as shown in Figure 1.6. Specifically, in the Case 3, the effect of action homophily on the network is stronger than clustering among

participants, leading i_1 to befriend i_4 instead of i_3 after changing her action (moving from (d) to (e)). This is common because participants tend to interact more with each other, making it easier to form or maintain friendships. The clustering effect then causes i_2 to befriend i_4, when action homophily on the network is minimal for nonparticipants. Eventually, i_2 may conform to the actions of i_1 and i_4 due to peer effects, illustrating that i_1's action can influence others through connected links. Additionally, changing i_1's action might lead to more participants (e.g., i_2), demonstrating a social multiplier effect of 2.

Conversely, if the clustering effect is stronger than any other effects, and the peer effect is fairly strong and comparable among both participants and nonparticipants (as illustrated in Case 1), i_1 might initially have enough incentive to participate. However, she may still conform to i_2 and i_3 in the long run (moving from (b) to (c)). In particular, since the clustering benefit is too high to forgo, the network is relatively fixed. Then the actions of i_2 and i_3 influence i_1 through their links, leading i_1 to stop participating, as the peer effect makes participating in the treatment less desirable than aligning with her linked peers. In this scenario, it becomes challenging for i_1 to change her behavior, let alone influence the actions of others, effectively negating the possibility of any social multiplier effect.

On the other hand, if both the clustering effect and social influence are small (as illustrated in Case 2), then when i_1 changes her action, her links with i_2 and i_3 break (moving from (e) to (f)). As a result, the policy effect is limited only to i_1, leading to a social multiplier of 1.

The examples presented in Figures 1.5 and 1.6 highlight the importance of considering network formation and understanding the relative magnitudes of all effects influencing participation decisions. Real-world dynamics align with the parameters demonstrated in Case 3, achieving a social multiplier is feasible, and policymakers are encouraged to exploit social networks accordingly. Conversely, if dynamics correspond to Case 2, efforts to leverage social networks should be accompanied by initiatives to strengthen the connections between peers.

1.3.2 Identification

Including peers' treatment participation as one of the variables explaining an individual's treatment participation is challenging. The specification of I_D needs to address two potential threats: *correlated effects* and the *reflection problem*.

To exploit social network interactions as exogenous variation for estimation, one must first address the problem of *correlated effects*. Essentially, since individuals and their peers tend to have similar tastes, the correlation in actions (e.g., participation in treatment) among peers may be attributed to (1) their similar characteristics and preference due to endogenous

peer selection (i.e., *network endogeneity*), and/or (2) common environment they share.[18] In other words, under correlated effects, it is difficult to determine whether linked individuals – people in the same peer group – engage in the same actions because they are influenced by the actions of each other (i.e., assimilation) or because they share similar characteristics and/or environments that lead them to act similarly (i.e., selection), without being in the same peer group.

To address the correlated effects, I follow Badev (2021)'s approach to account for network endogeneity by integrating a model of peer effects in networks with a model of network formation.[19] This approach introduces a mutual sharing term—the number of peers participating in the treatment—that controls for both peer effects conditional on the network and action homophily in network formation. This model specification explicitly captures the mutual dependence between network formation and action. It includes distinct components for the factors driving network change and those driving action change, enabling us to draw separate conclusions about selection and influence, even as these processes occur simultaneously.

Furthermore, by incorporating an environmental term that characterizes the overall participation rate at each school, I capture the effect of school-specific influences on individual actions. This allows us to address correlated effects driven by common shocks or environmental factors.

When correlated effects are addressed, the network of interactions can then be considered exogenous (Bramoullé et al. 2020). This allows the reflection problem to be resolved through these network structures and interactions: endogenous peer effects can be identified through the indirect interaction from intransitivity and the nonlinearity generated by variation in non-overlapping individual peer groups.[20]

In my approach, the degree of homophily between individuals based on exogenous characteristics (i.e., proximity) provides functionally exclusive variations – affecting links but not actions directly. An individual's action is indirectly influenced by the proximity between herself and her peers through their connections. Unlike the linear influence of peer actions on individ-

[18] Friendships are form based on homophily, which can include unobserved traits. Jackson et al. (2023) highlight that malleable traits, such as risk aversion and studying habits, significantly influence the selection of friends. These characteristics also play a crucial role in shaping decisions and actions. Additionally, individuals who share common environments (e.g., schools), are subject to similar influences (e.g., teachers, facilities and classmates), which lead to comparable decisions and actions.

[19] Strategies to address correlated effects and causally identify peer effects include random peers, random shocks, structural endogeneity, and panel data. Badev (2021)'s approach is classified as structural endogeneity. See Bramoullé et al. (2020) for details.

[20] In Bramoullé et al. (2009), Theorem 1 demonstrates that, once correlated effects are addressed, the exogenous characteristics of friends of friends who are not directly connected to an individual influence that individual's actions only through the actions and links of their mutual friends. The variation in non-overlapping individual peer groups introduces the nonlinearity necessary for identifying the endogenous peer effect.

ual behavior, the proximity between an individual and her peers exerts a nonlinear influence on her actions. This nonlinearity is crucial for identifying changes in individual action driven by changes in peer actions, thereby isolating the peer effect.

1.3.3 Model Limitation and Threat

There are two specific issues I have excluded from my current analysis for future consideration, as addressing them would complicate the study without offering significant contributions to its novelty.

First, unobserved idiosyncratic characteristics might affect network endogeneity. Although I base my model of network formation on observable homophily, there could be unobserved factors influencing network formation as well. If these unobserved factors also impact individual actions, the resulting network endogeneity could undermine the identification of endogenous peer effects.[21]

The second issue pertains to inaccuracies in social network observations (Marsden 2005) and missing network data. The latter can potentially confound estimation if it is nonrandom (Chandrasekhar and Lewis 2011). To mitigate the network data concerns, I restrict my analysis to the saturated sample from Add Health, which provides complete social network data collected in saturated field settings.

To alleviate concerns about model assumptions and related issues, I conduct a cross-validation test using estimated parameters from my model to predict outcomes and networks outside my estimation sample. The test results are discussed in Section 1.6.1. This test provides evidence of the model's credibility, similar to the predictive power test conducted in Griffith (2024). Additionally, as a robustness check to assess the extent to which the results are driven by correlated effects caused by common environmental factors, I estimate the model with the global peer effect–controlling for school-specific participation effects–switched off. This restricted model allows me to determine how much of the effect is controlled by these factors. Further details on the robustness checks and cross-validation tests, which verify the stability and reliability of the results, are provided in the next section and appendix.

1.4 Estimation

I closely follow the econometric framework of Badev (2021) to estimate the parameters in the treatment participation equations (1.1)-(1.2) (i.e., θ, as defined in section 1.3) using a

[21] See Moffitt (2001) and Mouw (2006) for a more detailed discussion on the challenges posed by correlated unobservables in the estimation of social influence.

Bayesian MCMC estimation method. With estimated parameters $\hat{\theta}$, we are able to understand the composition of individual treatment participation incentive and simulate their decisions that respond to certain policy, which can be used to evaluate a certain policy's efficiency to encourage population treatment participation.

1.4.1 Estimation Framework for Treatment Equation

In this paper, the decision environment is characterized as a discrete game with complete information. Individuals are assumed to observe the entire network and the individual attributes (e.g., characteristics and actions) of everyone within the population.[22] Both networks and actions (i.e., treatment participation) are adapted dynamically through a stochastic best-response process.

In particular, individuals meet sequentially at random, with each meeting consisting of an arbitrary subgroup of the population with more than two people, denoted as I_k. During each meeting, an arbitrary individual myopically updates her action and links to maximize their current utility, conditional on the meeting population and an idiosyncratic shock that varies by meeting, while treating the observed network and actions of others at that time as given and fixed.[23]

For example, consider an environment with three individuals $I = \{i_1, i_2, i_3\}$. There are nine different possible meetings in this case.[24] Each meeting out of these nine occurs randomly in turns at different times, and each individual's decision on her actions/links at each meeting results in a distribution of states (action and network).

Let S be a profile of actions and a network such that $S_{(i)} = (D_i, \{g_{ij}\}_{j \neq i})$ which individual i choose from her choice set $\mathbb{S}_{(i)} = \{0,1\}^n$; $S = (S_{(1)}, ..., S_{(n)}) \in \prod_i \mathbb{S}_{(i)} = \mathbb{S}$ and $X = (X_1, ..., X_n) \in \mathbb{X}$. Mathematically, each individual solves the following maximization problem at each meeting:

$$\max_{D_i, \{g_{ij}\}_{j \in I_k \setminus i}} I_{D_i}\left(D_i, \{g_{ij}\}_{j \neq i}; S^*_{-i}\right),$$

where $1 < k \leq n$, $I_k = \{i\} \cup \{i_1, \ldots, i_{k-1}\}$ and $i \notin \{i_1, \ldots, i_{k-1}\}$, for all i and I_k. S^*_{-i} is denoted as the

[22] The estimation of network formation processes in discrete games of complete information is challenging due to the curse of dimensionality and the existence of multiple equilibria. To overcome these challenges, the literature on network formation often relies on a sequential adaptive process (Christakis et al. 2020; Blume 1993).

[23] Individuals are assumed to be myopic in their decision-making, meaning they do not consider the long-term impact of their strategies on the network or the actions of others. They focus solely on current benefits, without attempting to manipulate the network's evolution or influence the behavior of others through their actions and linking decisions.

[24] Specifically, $(i_1, \{i_1, i_2\})$, $(i_1, \{i_1, i_3\})$, $(i_1, \{i_1, i_2, i_3\})$, $(i_2, \{i_1, i_2\})$, $(i_2, \{i_2, i_3\})$, $(i_2, \{i_1, i_2, i_3\})$, $(i_3, \{i_1, i_3\})$, $(i_3, \{i_2, i_3\})$, $(i_3, \{i_1, i_2, i_3\})$. Meeting $(i_2, \{i_1, i_2, i_3\})$ means that i_2 meets with i_1 and i_3 and determines whether to update her action/link or not. Each meeting occurs with equal probability.

optimal action/network decisions of every individuals except for i. Here, S also represent the state of this model.

Since individuals meet randomly, and each individual encounters a stochastic term (i.e., an idiosyncratic shock) in her utility function representing uncertainty over time, each meeting results in a distribution of states rather than a single state. Consequently, each state corresponds to a probability of occurrence, which is determined by the probability of each meeting, the distribution of V_i and the optimal solutions that individuals derive from this stochastic distribution.

For example, at some time t and given the current state S, suppose S_1^* is i_2's optimal solution under $V_{i_2,\mu_t} > 0.5$, and S_2^* is i_2's optimal solution under $V_{i_2,\mu_t} \leq 0.5$, when the meeting $(i_2, \{i_1, i_2, i_3\})$ occurs, conditional on the current state. Then, at time t, the probabilities of states S_1^* and S_2^* occurring would be given by the CDF of the corresponding V_{i_2,μ_t} that leads to them (i.e., $F_V(V_{i_2,\mu_t})$), as well as the probability of the meeting occurring (i.e., $\Pr(\mu_t = (i_2, \{i_1, i_2, i_3\}) | S, X)$).

Therefore, this process generates a set of probabilities for each state at each time, which coincides with a Markov chain with transition probabilities determined by the meeting process and individual optimal network-action choices. This process entails stochastic meetings, and individuals solve for optimal network-action, so this process is referred to as a stochastic best-response process.

Note that evaluating individual optimal network-action choices entails a great computational burden: even for a sample with a size of ten, we need to evaluate 2^{100} potential alternative network-action combinations before determining the optimal solution. Luckily, with some restrictions on the treatment participation utility specified in (1.1)-(1.2) and the stochastic best-response process, this process can be characterized as *a potential game* (Monderer and Shapley 1996). In such a game, any individual's incentives for participation at any state can be summarized by a single *potential function*, reducing the problem of finding the optimal solution for each individual to finding the local maxima of the potential function.

To validate this stochastic best-response process as a potential game, the following assumptions must be satisfied:

Assumption 1. *Friendship links are formed with mutual consent. Specifically, for any individuals $i, j \in I$, a link between i and j will only be established if both parties are assured of strictly higher utilities by changing their link status from $g_{ij} = 0$ to $g_{ij} = 1$, conditional on the rest of the network and actions of others (Jackson and Wolinsky 1996).*

Assumption 2. *If individuals i and j form a link, the degree of homophily based on their*

exogenous pretreatment characteristics provides them with equal benefits, such that:

$$\omega(W_i, W_j) = \omega(W_j, W_i)$$

Note that this equality pertains only to the utility components derived from homophily, not to their overall utilities.

Assumption 3. *All meetings have an equal probability of occurring, such that:*

$$Pr(\mu_t = (i, I_k)|S, X) = \frac{1}{n}\frac{1}{\binom{n-1}{k-1}} > 0$$

where $\mu = \{\mu_t\}_{t=1}^{\infty}$ is a stochastic meeting process. In this process, at each period $t = 1,2,\ldots$, μ_t represents the meeting at time t that selects an individual $i \in I$ and a subpopulation $I_k \subset I$ with $k \in (1,|I|]$. Since μ_t randomly chooses both the individual and the population subset, each i or I_k has an equal probability of being selected.[25]

Assumption 4. *V_i is an additive preference shock that is i.i.d. across time and states (i.e., profiles of the network and actions). Additionally, it follows a Gumbel distribution.*

Assumption 5. *The transition probabilities from the current state to the next depend only on the current state and on the meeting. (Holland and Leinhardt 1977)*

Following Assumptions 1 and 2, this stochastic best-response dynamic is a potential game. There exists a potential function, denoted as $\mathscr{P}$, that captures all the incentives driving any individual in any state of the network and actions:

$$\begin{aligned}\mathscr{P}(S,X) = &\sum_i D_i * \left(W_i'\beta_w\right) + \frac{1}{2}\phi_{(D=1)} * \sum_{i,j} g_{ij} * D_i * D_j + \frac{1}{2}\phi_{(D=0)} * \sum_{i,j} g_{ij} * (1-D_i) * (1-D_j) \\ &+ \frac{1}{2}\phi * \sum_{i,j;j\neq i} D_i * D_j + \frac{1}{2} * \sum_{i,j} g_{ij} * \omega(W_i, W_j; \beta_{homo}) + \frac{1}{6}q * \sum_{i,j,k} g_{ij} * g_{jk} * g_{ki}\end{aligned} \tag{1.4}$$

Under a potential game, any unilateral change in an individual's state that increases or decreases that individual's utility also increases or decreases the potential function by the same amount.

Under Assumption 5, the network and action jointly evolve as a Markov chain (Norris 1997). In addition, under Assumptions 3 and 4, the Markov chain generated by the stochastic best-

[25] Assumption 2 guarantees that any equilibrium network can be reached with positive probability.

response process converges to a unique stationary equilibrium distribution in the long run.[26] This distribution, denoted as π, is specified as follows:

$$\pi(S,X;\theta)=\frac{\exp\{\mathscr{P}(S,X)\}}{\sum_{S\in\mathbb{S}}\exp\{\mathscr{P}(S,X)\}}$$

The stationary equilibrium distribution π characterizes the likelihood of observing a specific network-action configuration in the long run and aligns with the Exponential Random Graph Model (ERGM). π provides a probabilistic ranking for every state $S\in\mathbb{S}$, ensuring that all states receive a positive probability. It implies that any equilibrium state can be reached with positive probability, and in the long run, the equilibrium states are observed more frequently, hence they entail higher probabilities. Additionally, it functions as the likelihood function in my estimation framework.

Note that the normalizing constant $\sum_{S\in\mathbb{S}}\exp\{\mathscr{P}(S,X)\}$ is computationally infeasible.[27] Therefore, the posterior distribution of the structural parameters is estimated using an approximate version of the exchange algorithm (Murray et al. 2012). This approximate algorithm employs a double Metropolis–Hastings sampler (Liang 2010) to avoid the computation of the normalizing constant in the likelihood.[28]

The following steps explain the algorithm and estimation process:

Step 0: Initialize $\theta=\theta_0$, where θ_0 is determined by the user.

Step 1 (Outer Sampler Begins): For each iteration $t\in[1,T]$, propose a new parameter vector θ' conditional on θ, such that θ' is drawn from a random walk proposal distribution $q(\theta';\theta,S)$.

Step 2 (Inner Sampler Begins): Initialize $S_0=S$, where S represents the observed state in the data.[29] Recall that a state is a profile of actions and a network.

Step 2.1: For each iteration $r\in[1,R]$, perform the following steps:

1. Draw a random meeting size k from a uniform distribution q_k and select an individual i.

[26] Note that this stationary distribution π is unique and independent of the meeting process μ, the choice of k, and the initial states or network before the meeting process.

[27] For a network of n players, this computation involves summing the potential function over all 2^{n^2} possible network configurations

[28] Several papers have proposed similar algorithms in the ERGM literature. See Liang (2010), Koskinen (2008) and Caimo and Friel (2011) for examples.

[29] Assuming that the observed network data are a draw from the stationary distribution, the structural parameters can be estimated using only one network observation.

2. Draw a random subpopulation I_k (i.e., random meeting) according to the size k.
3. Sample a new state S' (the auxiliary variable), such that S' is drawn from a uniform distribution q_S over the permissible neighborhood conditional on S_{r-1}, i, and I_k. Note that S_{r-1} is the simulated state in the previous iteration.
4. Accept S' with a probability α_{inner}, where:

$$\alpha_{inner} = \frac{\exp\{\mathscr{P}(S';\theta')\}}{\exp\{\mathscr{P}(S_{r-1};\theta')\}} \frac{Q(S_{r-1}|S';q_S,q_k,I_k)}{Q(S'|S_{r-1};q_S,q_k,I_k)},$$

   where $\mathscr{P}(.)$ is the potential function defined in (1.4) and $Q(.)$ is the unconditional proposal distribution.
5. Set $S_r = S'$ if accepted; otherwise, set $S_r = S_{r-1}$.

Step 2.1 ends at iteration $r = R$. Collect the last simulated state S_R, assuming it is the equilibrium state.[30] Inner sampler ends.

Step 3: Given the simulated state S_R, the proposed parameter θ' is accepted with a probability α_{outer}, where:

$$\alpha_{outer} = \frac{q(\theta;\theta')}{q(\theta';\theta)} \frac{p(\theta')}{p(\theta)} \frac{\exp\{\mathscr{P}(S_R;\theta)\}}{\exp\{\mathscr{P}(S;\theta)\}} \frac{\exp\{\mathscr{P}(S;\theta')\}}{\exp\{\mathscr{P}(S_R;\theta')\}},$$

where $p(.)$ is the prior distribution determined by the user and $q(.)$ is the proposal distribution defined in Step 1.

Set $\theta_t = \theta'$ if accepted; otherwise, set $\theta_t = \theta$. Update θ to θ_t and repeat Steps 1 to 3.

Outer sampler ends at iteration $t = T$.

Essentially, this algorithm starts with a guess of θ. Based on this guess, a long trajectory of states is simulated. Specifically, I first draw a meeting – a random individual and a random subgroup of people – and then propose a state as a result of the drawn meeting. The algorithm is more likely to accept this proposed state as the resulting state from the meeting if the corresponding potential of the proposed state, based on the guessed θ, suggests that the individual who makes the decision at this meeting benefits from moving from the observed state (assuming the observed state is the initial equilibrium) to the new state. Note that a proposed state with better potential than the current state will not always be accepted due to

[30] In practice, the algorithm produces good network equilibrium sample as long as the number of steps in the network simulation algorithm, i.e., R, is sufficiently large and the algorithm is run for enough iterations.

the presence of uncertainty (stochastic term), so better potential only means the chances of acceptance are higher.

Conditional on this new state, the algorithm continues to draw meetings and propose the next state until a large number of states have been simulated. These sequences of simulated states represent the evolution of the network-action determination process and characterize how states move from one equilibrium state (the observed state) to another (the last simulated state). Different sets of θ will drive the state in different directions. Note that, by assumption, equilibrium should be reached in the long run, so a large simulation should provide a good approximate equilibrium state based on the guessed θ.

Collecting the last simulated state and assuming it as another equilibrium state, the algorithm then calculates the transition probability from the observed state to the simulated state and the transition probability from the simulated state to the observed state using the guessed θ. The algorithm accepts the guessed θ if the transition probabilities and the stationary distribution based on the guessed θ are more likely to satisfy *detailed balance* – a condition where, for every pair of states S and S', the probability of moving from state S to state S' is equal to the probability of moving from state S' to state S – than the previous guess.[31] The algorithm continues this entire process and keeps updating θ using newly simulated states to refine the transition probabilities and stationary probabilities based on θ to optimally satisfy detailed balance.

1.5 Evaluating the Decision to Participate in Positive Youth Development Programs

While numerous studies have highlighted the success of various PYD programs in promoting positive outcomes, they also acknowledge that individuals with socioeconomic disadvantages are much less likely to participate.[32] Understanding the participation decisions that generate the current samples researchers focus on when evaluating PYD program effectiveness is essential for interpreting existing evidence and assessing the efficacy of PYD programs. This section aims to investigate one key aspect of PYD programs: Who are more likely to participate in PYD

[31] When a Markov chain converges to a stationary distribution, the flow of the Markov chain is the same looking toward the future and looking toward the past. Therefore, the probability from the observed state to the simulated state and the probability from the simulated state to the observed state should be the same, indicating detailed balance, if the simulated state and observed state are both draws from the stationary distribution and θ is an accurate guess.

[32] Children with college-educated mothers are three times more likely to engage in extracurricular activities than those with mothers who did not graduate high school. Additionally, low-income youth tend to face barriers such as financial constraints, childcare duties, and safety concerns. See descriptive analysis in Lerner and Lerner (2013) and Vandell et al. (2015).

programs? How are individual decisions to participate in PYD programs influenced by peers?

1.5.1 Main Results

I estimated the parameters from Equations (1.1) and (1.2) using Bayesian MCMC strategies as detailed in Section 1.4. The results are presented in Table 1.3.[33]

Column (1) displays the estimated effects (in terms of marginal probabilities) of individual exogenous characteristics, peer participation, and overall participation at school on individual participation in the PYD program. The results suggest that individuals with a 1-point higher math GPA have a 5.1 percentage point increase in the probability of participating in some PYD program. Similarly, a 1-point higher English GPA increases the probability by 3.8 percentage points, while a one standard deviation higher self-efficacy scale increases the probability by 3.9 percentage points. These findings align with existing literature (Schunk and Zimmerman 2006), which indicates that students who are more capable or have a greater desire for self-fulfillment and self-belief are more likely to engage in extracurricular activities.

In addition, students identifying as white were 7.3 percentage points more likely to participate in the program, while males were 6 percentage points more likely, holding other factors constant. The probability of participation decreases with age (a decrease of 3.4 percentage points for each additional year). Although having a parent with a high school or college degree or a 1% higher household income also increases the probability of participation, these factors are not statistically significant. Attending a school with more than 10% PYD program participants increases the probability by 2.6 percentage points. The base PYD probability, transformed from the constant of the participation model, represents the baseline tendency for students to participate in the PYD program. It suggests that participating in the PYD program generally does not yield significant utility for students without any incentives.[34]

One important finding is the significant impact of social influence on participation. With each additional friend participating in PYD programs, an individual's probability of partici-

[33] Note that the estimates in Column (1) and (2) are transformed parameters: instead of displaying the value in terms of utility, it is transformed into marginal probabilities and relative marginal probabilities using logistic functions. For example, let β_w represents a vector of parameters denoting the effect of individual exogenous characteristics W_i on individual participation utility. Let $\beta_{w:se}$ be the effect of self-efficacy on participation utility, and β_w^0 be the constant representing baseline participation utility. Marginal probabilities in terms of percentage points can be calculated as $\frac{e^{\beta_w^0+\beta_{w:se}*SelfEfficacy_i}}{1+e^{\beta_w^0+\beta_{w:se}*SelfEfficacy_i}} - \frac{e^{\beta_w^0}}{1+e^{\beta_w^0}}$, and relative marginal probabilities in terms of percentages can be calculated as $\frac{(e^{\beta_w^0+\beta_{w:se}SelfEfficacy_i})(1+e^{\beta_w^0})}{(1+e^{\beta_w^0+\beta_{w:se}SelfEfficacy_i})(e^{\beta_w^0})} - 1$.

[34] Baseline PYD program participation utility can be transformed into baseline participation probability with $\frac{e^{\beta_w^0}}{1+e^{\beta_w^0}}$.

pation increases by 4.8 percentage points,[35] a magnitude similar to having a 1-point higher math GPA. This effect from an additional participating friend indicates that individual utility increases when their decision to participate (or not) aligns with their friend's decision, resulting in both participating (or not participating). This effect also reflects the probability or incentive for two participants to connect.

Another important observation regarding the estimation results in Column (1) is the notable difference in peer effect externalities between participants and non-participants. Participants experience a significant peer effect of 4.8 percentage points, while non-participants show an insignificant effect of 0.2 percentage points. This indicates that social influence is much stronger among participants than among non-participants. This finding is intuitive: having friends who participate in PYD programs creates an incentive or motivation to follow their peers' actions, whereas it is unlikely that students would intentionally quit PYD programs simply because their friends do not share the same experience. In addition, individuals are more likely to connect when they share the same extracurricular experiences. In contrast, lacking certain experiences typically does not facilitate stronger connections between people. Moreover, their lack of involvement in such programs may limit their opportunities to interact with one another. Since they do not share the common location or structured environment provided by the program, they may not have a natural setting where they can build or maintain friendships.

Column (2) presents the estimated effects (in terms of relative marginal probabilities) of homophily based on exogenous characteristics, clustering, and the cost of forming/maintaining friendships on individual friendship links. The results suggest that, consistent with network formation literature, individuals are more likely to form friendships with people of similar demographics: sharing the same sex increases the probability of friendship by 69.8 percent, sharing the same grade by 65 percent, and sharing the same race by 43.8 percent. Malleable characteristics also play an important role in friendship selection: having similar levels of self-efficacy and risk aversion both increases the probability of forming a friendship by about 10 percent. Moreover, having a mutual friend significantly increases the probability of connection by 62.9 percent. The base friendship links of 1.835, transformed from the constant of the participation model, represent the frequency with which an individual connects with another without any other incentives.[36]

[35]The model I consider assumes that peer effects increase linearly with the number of friends participating in PYD programs, meaning that the marginal increase in peer influence remains constant and does not diminish or intensify as more friends join.

[36]Let β^0_{homo} be the constant of the network formation component, then base friendship links can be calculated with $(n-1)\frac{e^{\beta^0_{homo}}}{1+e^{\beta^0_{homo}}}$, where n is the size of population in a network.

Note that individuals who are more likely to participate in the PYD program on average (e.g., male, white, younger, or with a high GPA) tend to receive more utility from participation. As a result, rather than altering their actions, they are more likely to adjust their social links to optimize utility, making their connections relatively more dynamic compared to their counterparts.

This finding aligns with existing literature on gender differences in participation in voluntary organizations and its impact on social ties. McPherson (1982) notes that men generally have larger and more diverse social networks than women, largely due to their higher likelihood of participating in larger organizations, which offer more opportunities to form *weak ties*.[37] Although their sample included older individuals (aged 18 to 75), the study suggests that men are more likely to exhibit weak ties, which potentially leads to more dynamic network behavior compared to women – a comparable conclusion to mine.

This also correspond to previous research showing differences in social networks between blacks and whites throughout life (Ajrouch et al. 2001). Race affects opportunities and challenges, often leading minorities to face more hardships and fewer opportunities than the dominant group. This can make social networks among minorities less spread out and less dynamic. During adolescence, social norms and stereotypes may influence preferences for participation in national youth organizations, such as the PYD programs discussed in this paper. As a result, non-white or minority adolescents may be less likely to participate in PYD programs. These experiences also shape their overall social networks, which are more likely to remain restricted to certain groups and less likely to expand beyond the classroom. My findings suggest that white individuals are more likely to participate in PYD programs and connect with other participants, leading to the formation of more weak ties. Unlike strong ties formed with family, weak ties involve less frequent contact. This pattern is supported by Ajrouch et al. (2001), who found that Black individuals generally have more frequent interactions with their network members. This is likely because their networks are primarily composed of family members, leading to more consistent contact. These racial differences in interaction patterns may diminish with age, as weak ties and social network size generally decrease over time, gradually narrowing to strong ties with family (Wrzus et al. 2013; Bruine de Bruin et al. 2020; Morgan 1988).

[37] Weak ties, a concept introduced by Granovetter (1973), refer to less close or infrequent relationships within a social network. Despite their lower level of emotional closeness, weak ties are crucial because they act as bridges between different groups.

1.6 Simulation

This section explores the potential magnitude of the social multiplier effect if policymakers seek to leverage social networks to encourage overall participation in an intervention or program.

There is a growing demand for youth interventions to address multiple risk behaviors due to their tendency to cluster and the potential efficiency of combined interventions (Buck and Frosini 2012; Kipping et al. 2012). PYD programs have emerged as a powerful intervention approach. However, to ensure PYD programs cover a large population and improve the well-being of as many individuals as possible, policymakers may face the challenge of increasing overall participation with a limited budget. One tempting strategy is to use social networks to encourage participation behavior change by initiating changes in the actions of a subpopulation and allowing social influence to bring in more people. Two potential methods can be used to initiate changes in some individuals' actions: one involves mandatory assignment, requiring selected individuals to participate, and the other involves waiving activity fees and addressing financial barriers for targeted individuals to enhance their incentives for participating in PYD programs.[38]

Given the estimated parameters, I conduct counterfactual exercises to simulate the impacts of changes in three scenarios: (i) changing the participation behavior of a subset of students by assigning them to a PYD program and mandating their participation (100% compliance), (ii) providing a subset of students with an incentive boost that increases their utility by a fixed amount, thereby increasing their participation probability by 50 percentage points, and (iii) targeting the friends of marginal participants and offering them an incentive boost. Simulation results are displayed in Table 1.7, 1.8 and 1.9. These simulations estimate the potential magnitude of social multiplier effects of policies that alter the participation behaviors of a subset of students. Specifically, when individual participation behaviors change, the actions of their peers may be influenced and change accordingly, or the peer connections may dissolve. The outcome depends on how much these connections rely on common actions and how strong these friendship links are. Individual actions are subject to those of her linked peers only when the links are strong. This counterfactual analysis provides policy implications: if policymakers want to use PYD programs as policy instruments to enhance student well-being or other objectives these programs effectively address, but have limited budgets that cannot

[38]According to the National Association of Secondary School Principals (2018), activity fees for extracurricular activities or participation in school clubs can vary widely, ranging from $15 to $1,500 a year. The fee amount may depend on the nature and type of activity, as well as the school's policies and resources. For example, Auten Road Intermediate School requires a one-time fee of $25 for club participation, covering multiple clubs or activities (Auten Road Intermediate School 2023). Menomonee Falls High School charges an annual $20 per student activity fee for the first club or activity, with no additional charges for further clubs in the same school year (Menomonee Falls High School 2023).

include every student, it is crucial to understand how effective it would be to target a subgroup and exploit the social multiplier effect to encourage participation without the high cost of universal inclusion. This analysis provides evidence on whether social networks can be leveraged to change individual behaviors or if interventions will only lead to changes in the network structure.

In the first simulation scenario, a random subset of students is assigned to PYD programs with mandatory participation. This assignment is not limited to current nonparticipants. The random assignment (i.e. the portion of individual getting assignments) ratio ranges from 5% to 50%. Using the estimates from Table 1.3, I simulate other individuals' responses to changes in participation behaviors among targeted individuals. Simulation results in Table 1.7 suggest that upon assignment, the assigned nonparticipants now opt into PYD programs, increasing the overall participation rate from 12.9% to 17.26% when 5% of the population is assigned to PYD programs with mandatory participation. The baseline effect is 4.36, meaning it increases overall PYD participation by 4.36 percentage points. However, this is an underestimation. If we consider a fixed social network (i.e., the network does not change upon changes in actions) and allow individual actions to influence peers through the network, the simulation results suggest that the intervention will increase the overall effect by 5.15 percentage points, resulting in a social multiplier of 1.18. The multiplier effect increases as the policy magnitude (i.e., assignment ratio) increases. Yet, this is still an underestimation. When considering an action-network model (i.e., network and actions are interdependent), the simulation results suggest that the intervention will increase the overall effect by 6.42 percentage points, leading to a social multiplier of 1.47. This suggests that the intervention not only encourages behavior change through social networks by initiating changes in some individuals' actions but also facilitates the formation of new links that allow social influences to magnify and impact more people. These results provide evidence that interventions aimed at changing individual participation in PYD programs can be more efficient and achieve a social multiplier through dynamic social networks.

In the second simulation scenario, a random subset of students receives an incentive boost that increases their utility by a fixed amount, raising their participation probability by 50 percentage points. This scenario mimics reality in which individual participation is not guaranteed by assignment. Simulation results in Table 1.8 suggest that upon receiving the incentive boost, the PYD program participation rate increases from 12.9% to 15.08% when 5% of the population is targeted. The baseline effect of this intervention (without considering social influence) is 2.18 percentage points. Again, this is an underestimation. If we consider a fixed social network and allow individual actions to influence peers, the simulation results suggest the intervention will increase the overall effect by 2.44 percentage points, resulting in a

social multiplier of 1.12. However, when considering an action-network model, the simulation results indicate an increase in the overall effect by 2.57 percentage points, leading to a social multiplier of 1.18. The multiplier effect increases as the policy magnitude increases. These results are consistent with those from the first simulation, supporting the idea that under real-world dynamics, policy efficiency can be enhanced through social multipliers.

Combining the findings from both tables suggests that interventions aimed at changing individual participation in PYD programs may significantly impact overall participation prevalence without directly engaging a large portion of the student population. The multiplier factor–the ratio between the baseline effect and the predicted effect –indicates substantial spillover effects, reaching around 1.5 or more as policy size increases. It is important to note that in these simulations, assignments were random. Targeting friends of marginal participants who are not currently participating would likely result in a larger social multiplier.

In addition, the results highlight the importance of considering network formation in the participation choices for PYD programs. Ignoring the network formation process can lead to significant biases in the predicted social multiplier impact of public policies. Specifically, 25%-47% downward bias in the first policy scenario and 5%-33% downward bias in the second policy scenario, depending on the population size that the policy targets – the larger the target population, the greater the bias. Although the current literature often finds that estimated peer effects, after controlling for the endogeneity of the network, are similar to when the network is assumed to be exogenous (Badev 2021; Boucher 2016; Fortin and Boucher 2015), it remains crucial to account for these dynamics, particularly in large-scale interventions.

In the third scenario, I target a subset of marginal participants' friends and provide them with incentives that increase their participation probability by 50%. The simulation results, presented in Table 1.9, indicate that social multipliers are higher in this scenario compared to the previous two across all assignment ratios. In general, the targeted approach leads to significantly higher participation compared to the non-targeted strategies, with social multiplier effects ranging from 2.01 to 2.51. The third set of results underscores that even with limited incentives, policymakers can nearly double a policy's impact by using targeted strategies aimed at marginal participants. These strategies enable them to maximize the social multiplier effect and effectively leverage social networks for optimal outcomes.

1.6.1 Model Fit and Cross-Validation

To evaluate how well the predictions produced by this model align with the observed data, I conducted simulations of 1,000 states (i.e., action-network configurations) using the estimates from Table 1.3 and individual characteristics from the sample used for estimation. Each

simulated state was generated through a long trajectory of random meetings, based on the assumption that an equilibrium state would be reached in the long run. During each random meeting, the selected individual optimizes their utility based on the estimated parameters in Table 1.3. The descriptive statistics of these simulated states are presented in Table 1.4. The results indicate that the average participation rate suggested by these simulated states (11.7%) is closely aligned with the observed mean participation rate across the sample (12.7%), suggesting that the model performs well in predicting overall individual participation. This also indicates that the estimates effectively explain the incentives of individual participation choices. However, the model slightly overpredicts the frequency of individuals and their peers both participate in PYD program – whether due to assimilation or selection – by about 11 percent higher compared to the observed average, while the median prediction (0.069) is very close to the observed value.

Regarding network formation, the median predicted network statistics are close to the observed statistics, though the mean values are generally higher than observed. Overall, this model performs well in predicting both participation decisions and friendship patterns in the data. However, similar to Badev (2021)'s model fit results, the mean number of triangles in the predicted states is much higher than the observed number. According to Badev, this discrepancy may be due to a few draws with very densely connected networks where this metric is disproportionately high.

In addition to fundamental network statistics characterizing overall link formation, I also report three indices that quantify the tendency of individuals to connect based on homophily, further characterizing the network dynamics: the Homophily Index, Coleman's Inbreeding Homophily Index, and the Freeman Segregation Index.

The Homophily Index is a general measure that quantifies the extent of homophily within a social network, capturing how often individuals with similar characteristics or behaviors link with each other compared to what would be expected by random chance. Coleman's Inbreeding Homophily Index enhances the Homophily Index by accounting for the population composition of individuals with different characteristics within the network. Meanwhile, the Freeman Segregation Index measures the extent to which the observed network deviates from a situation of complete integration, where individuals would be equally likely to connect regardless of their characteristics or behaviors.[39]

Overall, the model performs well in predicting the magnitude of the effect of action homophily on network formation. In addition, the findings also highlight a relatively high tendency for individuals to connect with others who share the same participation action.

[39] Details on the construction of these indices can be found in the appendix.

Out-of-Sample Validation The sample I used to estimate the parameters for Equations (1.1) and (1.2) is only a subset of the Add Health dataset, leaving a substantial portion of the data unexploited. This out-of-sample dataset provides an opportunity to test the predictive power of my model. Specifically, I selected individuals from 8 schools not included in the original estimation sample and simulated 10,000 states to predict their actions and networks based on their exogenous characteristics. This approach is similar to the exercise conducted by Griffith (2024).

Table 1.5 reports descriptive statistics comparing the model's predictions to the realized states. Overall, the median of the predicted actions and network configurations is close to the observed states, which increases my confidence in the model's predictive power. However, the mean predicted participation rate (14.2%) is about 16% higher than the observed participation rate (12.9%). Additionally, the mean predicted frequency of individuals and their peers both participate in PYD program within each network (12.5%) is approximately 28% higher than the observed average (9.7%). These indicate that the model may be slightly over-optimistic regarding overall PYD program participation. Regarding network formation patterns, the model performs well in capturing the effect of action homophily on network formation.

Bounding the Impact of Correlated Effects on Participation and Network Formation In my model, the overall participation rate at each school controls for common environmental effects on participation as well as network formation, distinguishing the peer effect from shared school-specific influences. To better understand the influence of correlated effects on participation decisions and network formation, I run simulations with this school-specific effect switched off. This allows me to turn off the impact of correlated effects, with the simulated results reflecting only peer effects. By comparing these results, I can bound the peer effect net of correlated effects and establish the extent to which observed impacts are driven by pure peer influences.

The results from this restricted model are presented in Table 1.6. Compared to the prediction from the full model with the school-specific effect (Table 1.4), the mean predicted overall participation rate decreases by 21 percent, and the mean predicted frequency of individuals and their peers both participating in the PYD program drops by approximately 26 percent. Conversely, the predicted frequency of non-participation links greatly increases by over 50%, resulting in a denser and more segregated network.

Note that although the mean predicted frequency of individuals and their peers both participating in the PYD program may seem close to the observed value, the restricted model induces more bias in the overall participation rate, network segregation level, and the tendency for nonparticipants to connect.

These findings suggest that although the school-specific effect on participation is modest,

its absence may lead to a noticeable change in network. Specifically, the resulting reduction in participation, while not drastic, diminishes the likelihood of program participants forming friendships with each other. Furthermore, the decline in participation may result in a higher concentration of non-participants within the network, which could encourage the formation of friendships among non-participants and contribute to a more segregated and dense network structure. Therefore, the network dynamics induced by the policy are unlikely to facilitate the spread of its effects, as there are fewer channels of interaction among participants.

Next, I reran the counterfactual analysis across the same three scenarios as before, with the results presented in Table A.7. The simulations show that excluding the school-specific effect reduces the social multiplier. Nonetheless, this effect persists as significant, particularly for larger policy size. In Appendix A, I estimate an alternative model specification that excludes the overall participation rate at each school. The new estimates and simulations from this model show a pattern of results consistent with the findings discussed above.

1.7 Conclusion

With the rising prevalence of mental health disorders and risky behaviors among children and adolescents, behavioral health interventions for the general population, such as Positive Youth Development (PYD) programs, have gained increasing attention. These programs focus on building positive developmental assets and fostering connections that can support participants as they face life's challenges. However, the literature on PYD programs and behavioral health interventions often overlooks the crucial role of network dynamics that these programs encourage. According to my estimates, this oversight could lead to a bias of 25% to 47% in predicting a policy's effects.

Behavioral health interventions not only influence cognitive, mental, and behavioral outcomes but also reshape social network dynamics. This paper examines the role of social networks and peer influences in the decision to participate in PYD programs and offers policy implications on how to improve policy efficiency by leveraging social networks. Following Badev (2021)'s structural action-network model, I incorporate aspects of network formation and a game-theoretic framework into the individual treatment decision model and capture the interdependence between participation decisions and social networks. This approach allows me to identify which demographics are influenced by social networks – the marginal participants subject to peer effect – and who participates due to intrinsic preferences. By recognizing those whose participation decisions are subject to social influence, policymakers can significantly improve intervention uptake by targeting the peers of these marginal participants for incentive boosts.

My empirical findings underscore the significant role of social networks in shaping individual participation in PYD programs. Moreover, participation in PYD programs fosters network formation, which, in turn, encourages further participation. This offers a nuanced and fresh perspective on social capital formation, and suggests potential for social multiplier effects. Specifically, my counterfactual analysis indicates that with targeted strategies, for every incentivized individual whose participation is encouraged by a targeted policy incentive, at least an additional person is likely to be drawn into the program alongside them. These findings highlight the substantial spillover effects of targeted interventions and the critical role of social networks in amplifying policy impacts. Furthermore, they offer insights on how to enhance social capital by strategically altering individual participation in behavioral health interventions. Exploiting network-based target strategies, practitioners can effectively foster environments that not only support individual well-being growth but also promote social mobility.

The empirical structural model presented in this paper can be extended to study the Marginal Treatment Effect (MTE) of PYD programs, as it addresses the endogenous nature of treatment participation choices. This framework allows for the identification of the average treatment effect for marginal participants whose treatment utility changes incrementally due to the additional participation of a friend in the program. Future work will focus on extending this model further by incorporating it into a generalized Roy model.

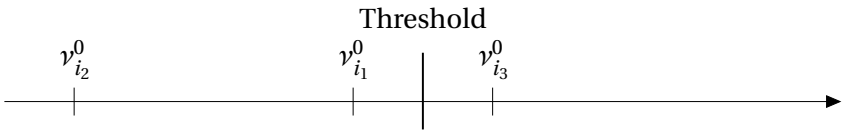

Figure 1.1: Intrinsic Participation Utility.

Note. The "Threshold" mark the lowest utility level for an individual to actually participate in treatment.

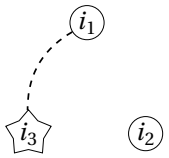

Figure 1.2: A Simple Decision Environment.

Note. The star shape denotes that i_3 has an intrinsic participation utility higher than the threshold, implying that she is likely to participate before any social influences. The dashed line denotes high characteristic homophily between two individuals, implying they likely will form a link with each other.

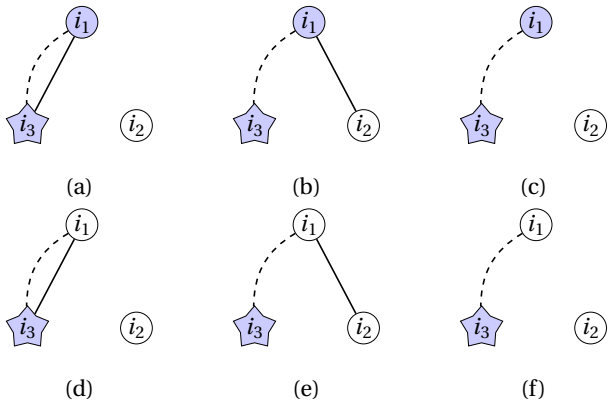

Figure 1.3: Possible Realized States When i_1 Make Decisions.

Note. The color indicates the realized action: blue implies an individual participated in the treatment, while no color means they did not. The solid line represents a realized friendship link.

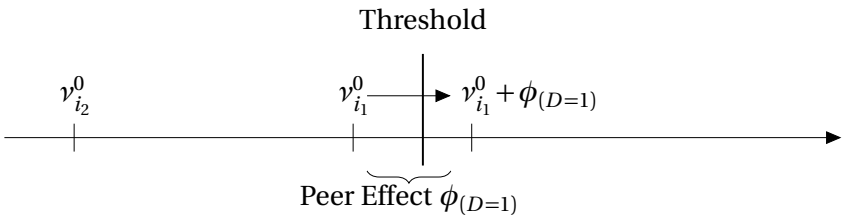

Figure 1.4: Intrinsic Participation Utility + Peer Effect.

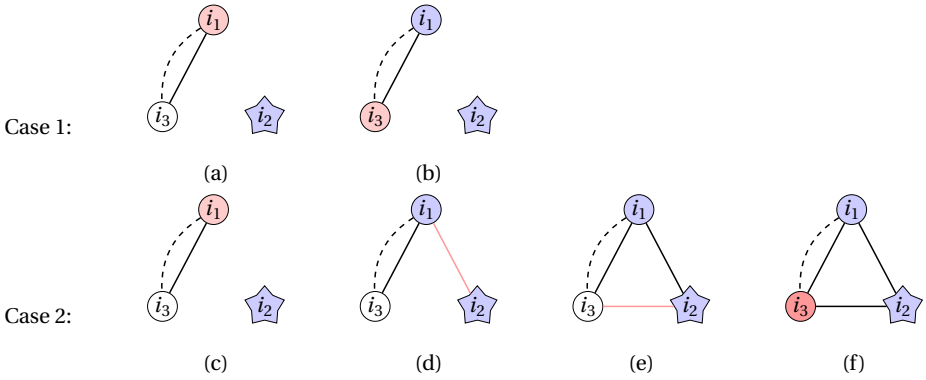

Figure 1.5: The Magnitude of Social Influence - With Network Formation Aspect vs. Without.

Note. Red nodes implies incentivized participate; red lines indicate newly formed links due to peers' actions. Case 1 presents an example state evolution where the network structure is relatively fixed; case 2 presents an example state evolution where the endogenous structure of the network significantly impacts the effectiveness of public policies.

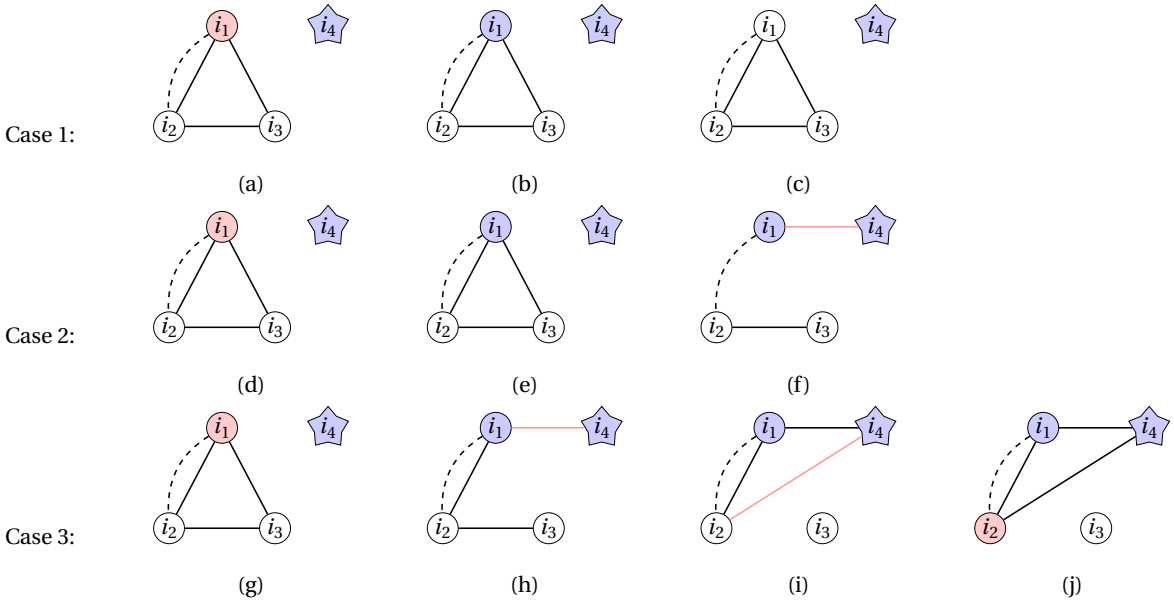

Figure 1.6: Social Multiplier, or Not.

Note. The color indicates the realized action: blue implies an individual participated in the treatment, while no color means they did not; red implies an individual is incentivized to participate. The solid line represents a realized friendship link; the red line represents newly formed link due to change in peers' action. Case 1 presents an example state evolution where the network structure is relatively fixed, resulting a social multiplier of 0. Case 2 presents an example state evolution where the network structure is relatively dynamic but social influences are small, resulting a social multiplier of 1. In contrast, Case 3 presents an example state evolution where the endogenous structure of the network significantly impacts the effectiveness of public policies, resulting a social multiplier of 1 to 2.

Table 1.1: Descriptive Statistics for Analysis Sample - Individual Level.

	Mean	SD
PYD Programs Participation	0.126	(0.332)
Female	0.47894	(0.500)
Male	0.52106	(0.500)
Age (94-95)	16.519	(1.516)
White	0.916	(0.278)
Black	0.055	(0.227)
Hispanic	0.020	(0.140)
Asian	0.004	(0.061)
Other Race	0.006	(0.077)
Self Efficacy [a]	25.795	(4.117)
Risk Aversion [b]	3.891	(1.682)
English GPA	2.738	(1.013)
Math GPA	2.475	(1.218)
HH Income (94-95)	45.925	(28.387)
Parent HSD/College	0.730	(0.444)
Num Friends Per Individuals (Degrees)	0.971	(1.194)
12th Grade	0.235	(0.424)
11th Grade	0.162	(0.368)
10th Grade	0.228	(0.419)
9th Grade	0.242	(0.429)
8th Grade	0.067	(0.249)
7th Grade	0.067	(0.249)
Observations	1,353	

Note. The summary statistics presented are unweighted because, in the network estimation process, each individual is treated as a unique node. As a result, the estimation sample is unweighted, reflecting the individual-level nature of the analysis.

[a] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

[b] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table 1.2: Descriptive Statistics for Analysis Sample - Network Level.

	Mean	Median	Min	Max
Num Students	169.125	157	111	234
Num Links	164.250	147	34	357
Pct PYD Programs Participation	0.127	0.113	0.074	0.246
Pct Female	0.47939	0.473	0.407	0.586
Pct Male	0.52061	0.527	0.414	0.593
Ave. Age (94-95)	16.467	16.248	15.607	18.429
Pct White	0.908	0.978	0.421	0.988
Pct Black	0.061	0.005	0.000	0.454
Pct Hispanic	0.021	0.010	0.000	0.105
Pct Asian	0.004	0.002	0.000	0.011
Pct Other Race	0.006	0.005	0.000	0.013
Avg. Self Efficacy [a]	25.862	25.842	25.342	26.784
Avg. Risk Aversion [b]	3.898	3.968	3.571	4.010
Avg. English GPA	2.762	2.651	2.376	3.276
Avg. Math GPA	2.474	2.420	1.934	3.184
Avg. HH Income (94-95)	45.637	48.006	29.890	56.813
Pct Parent HSD/College	0.728	0.738	0.553	0.842
Avg. Degrees	0.897	0.831	0.276	1.526

Note. My analysis sample consists of 8 distinct networks.

[a] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

[b] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table 1.3: Treatment Utility Parameter Estimates.

	(1) Action[a] (Treatment Participation)		(2) Network Formation[b]
Log HH Income	0.035	Same Sex	0.698***
	(0.037)		(0.048)
Parent HSD/B.A	0.049	Same Race	0.438***
	(0.062)		(0.246)
White	0.073***	Same Grade	0.650**
	(0.017)		(0.101)
Age	-0.034***	Same SE[e]	0.128***
	(0.005)		(0.045)
Male	0.060***	Same RA[f]	0.111**
	(0.023)		(0.045)
Self Efficacy	0.039***	Friendship Cost	-0.129***
	(0.012)		(0.029)
Risk Aversion	-0.000	Clustering	0.629***
	(0.003)		(0.159)
English GPA	0.038***	Base Links	1.835***
	(0.009)		(0.247)
Math GPA	0.051***		
	(0.010)		
Part. Peer Effect[c]	0.048***		
	(0.011)		
NonPart. Peer Effect[d]	0.002		
	(0.003)		
10% School PYD	0.026***		
	(0.012)		
Base PYD Probability	0.104***		
	(0.014)		

Note. The posterior sample consists of 100,000 simulations, with the initial 20% discarded as burn-in.This table reports the posterior mean of the simulations, with asymptotic standard errors in parentheses. Each simulated confidence interval (90%, 95%, and 99%) is checked to determine whether it contains zero. Confidence intervals that do not include zero are indicated by */**/*** for the confidence intervals 90%, 95%, and 99%, respectively.

[a] Column (1) results are presented as marginal probabilities, expressed in percentage points.

[b] Column (2) results are presented as relative marginal probabilities, expressed as percentages.

[c] This represents the peer effect among participants from individual friends who are also participants. It also indicates the incentive for two individuals to form a connection if both are participants.

[d] This represents the peer effect among nonparticipants from individual friends who are also nonparticipants. It also indicates the incentive for two individuals to form a connection if both are nonparticipants.

[e] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

[f] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table 1.4: Model Fit.

	Distribution of Prediction			Observed
	Mean	Median	SD	Value
Actions				
$\sum_{i \in I} D_i / n$	0.117	0.112	0.019	0.127
Interaction between Action and Links				
$\sum_{i \neq j} (g_{ij} * D_i * D_j)/n$	0.076	0.069	0.014	0.068
$\sum_{i \neq j} [g_{ij} * (1-D_i) * (1-D_j)]/n$	0.757	0.758	0.045	0.755
Network				
Avg degree	1.267	0.967	0.216	0.970
Min degree	0.304	0.000	0.475	0.000
Max degree	5.325	4.844	2.397	5.295
Density	0.009	0.005	0.085	0.005
$\sum_{i>j>k} g_{ij} * g_{ik} * (1-g_{jk})/n$	0.521	0.467	0.098	0.497
$\sum_{i>j>k} g_{ij} * g_{jk} * g_{ik}/n$	3.020	0.022	3.751	0.066
Network Formation Patterns				
Homophily Index	0.105	0.084	0.029	0.093
Coleman's Inbreeding Homophily Index	-0.094	-0.092	0.005	-0.098
Freeman Segregation Index	0.468	0.475	0.042	0.418

Note. $\frac{\sum_{i \in I} D_i}{n}$ represents the average participation rate in an action-network configuration. The term $\frac{\sum_{i \neq j} g_{ij} \cdot D_i \cdot D_j}{n}$ represents the average number of occurrences where both an individual and her peer participate in the PYD program. Similarly, $\frac{\sum_{i \neq j} g_{ij} \cdot (1-D_i) \cdot (1-D_j)}{n}$ represents the average number of occurrences where both the individual and her peer do not participate in the PYD program. For network structure, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot (1-g_{jk})}{n}$ represents the average occurrence where an individual is a mutual friend of two unconnected individuals in the action-network configuration. Lastly, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot g_{jk}}{n}$ represents the average number of link triangles (or clustering among three individuals) in the action-network configuration.

Table 1.5: Out-of-Sample Validation.

	Distribution of Prediction			Observed
	Mean	Median	SD	Value
Actions				
$\sum_{i \in I} D_i / n$	0.143	0.135	0.016	0.129
Interaction between Action and Links				
$\sum_{i \neq j} (g_{ij} * D_i * D_j)/n$	0.125	0.102	0.033	0.097
$\sum_{i \neq j} [g_{ij} * (1-D_i) * (1-D_j)]/n$	0.606	0.598	0.075	0.582
Network				
Avg degree	0.841	0.836	0.423	0.756
Min degree	0.000	0.000	0.002	0.000
Max degree	3.567	3.423	3.684	3.732
Density	0.043	0.043	0.025	0.041
$\sum_{i>j>k} g_{ij} * g_{ik} * (1-g_{jk})/n$	0.354	0.335	0.145	0.369
$\sum_{i>j>k} g_{ij} * g_{jk} * g_{ik}/n$	0.042	0.038	0.027	0.052
Network Formation Patterns				
Homophily Index	0.079	0.042	0.031	0.055
Coleman's Inbreeding Homophily Index	-0.075	-0.072	0.042	-0.084
Freeman Segregation Index	0.573	0.574	0.067	0.545

Note. $\frac{\sum_{i \in I} D_i}{n}$ represents the average participation rate in an action-network configuration. The term $\frac{\sum_{i \neq j} g_{ij} \cdot D_i \cdot D_j}{n}$ represents the average number of occurrences where both an individual and her peer participate in the PYD program. Similarly, $\frac{\sum_{i \neq j} g_{ij} \cdot (1-D_i) \cdot (1-D_j)}{n}$ represents the average number of occurrences where both the individual and her peer do not participate in the PYD program. For network structure, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot (1-g_{jk})}{n}$ represents the average occurrence where an individual is a mutual friend of two unconnected individuals in the action-network configuration. Lastly, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot g_{jk}}{n}$ represents the average number of link triangles (or clustering among three individuals) in the action-network configuration.

Table 1.6: Bounding the Peer Effect.

	Distribution of Prediction			Observed
	Mean	Median	SD	Value
Actions				
$\sum_{i \in I} D_i / n$	0.092	0.086	0.048	0.127
Interaction between Action and Links				
$\sum_{i \neq j} (g_{ij} * D_i * D_j)/n$	0.056	0.054	0.014	0.068
$\sum_{i \neq j} [g_{ij} * (1-D_i) * (1-D_j)]/n$	1.176	0.800	0.594	0.755
Network				
Avg degree	1.180	0.804	1.536	0.970
Min degree	0.786	0.000	0.858	0.000
Max degree	7.475	4.552	7.072	5.295
Density	0.018	0.004	0.065	0.005
$\sum_{i>j>k} g_{ij} * g_{ik} * (1-g_{jk})/n$	1.952	0.408	1.869	0.497
$\sum_{i>j>k} g_{ij} * g_{jk} * g_{ik}/n$	6.095	0.031	6.538	0.066
Network Formation Patterns				
Homophily Index	0.105	0.00	0.062	0.093
Coleman's Inbreeding Homophily Index	-0.068	-0.064	0.048	-0.098
Freeman Segregation Index	0.754	0.529	0.538	0.418

Note. $\frac{\sum_{i \in I} D_i}{n}$ represents the average participation rate in an action-network configuration. The term $\frac{\sum_{i \neq j} g_{ij} \cdot D_i \cdot D_j}{n}$ represents the average number of occurrences where both an individual and her peer participate in the PYD program. Similarly, $\frac{\sum_{i \neq j} g_{ij} \cdot (1-D_i) \cdot (1-D_j)}{n}$ represents the average number of occurrences where both the individual and her peer do not participate in the PYD program. For network structure, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot (1-g_{jk})}{n}$ represents the average occurrence where an individual is a mutual friend of two unconnected individuals in the action-network configuration. Lastly, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot g_{jk}}{n}$ represents the average number of link triangles (or clustering among three individuals) in the action-network configuration.

Table 1.7: Counterfactual Analysis Scenario 1 - Mandatory Assignment.

Mandatory	Baseline		Predicted (Fixed Network)			Predicted (Endogenous Network)		
Assignment	% Participation	Effect	% Participation	Effect	Multiplier	% Participation	Effect	Multiplier
0%	12.9	-	-	-	-	-	-	-
5%	17.26	4.36	18.05	5.15	1.18	19.32	6.42	1.47
10%	21.61	8.71	23.84	10.94	1.26	25.75	12.85	1.48
20%	30.32	17.42	35.06	22.16	1.27	39.98	27.08	1.55
30%	39.03	26.13	46.18	33.28	1.27	58.01	45.11	1.73
40%	47.74	34.84	57.65	44.75	1.28	73.88	60.98	1.75
50%	56.45	43.55	70.5	57.6	1.32	96.97	84.07	1.93

Note. The first column lists the assignment ratio, which is the proportion of the population receiving assignments. The second and third columns display the baseline participation rate post-assignment and the change in participation rate, respectively, if peer effects are not considered. The fourth to sixth columns present the predicted participation rate, the predicted change in participation rate considering peer effects post-assignment given a fixed network, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change. The seventh to ninth columns show the predicted participation rate, the predicted change in participation rate considering both peer effects and dynamic networks post-assignment, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change.

Table 1.8: Counterfactual Analysis Scenario 2 - Fixed Amount Incentive Boost.

Incentive	Baseline		Predicted (Fixed Network)			Predicted (Endogenous Network)		
Assignment	% Participation	Effect	% Participation	Effect	Multiplier	% Participation	Effect	Multiplier
0%	12.9	-	-	-	-	-	-	-
5%	15.08	2.18	15.34	2.44	1.12	15.47	2.57	1.18
10%	17.26	4.36	17.98	5.08	1.17	19.29	6.39	1.47
20%	21.61	8.71	23.88	10.98	1.26	25.82	12.92	1.48
30%	25.97	13.07	29.51	16.61	1.27	32.63	19.73	1.51
40%	30.32	17.42	35.06	22.16	1.27	39.95	27.05	1.55
50%	34.68	21.78	41.87	28.97	1.33	51.41	38.51	1.77

Note. The first column lists the assignment ratio, which is the proportion of the population receiving incentive boost. The second and third columns display the baseline participation rate post-assignment and the change in participation rate, respectively, if peer effects are not considered. The fourth to sixth columns present the predicted participation rate, the predicted change in participation rate considering peer effects post-assignment given a fixed network, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change. The seventh to ninth columns show the predicted participation rate, the predicted change in participation rate considering both peer effects and dynamic networks post-assignment, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change.

Table 1.9: Counterfactual Analysis Scenario 3 - Fixed Amount Incentive Boost with Target Effort.

Incentive	Baseline		Predicted (Fixed Network)			Predicted (Endogenous Network)		
Assignment	% Participation	Effect	% Participation	Effect	Multiplier	% Participation	Effect	Multiplier
0%	12.9	-	-	-	-	-	-	-
5%	15.11	2.21	17.35	4.45	2.01	17.35	4.45	2.01
10%	17.69	4.79	21.08	8.18	1.71	22.89	9.99	2.09
20%	22.96	10.06	30.20	17.30	1.72	35.51	22.61	2.25
30%	27.15	14.25	41.64	28.74	2.02	48.64	35.74	2.51
40%	32.94	20.04	49.93	37.03	1.85	57.88	44.98	2.24
50%	38.17	25.27	59.51	46.61	1.84	73.65	60.75	2.40

Note. The first column lists the assignment ratio, which is the proportion of the population receiving incentive boost. The second and third columns display the baseline participation rate post-assignment and the change in participation rate, respectively, if peer effects are not considered. The fourth to sixth columns present the predicted participation rate, the predicted change in participation rate considering peer effects post-assignment given a fixed network, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change. The seventh to ninth columns show the predicted participation rate, the predicted change in participation rate considering both peer effects and dynamic networks post-assignment, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change.

CHAPTER

2

SCHOOL ASSIGNMENT WITH ENDOGENOUS CAPACITY

There is now a very large market design literature on assigning students to schools. With very few exceptions, these papers treat the capacity of a school as a fixed, exogenous parameter. We helped design the assignment procedure for the Wake County Public School System's Magnet program (hereafter WCPSS), and we were surprised to discover that the capacity available for assignment and even the choice function itself can depend on the assignment.[1] Our paper addresses this problem: how to assign students to schools when the capacity and choice function are endogenous to the assignment.

A key objective for WCPSS is participation in the magnet program. With this aim in mind, Wake County designed the selection process so that all families would have a chance of being admitted to a magnet. This includes families with no priority at any magnet school. Specifically, Wake County requires that 10% of a magnet school's new assignments are done via lottery (Wake County Public School System 2020a,b). Any condition on "new" assignments creates a technical challenge. A student who attended magnet school A in the previous year is guaranteed to be

[1]The current assignment process was developed by Umut Dur, Robert Hammond, and Thayer Morrill. Dur and Hammond were both instrumental in identifying the problem of endogenous capacities. We are grateful for their many contributions to the development of this project. We would also like to thank Umut Dur and Will Phan for their many helpful comments on this paper.

able to continue at school A in the next year; however, students are free to apply to different magnet programs, and they often do. Therefore, "10% of new assignments" depends on both a school's capacity and the number of students who received new assignments. In other words, the choice function is endogenous to the assignment itself. Note that this same problem occurs for any restriction on new assignments. For example, with affirmative-action quotas, both the number of reserved seats and the preferences of eligible applicants jointly determine assignment outcomes, creating the same endogeneity problem.

Second, the capacity itself can be endogenous to the assignment. The WCPSS student assignment consists of two parts. First, every student is assigned a "base" school. A student has no input on the base school they are assigned. WCPSS also has a large and successful magnet program. The original purpose of the magnet program was to facilitate the voluntary desegregation of schools. To do this, the school system took the majority-black schools and invested significant resources in them (Frankenberg et al. 2008; Frankenberg and Siegel-Hawley 2010). The magnet schools are now regarded as the best schools in the district.[2] The intention was that these schools serve as a "magnet" so that white students would choose to voluntarily attend a majority-black school.

The key point is that, by design, the capacity at a magnet school is assigned in two ways. First, some of the students are assigned to a magnet school as their base school ("base students"). The remaining capacity of each magnet school is assigned through a centralized assignment process.[3] However, magnet programs have unique and diverse curricula (such as language immersion or a focus on STEM subjects or performance arts). It is common for a student to have one magnet school as a base yet apply to attend a different magnet school. Therefore, although a magnet school has a fixed and exogenous total capacity, the capacity available for assignment through the application process is endogenous to the assignment itself. If a student has magnet school A as a base school but is assigned to magnet school B, then it is now possible to assign an additional student to school A. Magnet school A's capacity has effectively increased by one. This is not unique to Wake County. Consider any centralized assignment where a subset of the participating students have already received an assignment. In addition to magnet assignment, examples include waitlist assignments and NYC's supplemental assignment round.[4] In any

[2] For example, in 2023 Washington Elementary School was named the top magnet elementary school in the country. See Patterson (2023).

[3] Beginning with the 2015-16 academic year, the WCPSS implemented the Deferred Acceptance algorithm (Gale and Shapley, 1962) for centralized student-school assignments. See Dur et al. (2018) and Dur et al. (2022) for more details.

[4] In the 2003-2004 academic year, NYC high school matching process underwent reforms, improving waitlist assignments and introducing the Supplemental Assignment Round (SAR). The SAR allowed unmatched students to submit new preferences for schools with vacancies, ensuring nearly all students received a placement. For further details, refer to Abdulkadiroğlu et al. (2009), Abdulkadiroğlu et al. (2017), Pathak (2017), and Pathak and Sethuraman (2011).

such application, the assignment capacity of a school will be endogenous.

The endogenous capacity problem is fairly straightforward to solve. Our problem is closely related to housing assignments with existing tenants. The difference is that the current WCPSS procedure does not recognize that there are existing tenants. The solution is to add seats for applicants whose base school is a magnet school and to create priorities for these seats so that a base student will never be rejected. We define a school's *shadow capacity* to be the number of base students assigned to it. We modify DA by increasing the capacity at each school by that school's shadow capacity. Next, we guarantee that no student is rejected by her base school by giving base students the highest priority at the school (with ties broken arbitrarily). Given a base assignment, we define the algorithm *Shadow DA* to run DA on the augmented capacities and priorities. We show that Shadow DA is strategyproof and fair. In fact, it makes the student-optimal, fair assignment. Shadow DA is closely related to variations of DA previously introduced in the literature by Combe et al. (2022) and Combe and Schlegel (2024). We discuss the similarities in differences in more detail in the section on related literature.

The endogenous choice functions are a more challenging problem to solve. Intuitively, our approach is to define a tâtonnement process that determines the number of students who will receive a new assignment. We then run DA on these augmented capacities. Our challenge is first to find the correct capacities and then to show that the choice function is consistent with these capacities. Surprisingly, we show that our recursive version of DA, which we call *Decremental DA*, is strategyproof.

We start Decremental DA by assuming that all current students at a school will receive a new assignment. For motivation, assume there is a school with 50 current students and a total capacity for 100 students. Further, assume that all current students are applying to a different magnet program. We run DA assuming that all 50 students will receive new assignments. In other words, we assume that the available capacity for the school is 100, and when we choose among applicants, we choose 10 seats (10% of 100) via lottery. This is the maximum possible number of new assignments. Suppose, for illustration, that only 20 of the 50 current students were admitted to a new school. Our school is now oversubscribed: 100 new students were accepted and we must accept the 30 current students who did not receive a new assignment. We reduce the capacities of all schools by the number of current students who were not admitted to a new school. In this case, our school's capacity is reduced from 100 to 70. We then rerun DA. Note that every student's assignment is weakly worse off under the new assignment than the previous assignment as the capacities at all schools have (weakly) decreased. Therefore, there may now be fewer current students who receive a new assignment. At the end of each round, we decrease the capacities by the number of current students who do not receive a new assignment and then run DA on the new capacities. As capacities monotonically decrease, eventually, the

process converges. The assignment made by Decremental DA is the DA assignment under the final capacities.

We show that Decremental DA solves our problem of endogenous choice functions. In the final assignment, 10% of the new assignments are allocated via lottery. This approach will work for any similar restriction on "new" assignments, such as a percentage of new assignments must be made to people from a particular demographic group. Further, we show that Decremental DA is strategyproof and outputs the student-optimal fair assignment. Interestingly, the corresponding incremental tâtonnement process is not strategyproof.[5] Shadow DA modifies the priorities at schools to guarantee that existing students are accepted. This modification has been considered by a number of papers in the literature.

This paper is organized as follows: Section 2 reviews literature related to this paper. Section 3 describes WCPSS magnet schools and their assignment processes in more detail. In Section 4, we present our model's setup and definitions. Section 5 introduces variations of the DA algorithm, namely Shadow DA and Decremental DA, as potential solutions for addressing endogenous capacity issues. Additionally, this section discusses the integration of a 10% lottery in the assignment process in the WCPSS context. This paper concludes in Section 6.

2.1 Related Literature

This study contributes to the school choice literature led by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003). Other notable works that also examine flexible capacities include Dur and Van der Linden (2021) and Avataneo and Turhan (2021). Dur and Van der Linden (2021) investigate capacity design strategies that aim to allocate additional seats across schools in accordance with students' reported preferences. On the other hand, Avataneo and Turhan (2021) focuses on the capacity transfers across various types of slots within a specific school when specific priorities for each slot are present. A critical distinction between these works and ours is that in our model, the capacities are determined endogenously by assignments and outcomes themselves, and each student is initially endowed with a base school.

To the best of our knowledge, Decremental DA is a novel algorithm. Shadow DA algorithm is very closely related to other variations of DA where the priorities have been modified in order to guarantee individual rationality (Compte and Jehiel 2008; Guillen and Kesten 2012). This is a generalization of the classic housing assignment with existing tenants problem (introduced by Abdulkadiroğlu and Sönmez 1999) in order to allow for priorities. Examples include Guillen and Kesten (2012) (dorm room assignment at MIT), Pereyra (2013) (teacher assignment in

[5]We thank Umut Dur for this observation.

Mexico), and Combe et al. (2022) (teacher assignment in France). Each solution uses DA where the priorities have been modified in order to give current occupants the highest priority. This mechanism is further studied in Combe and Schlegel (2024).

Our problem of endogenous capacities is very similar both in spirit and in the solution. However, there are subtle differences. Our initial problem can be viewed as the assignment of students to unoccupied spots in dorm rooms. Our insight is a simple one: this problem can be extended to the housing problem with existing tenants by recognizing that some of the applicants currently live in a dorm room. Our solution is to adjust both the capacities and the priorities.

The difference between the assignment problem considered in Combe et al. (2022) and Combe and Schlegel (2024) is that the existing tenants are already accounted for, and consequently, capacities are fixed. In this model, it is impossible for an assignment to be individually rational and respect priorities (Combe et al. 2022, Proposition 1). This result is very intuitive. Once the priorities have been changed to give a current occupant the highest priority, if the current occupant is assigned to the object, then it may violate the priority of any student she was moved above. However, in our model, there is no conflict between IR and respecting priorities. The reason is, again, very natural. We have added seats for each current occupant, and conceptually, we have only changed the priorities for these artificial seats. If a current occupant is given a new assignment, the change in priorities at her original assignment is irrelevant. If she does not receive a new assignment, then she is assigned to one of the artificial seats. This cannot create a priority violation at any of the "real" seats.

Our paper also builds on the literature of dynamic matching and reassignment, which primarily focuses on kidney exchange and housing allocations, initiated by Shapley and Scarf (1974) with their study on the housing market problem and the core's non-emptiness. They introduced the Top Trading Cycles allocation rule for core outcomes. Abdulkadiroğlu and Sönmez (1999) expanded on housing allocations considering trades among existing tenants, introducing a mechanism ensuring Pareto efficiency, individual rationality, and strategyproofness. Roth et al. (2004) applied these concepts to kidney exchange designs, highlighting efficiency and incentive compatibility. Compte and Jehiel (2008) addressed position re-allocation in organizations, while Anderson et al. (2015) demonstrated how kidney exchange designs benefit renal disease patients by overcoming donor-recipient incompatibilities.

Other closely related literature includes studies focusing on school assignment problems with an initial endowment. Hamada et al. (2017) explore a school choice problem that involves imposed minimum quotas for each school, where each student has a default school she would have attended if not assigned others. Their study demonstrates that there does not exist a strategyproof mechanism that is both efficient and fair when minimum quotas are in place.

Ehlers et al. (2014) show that if these minimum quota constraints are hard constraints, no mechanism can satisfy fairness, constrained nonwastefulness, and strategyproofness. In our application, there are no minimum quotas, but each student has a "base assignment" which serves the same role as a default school.

2.2 School Assignment in Wake County

The Wake County Public School System (WCPSS) is a leading entity in the U.S., having one of the largest and oldest magnet programs in the country. With 159,995 students spread across 198 schools during the 2023-2024 academic year, Wake County is the 14th largest school district in the United States and is the largest district in North Carolina. Since the inception of its first magnet elementary school in 1978, WCPSS has expanded its magnet offerings to over 40 schools. This constitutes over a fifth of all schools within the district. Reflecting this, WCPSS's magnet school program is ranked 12th in the nation in terms of the number of magnet schools.[6]

The original goal of the magnet program was to decrease segregation in public schools by encouraging voluntary reassignment. This strategy involves enhancing schools in economically disadvantaged areas with innovative curricula and additional resources to attract students from wealthier neighborhoods. The district is currently experiencing rapid growth, with an increase of 3.3% over the past decade.[7] Therefore, capacity utilization is a priority of the school district, and the magnet program is a key component. WCPSS states that the four primary objectives of the magnet program are: to reduce high concentrations of poverty; promote school integration; maximize the use of school facilities; and provide innovative and/or expanded educational opportunities.

Students enroll in a magnet school through an application process. Each student is initially assigned to a default school (called a *base school*) based on school attendance zones, which connect a student's home address to a specific zoned school. Note that within magnet schools, approximately half of the students come from the local school attendance boundary (i.e. half of the students are assigned to it as a default school). The remaining students are assigned via the centralized application process.

Critical to this paper, students who are already zoned for one magnet school frequently apply to a different magnet school. Specifically, out of the 13,000 applicants for 2015-16 SY in

[6] For further information on these statistics, refer to the National Center for Education Statistics Common Core of Data, WCPSS website or the following links:
https://nces.ed.gov/programs/digest/d22/tables/dt22_215.30.asp
https://www.wcpss.net/domain/100

[7] See https://assistive.eboardsolutions.com/Meetings/Attachment.aspx?S=920&AID=385518&MID=15063 for details.

Wake County, approximately 8,000 have been assigned a magnet school as their base school.

Magnet school placements are allocated via DA. The school priority points are based on factors such as income, whether the student's base school is overcrowded, and whether the student has a sibling attending the school.[8] When applicants have equal priority points, a lottery system is used to break ties, though the lottery number remains undisclosed to students. For non-entry level grades, students already enrolled at a school are "pathway students." WCPSS guarantees these students can continue at their current school. Approximately 27.9% of pathway students seek enrollment in a different magnet school.

WCPSS specifies that 90% of magnet school seats are allocated based on the priority points, while the remaining 10% are filled through a pure lottery, independent of priority points. This lottery was introduced to encourage participation in the magnet application process. In particular, it ensures that students from neighborhoods not previously prioritized have the opportunity to attend magnet schools.

2.3 Model

We start with the canonical school assignment model. There is a set of students, I, who are to be assigned to a finite set of schools, S. Each student has strict preferences over all schools, and we denote student i's preferences by P_i.[9] We denote by R_i the weak preferences associated with P_i.

Each school s has a strict priority ranking of students denoted by $\succ_s$. Our results hold for more general choice rules, but we assume a strict priority ranking both for expositional convenience and because this assumption fits the WCPSS application. We let $P=(P_i)_{i\in I}$ and $\succ=(\succ_s)_{s\in S}$ denote the profile of preferences and priorities, respectively. For notational convenience, we will typically suppress P and $\succ$.

An assignment is a function $\nu: I \to S$. In a slight abuse of notation, given an assignment ν, we let $\nu(s)$ be the students assigned to school s. Mathematically, $\nu(s) := \{i \mid \nu(i) = s\}$. We will discuss later what constitutes a feasible assignment in our model.

The focus of our paper is on the centralized assignment of students to magnet schools. Our

[8]The priority points system varies slightly between elementary and secondary levels, factoring in considerations such as sibling attendance, the performance of the residential area, and whether the base school is overcrowded. For middle and high schools, priorities further include existing magnet pathway considerations and the level of overcrowding at the base school. See Board of Education Policy 6200 for more details on priority point determination rules.

[9]In WCPSS, every student is assigned to a school. Therefore, it is not possible for a student to declare some schools unacceptable. Of course, many students choose to attend a private school or a charter school if her assignment is "unacceptable", but this decision is outside the scope of the assignment process where, by law, all students must be given a public school assignment.

departure from the standard model is that there is an exogenously given **base assignment** μ of students to the schools. That is, every student i is initially assigned to a base school, denoted as $\mu(i) \in S$. We will refer to the students whose base assignment is s as the base students at s. A student who is not assigned to a magnet school through the centralized process is assigned to her base school. Therefore, given base assignment μ and magnet assignment ν, there is an induced final assignment

$$\phi^{\mu,\nu}(i) := \begin{cases} \nu(i), & \text{if } \nu(i) \neq \emptyset \\ \mu(i), & \text{otherwise} \end{cases}$$

In the centralized assignment (the magnet assignment), each school s has an exogenously given capacity q_s^a. We call this the school's **assignment capacity**. We define an assignment ν to be myopically feasible if for every school s, $|\nu(s)| \leq q_s^a$. This is the feasibility condition currently used in WCPSS. In actuality, each magnet school is the base school for some of the students. A student who is not assigned to another magnet school attends her base school. However, if a base student at s is assigned to a different magnet school, then there is an additional seat available for assignment at s. Therefore, the actual available capacity of a school is endogenous to the assignment itself.

We define a school s's shadow capacity, denoted $\tilde{q}_s$ to be the number of students assigned to it as a base school. More formally, for every school s, $\tilde{q}_s = |\mu(s)|$. The total capacity of the school s is its assignment capacity plus its shadow capacity. Therefore, given base assignment μ and magnet assignment ν, the final assignment $\phi^{\mu,\nu}$ is feasible if for every school s,

$$|\phi^{\mu,\nu}(s)| \leq q_s^a + \tilde{q}_s.$$

The final assignment is induced from the magnet assignment. Therefore, in order for the final assignment to be feasible, there is an implied feasibility constraint on the magnet assignment. We define a magnet assignment to be feasible if the induced final assignment is feasible. The mathematical condition is as follows. Given base assignment μ, magnet assignment ν is feasible if for every school s,

$$|\nu(s)| + |\{i \in \mu(s) | \nu(i) = \emptyset\}| \leq q_s^a + \tilde{q}_s. \tag{2.1}$$

Intuitively, an assignment is wasteful if a student desires a school and that school is below its capacity. As we have multiple capacities, we correspondingly have multiple definitions of wastefulness. A magnet assignment ν is myopically wasteful if there is a student i and a school s such that $s \, P_i \, \nu(i)$ and $|\nu(s)| < q_s^a$. A final assignment $\phi^{\mu,\nu}$ is wasteful if there is a

student i and a school s such that $s\ P_i\ \phi^{\mu,\nu}(i)$ and $|\phi^{\mu,\nu}(s)| < q_s^a + \tilde{q}_s$. A magnet assignment ν is wasteful if the final assignment it induces is wasteful. An assignment ν' **Pareto dominates** an assignment ν if for every student i, $\nu'(i)\ R_i\ \nu(i)$ and for some student i, $\nu'(i)\ P_i\ \nu(i)$. Assignment ν is **Pareto-efficient** if it is not Pareto-dominated by a feasible assignment.

For the centralized assignment (the magnet assignment), we only consider variations of DA. Therefore, the outcome will be fair when we restrict our attention to the magnet assignment and the magnet capacities. However, our objective is to make the final assignment fair. In WCPSS, a student is not allowed to apply to her base school; consequently, students do not have priorities at their base school. Therefore, we cannot use the standard definition of justified envy for our final assignment. Nonetheless, there is a straightforward generalization to our application: a student's base assignment is guaranteed; therefore, a student cannot have justified envy of a base student. Given a final assignment $\phi^{\mu,\nu}$ (induced from base assignment μ and magnet assignment ν), student i has justified envy of student j if (i) $\phi^{\mu,\nu}(j) \neq \mu(j)$ (in words, j is not a base student for $\nu(j)$) and (ii) $\phi^{\mu,\nu}(j)\ P_i\ \phi^{\mu,\nu}(i)$ and $i \succ_{\phi^{\mu,\nu}(j)} j$. The final assignment $\phi^{\mu,\nu}$ is fair if no student has justified envy.

A school $s \in S$ is **achievable** for student $i \in I$ if there exists a fair final assignment $\phi^{\mu,\nu}$ such that $\phi^{\mu,\nu}(i) = s$. A magnet assignment ν is **student-optimal fair** if in the final assignement $\phi^{\mu,\nu}$, every student is assigned to her favorite achievable school. A mechanism is **strategyproof** if, for all $i \in I$, reporting the truth is a weakly dominant strategy. Let Φ denote a mechanism, and $\Phi_i(P)$ be its output assignment for i given preference list P. Then, mathematically, a mechanism Φ is strategyproof if $\Phi_i(P)\ R_i\ \Phi_i(P_i', P_{-i})$ for every student in I and every other preference list P_i'.

2.4 Variations of Deferred Acceptance

There are two reasons why capacity is endogenous in the WCPSS assignment mechanism. First, if a base student at magnet school A is assigned to a different magnet school, more students could be assigned to A. The second stems from the WCPSS requirement that 10% of new assignments at a magnet school be done via lottery. For an entry-grade, i.e. kindergarten, 6^{th}, and 9^{th} grades, all assignments are new assignments and this requirement is well-defined. For all other grades, the number of "new" assignments inherently depends on the assignment itself.

To illustrate, consider second-grade assignment at elementary school A. All current first-grade students are guaranteed a second-grade assignment at A, but suppose that all of these students apply to other schools. If they are all admitted to other schools, then 10% of A's seats should be allocated via lottery. If 50% are admitted elsewhere, then 5% of A's seats should be allocated via lottery, and so on. The proportion of base students and/or pathway students

accepted influences the number of seats at A assigned randomly, leading to a varying profile of accepted students, which affects A's capacity further.

2.4.1 Shadow DA

We introduce a natural modification of DA and show that it is superior to the standard DA along a number of dimensions. DA is formally defined in the Appendix. Intuitively, we modify the capacity of each school to account for the school's base students, and we modify the priorities of each school so that no base student is rejected. Standard DA is strategyproof and produces a fair assignment. We will show that our modification is also strategyproof, also fair, but the resulting assignment Pareto dominates the standard DA assignment.

Our parameters are a base assignment μ, assignment capacities q^a, and magnet priorities $\succ$. In WCPSS, a student is not allowed to apply to her base school (this school is guaranteed); therefore, base students do not have priorities. There is a straightforward way to extend priorities to include base students. Intuitively, we put the base students at the top of the priority list and otherwise keep the priorities the same. More formally, given priorities $\succ$, we define school s's base-augmented priorities, denoted $\tilde{\succ}_s$, as follows. For any students i and j:[10]

$$i \tilde{\succ}_s j \Leftrightarrow \begin{cases} \mu(i) = s \text{ and } \mu(j) \neq s, \text{ or} \\ \mu(i) \neq s \neq \mu(j) \text{ and } i \succ_s j. \end{cases}$$

As a reminder, the shadow capacity for school s, $\tilde{q}_s$, is the number of students assigned to it as a base school. Given a base assignment μ, assignment capacities q^a, and magnet priorities $\succ$, let $\tilde{q}$ and $\tilde{\succ}$ be the corresponding shadow capacities and base-augmented priorities. We define the Deferred Acceptance with Shadow Capacities (hereafter Shadow DA) to be DA run where the capacity of each school s is set to $q^a_s + \tilde{q}_s$ and the priorities are set to $\tilde{\succ}_s$.

We provide a simple example to illustrate and motivate Shadow DA.

Example 1. *There are three magnet schools, $S = \{a, b, c\}$, $q^a = (1,1,1)$, i.e. each has one seat, and four students, $I = \{i, j, k, l\}$. Student i has school a as a base school; student j has school b as a base school; and the base schools of the remaining students are not magnet schools. The priorities of each school and the preferences of each student are as follows:*

Under DA, the resulting outcome ν is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{l} \\ b & \emptyset & c & a \end{pmatrix}$$

[10] In this definition, all base students are given equal priority. As the number of base students is less than the total capacity of the school, breaking ties between base students is never necessary.

Priority			Preference			
$\succ_a$	$\succ_b$	$\succ_c$	P_i	P_j	P_k	P_l
l	i	l	b	a	a	b
k	l	k			c	a
j	k	j			b	c
		i				

Therefore, j is assigned to her base school and the final assignment is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{l} \\ b & b & c & a \end{pmatrix}$$

The shadow capacities of the magnet schools are $(1,1,0)$. Therefore, the total capacities are $(2,2,1)$. The base-augmented priorities are as follows:

Priority		
$\tilde{\succ}_a$	$\tilde{\succ}_b$	$\tilde{\succ}_c$
i	j	l
l	i	k
k	l	j
j	k	i

Under Shadow DA, the resulting outcome $\tilde{\nu}$ is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{l} \\ b & a & a & b \end{pmatrix}$$

We highlight several points from Example 1. The Shadow DA assignment is feasible, fair, and Pareto dominates the standard DA assignment. Under the standard DA assignment, i is assigned to school a. Therefore, she will not use the seat allocated to her at magnet school b. This seat is not assigned under standard DA, but it is assigned under Shadow DA. Furthermore, this instance shows that the standard DA, despite being feasible, becomes wasteful in scenarios involving endogenous capacity.

Theorem 1. *Shadow DA:*

- *produces a feasible assignment;*
- *is both fair and non-wasteful; and*
- *is strategyproof.*

Proof. Let ν be the outcome under Shadow DA, and let $\phi^{\mu,\nu}$ be the final assignment induced by ν and base assignment μ. Feasibility, fairness, and non-wastefulness are defined relative to the induced final assignment and not the outcome of DA. However, we have constructed priorities so that each student i will be accepted to her base school if she applies. In particular, for every student i, $\nu(i)\, R_i\, \mu(i)$. This implies that $\nu(i) \neq \emptyset$. Therefore, $\phi^{\mu,\nu} = \nu$. Shadow DA is feasible because for every school s, the capacity of each school was set to $q_s^a + \tilde{q}_s$. As the induced final assignment is the same as the outcome from a particular implementation of DA, Shadow DA inherits the properties of DA. Shadow DA is fair, non-wasteful, and strategyproof because DA is fair, non-wasteful, and strategyproof. □

The main benefit of Shadow DA is that it maximizes the number of seats that are assigned using DA. A classic result in the matching literature is that in the marriage market, having additional women makes every man at least as well off in the male-optimal stable assignment (see Gale and Sotomayor 1985; Roth and Sotomayor 1990, for a full discussion) . The following classic lemma is the application of this result to the school assignment problem.

Lemma 1. *(Roth and Sotomayor 1990; Crawford 1991; Ehlers and Klaus 2004) If $q \leq q'$, then for every student i, $DA(q')(i)\, R_i\, DA(q)(i)$.*

Proof. When additional resources become available, each agent is weakly better off, a principle known as resource-monotonicity, originally introduced by Chun and Thomson (1988) and Crawford (1991). By Theorem 1 from Crawford (1991), we see that responsive DA is resource-monotonic in the context of college admissions and school assignment problems. Therefore, if the capacity of each school is weakly increased, then all students must become weakly better off. This concept is further explored in the context of house allocations by Ehlers and Klaus (2004, 2011) and Kesten (2009). □

A direct consequence of Lemma 1 is that students prefer their assignment under Shadow DA to under standard DA. We provide a proof for completeness.

Theorem 2. *Let ν be the outcome of DA, and let $\tilde{\nu}$ be the outcome of Shadow DA. Then for every student i, $\tilde{\nu}(i)\, R_i\, \nu(i)$.*

Proof. Assume for contradiction that $\nu(i) P_i \tilde{\nu}(i)$ for some student $i \in I$. Student i must have been rejected (under Shadow DA) by $\nu(i)$. Let i be the first student rejected by $s = \nu(i)$. This implies that $q^i_{\nu(i)} + \tilde{q}_{\nu(i)}$ many students have been temporarily accepted and that all of these students have higher priority at $\nu(i)$ than i. As $\mu(i)$'s capacity has weakly increased, one of these students was not assigned to $\mu(i)$ under DA. This student has not yet applied to her DA assignment or else i would not be the first student rejected from her DA assignment. As

she strictly prefers $\mu(i)$ to her DA assignment and she has higher priority than i, she forms a blocking pair to the DA assignment. This contradicts the fairness of DA. Therefore, all students weakly prefer $\tilde{\nu}(i)$ than $\nu(i)$. □

Theorem 2 shows that the Shadow DA assignment is never worse than the DA assignment, and Example 1 demonstrates that the Shadow DA assignment can Pareto dominate the DA assignment. In addition to its practical relevance, Theorem 1 has interesting theoretical implications. It is a stylized fact in the matching literature that DA is the unique strategyproof and fair assignment mechanism.[11]

Our results show that in practice there can exist more than one fair, strategyproof assignment. This does not contradict any existing result. Rather, it highlights dimensions of the assignment process that have been overlooked by the literature. First, in some contexts, a school's capacity is not fixed but rather endogenous to the assignment. And second, the standard definition of individual rationality may not apply to all assignment problems. The standard definition of individual rationality is that no student is assigned to a school she deems unacceptable. In our application, a student must receive an assignment. Therefore, in our context, individual rationality is that a student is not assigned to a school worse than her base assignment.

Corollary 1. *There are multiple fair and strategyproof assignment mechanisms.*

Proof. DA and Shadow DA are both fair and strategyproof. Example 1 demonstrates they are not the same mechanism. □

2.4.2 Decremental DA

In this section, we introduce an alternative implementation of Shadow DA. The algorithm is interesting in its own right, but its main purpose is to resolve the second source of capacity endogeneity: WCPSS requires that 10% of new assignments at a magnet school be done via lottery. For intermediate grades, the number of new assignments depends on the assignment itself; therefore, we cannot allocate 10% of the seats using the shadow capacity without potentially oversubscribing schools. Here, we introduce a decremental capacity "tâtonnement" process that solves this problem.

[11] Alcalde and Barbera (1994) was the pioneering work to characterize DA as a unique strategyproof and fair assignment mechanism. They introduced the top dominance condition and demonstrated that if top dominance is satisfied for each agent on one side, a unique fair and strategyproof rule exists. The findings of Balinski and Sönmez (1999) directly follow as an immediate corollary of their work. Akahoshi (2014) revisited this topic, identifying the "no-detour" condition as a necessary condition for the existence of a unique fair and strategyproof rule. Subsequent research by Afacan and Dur (2020), Sakai (2011), and Hirata and Kasuya (2017) generalize Alcalde and Barbera (1994)'s characterization, extending it to matching markets with contracts.

In the WCPSS magnet assignment, a student is not allowed to apply to her base school: she only applies to schools she strictly prefers to her base. Therefore, in our magnet assignment, we truncate each student's preferences above her base school, and a student who is unassigned during the magnet assignment is assigned to her base school. The most optimistic view is that every base student will receive a new magnet assignment. In that case, the capacity we have available is each school's initial magnet capacity (q_s^a) plus the number of base students ($|\mu(s)|$).

Decremental DA starts by setting each school s's capacity equal to $q_s^1 = q_s^a + |\mu(s)|$, running DA on these capacities, and checking to see how many base students at s did, in fact, get a new assignment. Call ν^1 the DA assignment on these (optimistic) capacities. Note that it is quite possible that we have assigned too many students to a school s. A base student at s who does not receive a magnet assignment ($\mu(i) = s$ and $\nu^1(i) = \emptyset$) is assigned to s in the final assignment. Therefore, if s is assigned $q_s^1 = q_s^a + |\mu(s)|$ students during the assignment phase and one or more of s's base students do not receive an assignment, then we have exceeded s's total capacity.

In this case, we decrease the capacity at s by the number of base students who did not receive a new assignment. There are two equivalent ways we can write this capacity: $q_s^2 = q_s^a + |\{i \in \mu(s) | \nu^1(i) \neq \emptyset\}|$ or $q_s^2 = q_s^1 - |\{i \in \mu(s) | \nu^1(i) = \emptyset\}$. Call ν^2 the DA assignment on these capacities. It is clear that for every school s, $q_s^2 \leq q_s^1$. Therefore, by Lemma 1, for every student i, $\nu^1(i) \, R_i \, \nu^2(i)$. As it is entirely possible that more students are unassigned by ν^2 than ν^1, we update the capacities at each school. For each school s, we set $q_s^3 = q_s^a + |\{i \in \mu(s) | \nu^2(i) \neq \emptyset\}|$, and we define ν^3 to be DA run on capacities q^3. We repeat this process by checking to see if any additional students are now unassigned, and if that is the case, we reduce the capacity at that student's base school. As the capacities decrease in each round and are bounded below by the assignment capacities, eventually, the process must stop. The assignment made by the Decremental DA is the DA assignment on these final capacities.

Decremental DA

Parameters: a base assignment μ, student preferences P, assignment capacities q^a, and magnet priorities $\succ$.

Initialize: We truncate each student's preferences immediately above her base school. Formally, for each student i, P_i' is defined by: (i) for every schools s and s', $s \, P_i' \, s'$ if and only if $s \, P_i \, s'$; and (ii) for every school s such that $\mu(i) \, R_i \, s$, $\emptyset \, P_i' \, s$. For each school s, define $\Delta_s^1 := |\mu(s)|$ and $q_s^1 := q_s^a + \Delta_s^1$.

Round k ($k > 1$): Let $\nu^k = DA(q^k)$. For each school s, define $\Delta_s^{k+1} := |\{i \in \mu(s) | \nu^k(i) \neq \emptyset\}|$. Note that Δ_s^{k+1} is the number of base students assigned to a different school at step k.[12] For

[12] As a reminder, we truncated preferences above a student's base school. Therefore, if $\nu^k(i) \neq \emptyset$, then i was assigned to a school different than her base school.

each school s, set $q_s^{k+1} := q_s^a + \Delta_s^{k+1}$.

Output: Let n be the first round where $q^n = q^{n+1}$. The assignment made by Decremental DA is $\nu^n = DA(q^n)$.

Notation: We will refer to Δ^k, q^k, and ν^k as defined in the algorithm above. We let ν denote the outcome of Decremental DA (i.e. ν^n defined on the final capacities). We will denote the final assignment induced by Decremental DA as δ. Specifically, for each student i, $\delta(i) = \nu(i)$ if $\nu(i) \neq \emptyset$ and $\delta(i) = \mu(i)$ otherwise.

We give a formal proof in the Appendix that Δ^k is decreasing in k. As it is bound below by $\bar{0}$, this implies that, eventually, the algorithm must terminate. Below is an example of Decremental DA.

Example 2. *There are seven magnet schools, $S = \{a,b,c,d,e,f,g\}$, $q^a = (0,1,0,0,0,0,1)$, and seven students, $I = \{i,j,k,\ell,m,n,p\}$. The student priority for each school $s \in S$, and the preferences and base schools of each student $i \in I$ are as follows:*

Priority

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$	$\succ_f$	$\succ_g$
m	m	ℓ	m	p	j	j
n	n	m	j	n	ℓ	ℓ
p	i	j	k	ℓ	k	k
k	ℓ	i	i	i		n
	k			k		
				j		

Preference

	P_i	P_j	P_k	P_ℓ	P_m	P_n	P_p
	c	e	b	c	b	b	a
	e		e	a	e	a	e
				f	f	d	
μ	a	b	c	d	e	f	g

At the outset of the Decremental DA process, we set $\Delta_s^1 = (1,1,1,1,1,1,1)$ and set $q^1 = q_s^a + \Delta_s^1 = (1,2,1,1,1,1,2)$. The DA assignment on these capacities, ν^1, is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ e & \emptyset & \emptyset & c & b & b & a \end{pmatrix}$$

We update the capacities by the number of students who did not receive a magnet assignment. In this case, we set $\Delta_s^2 = (1,0,0,1,1,1,1)$ and set $q^2 = q_s^a + \Delta_s^2 = (1,1,0,1,1,1,2)$.[13] The assignment from the second round, ν^2, is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & f & b & a & e \end{pmatrix}$$

[13]This shows the motivation for this adjustment. If we used ν^1 as the final assignment, j, m, and n would all be assigned to b, which exceeds b's total capacity of two.

Now $\Delta_s^2 = (0,0,0,1,1,1,1)$ as i no longer receives a new assignment. We set $q^3 = (0,1,0,1,1,1,2)$, and the outcome from the third round, ν^3, is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & f & b & d & e \end{pmatrix}$$

Note that the same number of students receive assignments in round 3 as in round 2. That is $q_s^4 = q_s^3$ for every school s. Therefore, the outcome produced by Decremental DA, ν, will be ν^3.

Our main result is to show that Decremental DA is strategyproof and that it makes the student optimal fair assignment. In other words, Decremental DA and Shadow DA produce the same assignment. However, the proof that Decremental DA is strategyproof is not as straightforward as the proof for Shadow DA. The reason is that there is additional scope for manipulation in Decremental DA because a student's report can change the assignment capacity of a school. We illustrate this with the following example. There is a natural algorithm that is symmetric to Decremental DA. Under **Incremental DA**, we start with a school's assignment capacity and run DA. In each round, we increase the capacity of a school by the number of base students that receive a magnet assignment in this round but did not in the last. We provide a formal definition in the Appendix. Even though Incremental DA uses DA to make its final assignment, the example below shows that Incremental DA is not strategyproof.[14] To see how manipulation is possible under Incremental DA, let's reconsider Example 2.

Under Incremental DA, DA was first run on assignment capacity vector $q^a = (0,1,0,0,0,0,1)$. The resulting outcome for the first round, ν_{Incre}^1, is

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & \emptyset & b & \emptyset & \emptyset \end{pmatrix}$$

Since m is assigned to b, the capacity for m's base school e is thereby increased by 1. We then rerun DA on updated capacity vector $(0,1,0,0,1,0,1)$ and obtain outcome for the second round, ν_{Incre}^2:

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & \emptyset & b & \emptyset & e \end{pmatrix}$$

We adjust the capacity and rerun DA to reflect the release of p's base seat from school g. Nevertheless, the assignment outcome remains the same as no students apply to school g. Therefore, Incremental DA yields an assignment where ν_{Incre} is equivalent to ν_{Incre}^2.

[14] We are extremely grateful to Umut Dur, who recognized that Incremental DA is manipulable. Our example is based on the example he showed us.

Let the assignment from Decremental DA be denoted as ν_{Decre} and the outcome from Incremental DA be denoted as ν_{Incre}, it is evident that every student i weakly prefer $\nu_{Decre}(i)$ over $\nu_{Incre}(i)$. That is, the outcome from Decremental DA Pareto dominates the one from Incremental DA.

However, some students may benefit from misreporting their preferences. Specifically, both n and ℓ can improve if either one of them, or both, include school g in their preference lists and report the following revised preferences:

Misreported Preference

	$\tilde{P}_\ell$	$\tilde{P}_n$
	c	b
	a	a
	f	d
	g	g
μ	d	f

Let's consider a case where only ℓ misreports. With the misreported preference list, the resulting assignment for the first round, $\tilde{\nu}^1_{Incre}$, becomes:

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & g & b & \emptyset & \emptyset \end{pmatrix}$$

Now, two students have been assigned to new schools, resulting in an increment of one in the capacity for both schools d and e. We subsequently rerun DA using the updated capacity vector $(0,1,0,1,1,0,1)$ and derive the outcome for the second round, $\tilde{\nu}^2_{Incre}$:

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & g & b & d & e \end{pmatrix}$$

Since n is assigned to d, the capacity for n's base school f is thereby increased by 1. We then rerun DA on the updated capacity vector $(0,1,0,1,1,1,1)$ and obtain outcome for third round, $\tilde{\nu}^3_{Incre}$:

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} & \underline{\ell} & \underline{m} & \underline{n} & \underline{p} \\ \emptyset & \emptyset & \emptyset & f & b & d & e \end{pmatrix}$$

Since no more new students are assigned, and thus $q^3_s = q^4_s$ for all school s, Incremental DA terminates. The final assignment is $\tilde{\nu}^3_{Incre}$.

Although students cannot benefit from misreporting preferences in a single run of DA, changes in capacities that trigger a new run of DA can impact their final assignments in such cases. If capacities are adjusted, a student's final assignment, made with these larger capacities, can be strictly better than their interim assignment. For instance, consider student ℓ. Initially, ℓ's outcome appears to be worse: at $\nu^2_{Increase}$, she is assigned to school g, which is worse than her base school. However, this assignment indirectly increases capacity at school d due to the acceptance of student n. This, in turn, facilitates ℓ's own acceptance at school f.

We establish some technical properties of Decremental DA that will be useful in later proofs.

Lemma 2. *For any $k < k'$ and any student i,*

$$\nu^k(i)\, R_i\, \nu^{k'}(i).$$

Moreover, $\{i : \nu^k(i) = \emptyset\} \subseteq \{i : \nu^{k'}(i) = \emptyset\}$.

Proof. This is a straightforward consequence of Lemma 3, the details of which, including its proof, can be found in the appendix. Since every school's capacity has weakly decreased, every student is made weakly worse off. A straightforward consequence of this is that if, in any round, a student is not assigned to a school, then her Decremental DA assignment is her base school. □

We first show that Decremental DA always selects the student optimal fair assignment while allowing capacity to depend on the assignment itself. This will also be the basis of our proof that Decremental DA is strategyproof: since a student receives her best possible achievable assignment, she has no incentive to misreport.

Theorem 3. *The final assignment induced by Decremental DA, δ, is:*

- *fair;*
- *nonwasteful;*
- *and for any fair and nonwastful assignment λ, $\delta(i)\, R_i\, \lambda(i)$.*

Proof.
Fairness: Fairness is straightforward. As a reminder, in the final assignment, a student cannot have justified envy of a base student. If student i envies student j, and j is not a base student, then both i and j applied to $\delta(j)$ during Decremental DA. As j was chosen and i was not, j must have higher priority, and i's envy is not justified.
Nonwasteful: Fix a school s and suppose there is a student j such that $s\, P_j\, \delta(i)$. Let q^*_s be the capacity of s in the final round of Decremental DA. By construction, $q^*_s = q^a_s + |\{i | i \in$

$\mu(s)$ and $\nu(i)\neq\emptyset\}|$. As j applied and was rejected by s, s was assigned to its full capacity. Specifically, $|\nu(s)|=q_s^a+|\{i\in\mu(s)|\,\nu(i)\neq\emptyset\}|$. For every student i such that $\mu(i)=s$ and $\nu(i)=\emptyset$, i's final assignment is s. Therefore, in the final assignment, the number of students assigned to s is:

$$\begin{aligned}&[q_s^a+|\{i|i\in\mu(s)\text{ and }\nu(i)\neq\emptyset\}|]+[|\{i|i\in\mu(s)\text{ and }\nu(i)=\emptyset\}|]\\&=q_s^a+[|\{i|i\in\mu(s)\text{ and }\nu(i)\neq\emptyset\}+\{i|i\in\mu(s)\text{ and }\nu(i)=\emptyset\}|]\\&=q_s^a+|\{i|i\in\mu(s)\}|\end{aligned}$$

This is s's total capacity. Therefore, if a student prefers s to her final assignment, s has been assigned to its full capacity.

Student-optimal: Suppose for contradiction that there is a student i such that $\sigma(i)\,P_i\,\delta(i)$. Let k be the first round of Decremental DA where a student is rejected by her Shadow DA assignment. In the k^{th} round of Decremental DA, fix i as the first student who is rejected by $\sigma(i)$ during the running of DA. Set $s=\sigma(i)$. Let X be the students that s temporarily holds when i is rejected. Note that

$$|X|=q_s^k.$$

As i is the first student rejected by her Shadow DA assignment, for every $j\in X$,

$$s\,R_j\,\sigma(j).$$

Since s rejected i but held onto j, $j\succ_s i$. As $\sigma(i)=s$, for every $j\in X$,

$$\sigma(j)\,R_j\,s$$

or else j would have justified envy of i. Combining these two, we conclude that for every $j\in X$, $\sigma(j)=s$.

Therefore,

$$\begin{aligned}|\sigma(s)|&\geq 1+|X|+|\left\{j\in\mu(s)|\,\nu^{k-1}(j)=\emptyset\right\}|\\&=1+q_s^k+|\left\{j\in\mu(s)|\,\nu^{k-1}(j)=\emptyset\right\}|\\&=1+q_s^a+|\left\{j\in\mu(s)|\,\nu^{k-1}(i)\neq\emptyset\right\}|+|\left\{j\in\mu(s)|\,\nu^{k-1}(j)=\emptyset\right\}|\\&=1+q_s^a+|\mu(s)|\end{aligned}$$

As this exceeds the capacity of school s, this is a contradiction. □

An immediate corollary of Theorem 3 is that Decremental DA makes the same assignment as Shadow DA.

Corollary 2. *For any preference profile P, let $\phi^{DDA}(P)$ and $\phi^{SDA}(P)$ be the assignments made by Decremental DA and Shadow DA, respectively. For any preference profile P,*

$$\phi^{DDA}(P) = \phi^{SDA}(P).$$

Decremental DA is a complicated mechanism, and it is not straightforward to show directly that it is strategyproof. The reason is that by changing her report, a student can change the number of people assigned, and therefore, can change the path of capacities. However, as Shadow DA is strategyproof, Corollary 2 provides a simple indirect proof that Decremental DA is strategyproof.

Theorem 4. *Decremental DA is strategyproof.*

Proof. For any preference profile P, let $\phi^{DDA}(P)$ and $\phi^{SDA}(P)$ be the assignments made by Decremental DA and Shadow DA, respectively. By Corollary 2, for any preference profile P,

$$\phi^{DDA}(P) = \phi^{SDA}(P).$$

Fix a student i, and consider any preference report P_i' that i can make. Let $P = (P_j)_{j \in I}$ and $P' = (P_i'; P_{-i})$. As Shadow DA is strategyproof,

$$\phi^{SDA}(P)(i) \, R_i \, \phi^{SDA}(P')(i).$$

Therefore, as

$$\phi^{DDA}(P)(i) = \phi^{SDA}(P)(i),$$
$$\implies \phi^{DDA}(P')(i) = \phi^{SDA}(P')(i)$$

, and

$$\phi^{SDA}(P)(i) \, R_i \, \phi^{SDA}(P')(i),$$

we conclude that

$$\phi^{DDA}(P)(i) \, R_i \, \phi^{DDA}(P')(i).$$

□

2.5 Conclusion

In this study, we address endogenous capacity in WCPSS magnet school assignments, which is driven by two factors: firstly, moving a student to other magnet schools can effectively raise the available capacity at her base school; secondly, WCPSS rules require that 10% of each magnet school's new assignments are allocated through a lottery. Endogenous capacities can lead to priority violations in the overall assignment, which cannot be resolved by a waitlist system.

We propose two solutions, each maintaining the desirable properties of the DA – strategyproofness and fairness. Shadow DA is a simple solution to the first problem: how to make a fair assignment when there already exists a partial assignment. Decremental DA has the same desirable properties as Shadow DA, but is far more complicated. The advantage of Decremental DA is that it can be used in situations where Shadow DA cannot. WCPSS's rule that 10% of *new* assignments at a school must be made via lottery means that Shadow DA cannot be used. Initially, we do not know how many new assignments there will be. Decremental DA dynamically tracks seat availability, adjusting capacities to reflect real assignable capacities accurately and avoid oversubscription, which enables it to address the dual challenges of endogeneity in WCPSS.

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APPENDICES

APPENDIX

A

APPENDIX FOR CHAPTER 1

A.1 Estimation and Sample Construction Details

Add Health sampled the multiple environments in which young people live their lives, including the family, peers, school, neighborhood, community, and relationship dyads, and provides independent and direct measurement of these environments over time. Add Health contains extensive longitudinal information on health-related behavior, including life histories of physical activity, involvement in risk behavior, substance use, sexual behavior, civic engagement, education, and multiple indicators of health status based on self-report (e.g., general health, chronic illness), direct measurement (e.g., overweight status and obesity), and biomarkers.

A.1.1 Sample and Variable Construction Details

My sample and variables are primarily drawn from Wave I in-home interviews of the Add Health dataset, which was conducted in 1994-95. This wave concentrated on various factors influencing adolescents' health, including personal traits, family dynamics, friendships, romantic relationships, peer groups, schools, neighborhoods, and communities. The core in-home sample is nationally representative, comprising adolescents in grades 7 through 12 during the 1994-95 school year. All students who completed the in-school questionnaire, as well as those

listed on a school roster but did not complete the questionnaire, were eligible for selection into the in-home interviews. Students were stratified by grade and sex within each school, and approximately 17 students were randomly selected from each stratum, resulting in a total of around 200 adolescents from each of the 80 paired schools. This process yielded a core sample of 12,105 adolescents who were interviewed.

The social network data used in my analysis are drawn from a saturated sample, which includes 16 out of the 80 schools in the dataset. Schools with populations below 100 or above 1000 students were excluded, leaving a final set of 5 schools. To ensure comparability in size across networks, the largest school in this subset was divided by grade (9th through 12th grades), with each grade treated as a separate network. In this social network dataset, each student nominated 5 female and 5 male friends. A mutual nomination (where both individuals nominate each other) is coded as a link in the network. The summary statistics for both my final sample and the full Add Health sample are presented in Table A.1.

Construction of Self-Efficacy Scale My measure of self-efficacy is constructed using nine items from the Wave I interviews, following the guidelines provided by Bandura (2006). Fundamentally, self-efficacy refers to an individual's belief in their abilities – both physical and cognitive abilities – to successfully perform tasks and overcome challenges. This concept is closely tied to how people approach goals, tasks, and obstacles, influencing their motivation and perseverance. Because self-efficacy is about confidence in handling various situations, items that assess an individual's responses to difficulties, problem-solving strategies, decision-making processes, and feelings of competence and social acceptance are effective measures. These items capture the different dimensions of self-efficacy, providing a comprehensive understanding of a person's perceived capabilities. In these selected items, students were asked to state their level of agreement with the following nine statements:

- *"Do you agree or disagree with the following statement? Difficult problems make you very upset."*
- *"Do you agree or disagree with the following statement? When you have a problem to solve, one of the first things you do is get as many facts about the problem as possible."*
- *"Do you agree or disagree with the following statement? When you are attempting to find a solution to a problem, you usually try to think of as many different ways to approach the problem as possible."*
- *"Do you agree or disagree with the following statement? When making decisions, you generally use a systematic method for judging and comparing alternatives."*

- *"Do you agree or disagree with the following statement? After carrying out a solution to a problem, you usually try to analyze what went right and what went wrong."*
- *"Do you agree or disagree with the following statement? You are well coordinated."*
- *"Do you agree or disagree with the following statement? You feel like you are doing everything just about right."*
- *"Do you agree or disagree with the following statement? You feel socially accepted."*
- *"Do you agree or disagree with the following statement? You feel loved and wanted."*

The permitted responses range from "strongly agree", "agree", "neither agree nor disagree", "disagree", and "strongly disagree," with each answer coded from 0 to 4, where a higher score indicates greater self-efficacy. This results in a self-efficacy scale that ranges from 0 to 36.

Construction of Risk Aversion Scale Based on the findings of Mishra and Lalumière (2011), impulsivity – the tendency to react without planning or considering the consequences – is a personality trait that is strongly associated with real-world risk-taking behaviors. The paper demonstrates that impulsive individuals are more likely to engage in behaviors with immediate rewards despite potential long-term negative consequences, which aligns closely with risk-averse tendencies. Therefore, my measure of risk-aversion is constructed using two survey questions that reveal individual impulsivity from the Wave I interviews:

- *"Do you agree or disagree with the following statement? You usually go out of your way to avoid having to deal with problems in your life."*
- *"Do you agree or disagree with the following statement? When making decisions, you usually go with your "gut feeling" without thinking too much about the consequences of each alternative."*

The permitted responses range from "strongly agree", "agree", "neither agree nor disagree", "disagree", and "strongly disagree," with each answer coded from 0 to 4, where a higher score indicates greater risk-aversion. This results in a risk-aversion scale that ranges from 0 to 8.

Construction of Homophily Index The Homophily Index and Coleman's Homophily Index (Currarini et al. 2010) are measures used to quantify the degree of similarity among linked individuals within a social network. These indices assess the extent to which individuals with same attributes (e.g., race, gender, or behaviors) are more likely to form connections with each other compared to what would be expected by chance.

Let s_i represents the average number of friendships that individuals of type i (i.e., individuals with a specific attribute i; in my paper, this refers to PYD program participants) have with

others of the same type; d_i represents the average number of friendships that type i individuals form with others of different types. The Homophily Index is given by:

$$\text{Homophily Index} = \frac{\sum_i s_i}{s_i + d_i}$$

The Homophily Index measures the proportion of connections in a network that occur between individuals who share same type/attributes. If the Homophily Index for type i is higher than the relative fraction of type i individuals in the population, it suggests inbreeding homophily – people prefer to connect with others who share the same attribute. If the index is equal to the relative fraction, it suggests baseline homophily, indicating that people are neutral in their connections regarding this attribute – connections are formed by chance.

Coleman's Homophily Index (Coleman 1958) builds upon the basic Homophily Index by adjusting for the overall prevalence of the attribute in the population. Essentially, frequent connections between individuals of the same type within a small portion of the population indicate a different level of homophily tendency compared to when these connections occur within a larger group. Let w_i represents the proportion of individuals who are of type i. The Coleman's Homophily Index is given by:

$$\text{Coleman's Homophily Index} = \frac{\text{Homophily Index} - w_i}{1 - w_i}$$

A Coleman's Homophily Index greater than 0 indicates inbreeding homophily; a negative index suggests that individuals are more likely to connect with others of a different type.

Construction of Segregation Index Freeman's Segregation Index (Freeman 1978) is a measure used to quantify the level of segregation within a network based on a particular attribute (e.g., race or behaviors). The index quantifies how much the observed connections between groups with different attributes deviate from what would be expected if links were formed randomly.

Let M_{ij} represent the number of links from individuals with attribute i to those with attribute j. The Freeman's Segregation Index is calculated by comparing the actual number of inter-group links $M_{ij} + M_{ji}$ to the expected number of such links if they were formed at random:

$$\text{Freeman's Segregation Index} = 1 - \frac{M_{ij} + M_{ji}}{\mathbb{E}(M_{ij} + M_{ji})},$$

where $\mathbb{E}(M_{ij} + M_{ji})$ represents the expected number of inter-group links under random mixing. The index ranges from 0 to 1, where 0 indicates no segregation (i.e., random mixing), and 1 indicates complete segregation (i.e., all connections are within the same group).

A.1.2 Estimation Details

The prior distributions for θ used in my estimation are normal distributions, with means and standard deviations detailed in Table A.4. The network and parameters in Equations (1.1)–(1.2) are sampled using the double Metropolis-Hastings algorithm. In the inner sampler loop, 7,000 states ($R = 7{,}000$) are drawn from a uniform distribution over the neighborhood of the previous simulated state, with the last simulated state treated as another equilibrium state, beside the observed state. The outer sampler loop draws 100K θ's following a random walk process ($T = 100{,}000$), with the first 20% of the posterior sample discarded from the analysis. The proposal distributions for the meeting size k, the meeting $\mu_t(i, I_k)$, and the auxiliary variable S are generally uniform distributions. Specifically, the meeting size is set to 2 with a 75% probability, while the remaining 25% is drawn from a discrete uniform distribution over $\{2, \ldots, n-1\}$. Each meeting is drawn from a uniform distribution over all possible meetings, where each occurs with equal probability, as required by the underlying assumptions.

A.2 School Club Involvement and Network Formation

According to Eccles et al. (2003), school clubs also align with the PYD framework and enhance students' well-being. These clubs are considered school-based extracurricular activities, in contrast to the PYD programs defined in my paper, which involve Scouting and BBBS participation and are categorized as outside-school extracurricular activities. This section explores the decision to participate in school clubs and its relationship with network formation. The variable of interest is now defined as participation in any school club (e.g., sports, arts, etc.) or organization within the school. The results are reported in Table A.5. The findings suggest that the peer effect on school club participation is less than half of that on PYD program participation. This indicates that individual school club participation is less likely to be influenced by peers, and both participating in some school clubs does not significantly incentivize individuals to connect with each other. This may be because school clubs can be classified into multiple categories—some related to sports, others to arts/literature. Students participating in sports clubs and arts clubs would both be identified as school club participants, but the participants from these two types of clubs could be fundamentally different from each other, making it harder for them to connect with each other. On the other hand, the magnitude of the effect of characteristics homophily on network formation is roughly the same as that suggested by Table 1.3.

My findings suggest that the utility derived from school club participation is relatively greater than that from friendships, indicating that individuals may be more inclined to alter

their friendship links than to change their school club participation choices upon shock. This aligns with Boucher (2016), who applied a model of conformism in social networks to study high school students' participation in extracurricular activities. His results suggest that policies aimed at changing individual school club participation would likely impact network structures rather than altering individual behaviors. However, his findings also indicate that conformism significantly influences school club choices. This contradicts my results, which suggest that social influence is minimal, school club participation is not significantly affected by others, and students do not form connections based on school club participation. The discrepancy may stem from differences in sample restrictions: while we both used the Add Health dataset, he drew data from 100 schools, whereas I focused on a saturated sample, ultimately concentrating on just five schools.

While this model seems to fairly predict the overall participation rate, it performs poorly in predicting the interaction between school club participation and network formation. Specifically, as shown in Table A.6, the mean predicted frequency of both individuals and their peers participating in the school clubs is approximately 72% higher than the observed average. Moreover, the mean predicted frequency of both individuals and their peers not participating in the school clubs is more than three times higher than the observed average. This results in a predicted network that is denser than the observed one, with a significantly overestimated tendency for individuals with the same actions to connect, as indicated by both homophily indices. This discrepancy suggests that, despite the small magnitude of the estimated effect of each participating peer on school club participation, the total peer effect may still be overstated. Given that more than half of the population participates in school clubs, and that the peer effect is assumed to increase linearly with the number of participating peers, this overestimation could lead to an inflated prediction of network connections. As a result, the model may falsely predict a stronger incentive for individuals to connect based on similar actions than what actually occurs.

Table A.1: Summary Statistics - Add Health v.s Analysis Sample.

	Full Add Health		Saturated Sample		Analysis Sample		
	All Students	PYD Participants	All Students	PYD Participants	All Students	PYD Participants	Non Participants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Variable of Interest							
PYD Program Participation	0.118	1	0.128	1	0.12	1	0
	(0.004)		(0.31)		(0.016)		
Individual Demographics							
Female	0.457	0.395	0.478	0.164	0.48	0.16	0.512
	(0.015)	(0.044)	(0.047)	(0.054)	(0.118)	(0.056)	(0.025)
Male	0.543	0.605	0.522	0.836	0.52	0.84	0.488
	(0.015)	(0.044)	(0.047)	(0.054)	(0.118)	(0.056)	(0.025)
Age (94-95)	15.94	15.6	15.79	15.44	16.519	16.05	16.51
	(1.023)	(1.062)	(1.065)	(1.153)	(1.516)	(0.169)	(0.076)
White	0.69	0.668	0.958	0.966	0.988	0.961	0.991
	(0.013)	(0.042)	(0.021)	(0.005)	(0.079)	(0.026)	(0.005)
Black	0.162	0.183	0.035	0.034	0.009	0.039	0.006
	(0.011)	(0.033)	(0.02)	(0.004)	(0.017)	(0.026)	(0.004)
Hispanic	0.095	0.085	0.002	0.000	0.003	0.000	0.003
	(0.009)	(0.009)	(0.024)	(0.002)	(0.003)	(0.001)	(0.003)
Asian	0.034	0.029	0.001	0	0.000	0	0.000
	(0.002)	(0.006)	(0.001)		(0.01)		(0.001)
Other Race	0.019	0.035	0.004	0.000	0.000	0	0.000
	(0.004)	(0.018)	(0.004)	(0.070)	(0.003)		(0.003)
Individual Health							
General Health	2.13	1.89	2.07	1.81	2.16	1.92	2.2

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Table A.1 (continued)

	Full Add Health		Saturated Sample		Analysis Sample		
	All Students	PYD Participants	All Students	PYD Participants	All Students	PYD Participants	Non Participants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	(0.011)	(0.028)	(0.032)	(0.081)	(0.041)	(0.11)	(0.044)
Happy	0.047	0.273	0.153	0.377	0.046	0.311	0.01
	(0.012)	(0.031)	(0.034)	(0.075)	(0.043)	(0.098)	(0.047)
Depressed	-0.055	-0.33	-0.162	-0.459	-0.028	-0.394	-0.022
	(0.012)	(0.028)	(0.35)	(0.076)	(0.048)	(0.105)	(0.052)
Freq. Exercise	1.62	1.78	1.59	1.72	1.55	1.73	1.53
	(0.013)	(0.035)	(0.039)	(0.102)	(0.051)	(0.141)	(0.054)
Freq. Hangout	2	2.01	1.96	2.09	2.03	2.16	2.01
w/ Friends	(0.013)	(0.032)	(0.036)	(0.086)	(0.044)	(0.114)	(0.047)
Individual Malleable Characteristics							
Self Efficacy[a]	-0.142	0.009	-0.126	-0.08	-0.134	-0.13	-0.134
	(0.023)	(0.072)	(0.074)	(0.168)	(0.086)	(0.257)	(0.091)
Risk Aversion[b]	-0.004	0.009	0.099	0.011	0.032	-0.04	0.04
	(0.023)	(0.068)	(0.069)	(0.127)	(0.078)	(0.16)	(0.086)
Expect B.A	-0.205	-0.028	-0.289	0.226	-0.288	0.215	-0.348
	(0.028)	(0.09)	(0.088)	(0.145)	(0.102)	(0.189)	(0.109)
Dislike Protection	0.715	0.65	0.851	0.703	0.818	0.668	0.836
	(0.025)	(0.078)	(0.092)	(0.214)	(0.1)	(0.297)	(0.106)
Individual Problematic Behaviors							
Freq. Skip Classes	1.86	0.897	1.34	0.851	1.85	1.2	1.94
	(0.086)	(0.176)	(0.204)	(0.363)	(0.323)	(0.584)	(0.358)
Individual GPA							
English GPA	2.74	3.11	2.94	3.13	2.82	3.07	2.78

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Table A.1 (continued)

	Full Add Health		Saturated Sample		Analysis Sample		
	All Students	PYD Participants	All Students	PYD Participants	All Students	PYD Participants	Non Participants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	(0.014)	(0.033)	(0.034)	(0.073)	(0.048)	(0.097)	(0.053)
Math GPA	2.53	2.89	2.67	2.88	2.51	2.73	2.48
	(0.015)	(0.036)	(0.041)	(0.11)	(0.053)	(0.148)	(0.057)
Individual School Club/Extracurricular Activities Participation							
Part Extracurricular activities	0.549	0.67	0.665	0.713	0.656	0.73	0.646
	(0.006)	(0.016)	(0.017)	(0.043)	(0.022)	(0.058)	(0.024)
Art Club	0.201	0.297	0.244	0.305	0.216	0.252	0.211
	(0.005)	(0.016)	(0.016)	(0.048)	(0.02)	(0.065)	(0.021)
Edu Club	0.134	0.192	0.178	0.223	0.175	0.242	0.166
	(0.004)	(0.013)	(0.015)	(0.046)	(0.018)	(0.063)	(0.019)
Sprt Club	0.404	0.543	0.545	0.651	0.523	0.683	0.501
	(0.006)	(0.017)	(0.018)	(0.046)	(0.023)	(0.061)	(0.025)
Student Org	0.156	0.229	0.165	0.135	0.147	0.095	0.154
	(0.004)	(0.014)	(0.014)	(0.028)	(0.017)	(0.03)	(0.019)
Other Club	0.14	0.186	0.136	0.092	0.146	0.113	0.15
	(0.004)	(0.013)	(0.014)	(0.029)	(0.017)	(0.045)	(0.019)
Individual Grade							
12th Grade	0.204	0.152	0.18	0.118	0.249	0.145	0.263
	(0.005)	(0.013)	(0.014)	(0.03)	(0.021)	(0.043)	(0.023)
11th Grade	0.134	0.116	0.119	0.069	0.146	0.074	0.156
	(0.004)	(0.01)	(0.011)	(0.022)	(0.017)	(0.031)	(0.018)
10th Grade	0.156	0.163	0.146	0.139	0.195	0.224	0.191
	(0.004)	(0.012)	(0.013)	(0.037)	(0.019)	(0.061)	(0.02)

Continue on the next page

Table A.1 (continued)

	Full Add Health		Saturated Sample		Analysis Sample		
	All Students	PYD Participants	All Students	PYD Participants	All Students	PYD Participants	Non Participants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
9th Grade	0.168	0.188	0.181	0.257	0.26	0.414	0.239
	(0.005)	(0.013)	(0.014)	(0.048)	(0.021)	(0.072)	(0.022)
8th Grade	0.163	0.191	0.181	0.201	0.073	0.076	0.073
	(0.005)	(0.014)	(0.014)	(0.039)	(0.01)	(0.023)	(0.011)
7th Grade	0.175	0.191	0.194	0.216	0.078	0.068	0.079
	(0.005)	(0.014)	(0.014)	(0.04)	(0.01)	(0.03)	(0.011)
Parent/Household Characteristics							
HH Income	46.15	58.41	47.54	52.07	47.14	49.22	46.87
(94-95)	(0.589)	(1.86)	(1.26)	(4.02)	(1.49)	(3.91)	(1.6)
HH Below FPL	0.161	0.082	0.099	0.083	0.095	0.055	0.101
(94-95)	(0.005)	(0.01)	(0.013)	(0.028)	(0.015)	(0.03)	(0.016)
No Parent	0.009	0.005	0.008	0.015	0.009	0.008	0.009
	(0.001)	(0.002)	(0.003)	(0.011)	(0.004)	(0.008)	(0.004)
Single Parent	0.288	0.239	0.236	0.199	0.232	0.18	0.238
	(0.006)	(0.017)	(0.018)	(0.042)	(0.022)	(0.055)	(0.024)
Biparent	0.703	0.756	0.757	0.786	0.759	0.812	0.753
	(0.007)	(0.017)	(0.018)	(0.043)	(0.022)	(0.055)	(0.024)
Parent LHS	0.153	0.099	0.085	0.031	0.094	0.043	0.101
	(0.005)	(0.013)	(0.012)	(0.013)	(0.015)	(0.019)	(0.016)
Parent HSD	0.608	0.516	0.697	0.633	0.738	0.637	0.751
	(0.007)	(0.02)	(0.019)	(0.057)	(0.024)	(0.082)	(0.024)
Parent B.A.	0.239	0.385	0.218	0.335	0.168	0.32	0.148
	(0.006)	(0.019)	(0.017)	(0.057)	(0.021)	(0.083)	(0.02)

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Table A.1 (continued)

	Full Add Health		Saturated Sample		Analysis Sample		
	All Students	PYD Participants	All Students	PYD Participants	All Students	PYD Participants	Non Participants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Num HH Members	4.44	4.36	4.43	4.48	4.51	4.48	4.52
	(0.021)	(0.052)	(0.054)	(0.151)	(0.07)	(0.183)	(0.075)
Parent Happy	0.959	0.979	0.959	0.977	0.959	0.957	0.959
	(0.003)	(0.005)	(0.01)	(0.018)	(0.012)	(0.034)	(0.013)
Parent Health	2.37	2.18	2.27	2.23	2.33	2.29	2.34
	(0.015)	(0.037)	(0.041)	(0.112)	(0.052)	(0.156)	(0.055)
Neighborhood/Environment Characteristics							
Neighbor Income	29.52	30.2	29.28	30.27	30.33	31.66	30.14
	(0.087)	(0.232)	(0.227)	(0.661)	(0.259)	(0.732)	(0.275)
Neighbor Unemployment	0.068	0.065	0.065	0.063	0.061	0.057	0.062
	(0.000)	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
School Connectness	0.158	0.21	0.23	0.261	0.202	0.306	0.189
	(0.004)	(0.014)	(0.016)	(0.045)	(0.019)	(0.064)	(0.02)
Neighbor Connectness	0.732	0.784	0.731	0.819	0.712	0.82	0.698
	(0.006)	(0.014)	(0.017)	(0.038)	(0.022)	(0.053)	(0.023)
Observations	20,745	1,852	1,882	206	1,353	138	959

Note. The summary statistics presented are weighted to account for the sampling design.

[a] The self-efficacy scale generated in this paper ranged from 0 to 36. Here, I display the standardized scale.

[b] The risk aversion scale generated in this paper ranged from 0 to 8. Here, I display the standardized scale.

Table A.2: Peer Characteristics Summary Statistics.

	(1) PYD Participants		(2) Non Participants	
	Mean	SD	Mean	SD
PYD Participation	0.161	0.349	0.116	0.320
Extracurricular Activities	0.737	0.442	0.696	0.460
Parent College+	0.814	0.390	0.822	0.383
HH Income	44.994	34.681	41.657	32.979
English GPA	2.865	0.909	2.815	0.961
Math GPA	2.776	1.139	2.586	1.187
Self Efficacy	25.404	3.471	25.031	3.737
Risk Aversion	4.019	1.461	4.067	1.626
Num Friends	2.506	1.346	2.423	1.304
Same Sex	0.582	0.494	0.537	0.499
Same Grade	0.678	0.468	0.610	0.488
Same Race	0.697	0.461	0.638	0.481
Same Self Efficacy[a]	0.202	0.402	0.212	0.409
Same Risk Aversion[b]	0.524	0.501	0.393	0.488

Note. Sample is restricted to individuals who have nominated friends (N=1,314). The summary statistics presented are unweighted because, in the network estimation process, each individual is treated as a unique node. As a result, the estimation sample is unweighted, reflecting the individual-level nature of the analysis.
[a] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.
[b] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table A.3: National Survey on Drug Use and Health (NSDUH).

	Full Sample				PYD Participants			
	Mean	SD	Min	Max	Mean	SD	Min	Max
PYD Participation	0.214	0.004	0	1	1	0	1	1
Participate Scouting	0.1	0.003	0	1	0.467	0.011	0	1
Participate BBBS	0.133	0.004	0	1	0.621	0.011	0	1
Age	14.753	0.017	12	17	14.521	0.036	12	17
Female	0.492	0.005	0	1	0.507	0.011	0	1
Male	0.508	0.005	0	1	0.493	0.011	0	1
White	0.694	0.005	0	1	0.713	0.01	0	1
Black	0.125	0.004	0	1	0.141	0.008	0	1
Asian	0.039	0.003	0	1	0.037	0.005	0	1
Hispanic	0.121	0.003	0	1	0.095	0.006	0	1
Other Race	0.021	0.002	0	1	0.013	0.003	0	1
Health	4.171	0.009	1	5	4.206	0.018	1	5
Overall GPA	2.937	0.009	1	4	3.055	0.019	1	4
Family Income	47.751	0.245	0	75	49.273	0.516	0	75
Observations	12,358				2,693			

Note. The means displayed are weighted to account for the sampling design. The sample is constructed from students in Grades 7-12, using data from the National Household Survey on Drug Abuse (2000). Scouting participation appears smaller in my sample drawn from the Add Health dataset, potentially due to differences in geographic area or time period.

Table A.4: Prior Distribution for θ.

	Mean	SD
Base PYD Utility	0.2	0.1
Log HH Income	0.1	0.1
Parent HSD/B.A	0.1	0.1
White	0.2	0.2
Age	-0.2	0.2
Male	0.2	0.2
Self Efficacy	0.05	0.1
Risk Aversion	0.005	0.01
English GPA	0.05	0.1
Math GPA	0.05	0.1
Part. Peer Effect[a]	0.05	0.05
NonPart. Peer Effect[b]	0.05	0.05
10% School PYD	0.05	0.1

	Mean	SD
Baseline Link Utility	2	2
Same Sex	0.7	0.5
Same Grade	0.7	0.5
Same Race	0.7	0.5
Same Self Efficacy[c]	0.2	0.2
Same Risk Aversion[d]	0.2	0.2
Friendship Cost	0	0.5
Clustering	0	2

Note.

[a] This represents the peer effect among participants from individual friends who are also participants. It also indicates the incentive for two individuals to form a connection if both are participants.

[b] This represents the peer effect among nonparticipants from individual friends who are also nonparticipants. It also indicates the incentive for two individuals to form a connection if both are nonparticipants.

[c] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

[d] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table A.5: ECA Participation Utility Parameter Estimates.

	(1) Action[a] (ECA Participation)		(2) Network Formation[b]
Log HH Income	0.015	Same Sex	0.696***
	(0.053)		(0.050)
Parent HSD/B.A	0.060	Same Race	0.486***
	(0.065)		(0.248)
White	-0.041	Same Grade	0.642**
	(0.068)		(0.100)
Age	-0.029**	Same SE[e]	0.146***
	(0.011)		(0.046)
Male	0.043	Same RA[f]	0.112**
	(0.041)		(0.046)
Self Efficacy	0.034*	Friendship Cost	-0.132***
	(0.016)		(0.032)
Risk Aversion	0.001	Clustering	0.652***
	(0.004)		(0.179)
English GPA	0.044***	Base Links	1.689***
	(0.015)		(0.603)
Math GPA	0.048***		
	(0.012)		
Part. Peer Effect[c]	0.021**		
	(0.012)		
NonPart. Peer Effect[d]	0.001		
	(0.008)		
50% School ECA	0.023**		
	(0.012)		
Base Participation Utility	0.182***		
	(0.015)		

Note. The posterior sample consists of 100,000 simulations, with the initial 20% discarded as burn-in.This table reports the posterior mean of the simulations, with asymptotic standard errors in parentheses. Each simulated confidence interval (90%, 95%, and 99%) is checked to determine whether it contains zero. Confidence intervals that do not include zero are indicated by */**/*** for the confidence intervals 90%, 95%, and 99%, respectively.

[a] Column (1) results are presented as marginal probabilities, expressed in percentage points.

[b] Column (2) results are presented as relative marginal probabilities, expressed as percentages.

[c] This represents the peer effect among participants from individual friends who are also participants. It also indicates the incentive for two individuals to form a connection if both are participants.

[d] This represents the peer effect among nonparticipants from individual friends who are also nonparticipants. It also indicates the incentive for two individuals to form a connection if both are nonparticipants.

[e] The self-efficacy scale generated in this paper ranged from 0 to 36, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

[f] The risk aversion scale generated in this paper ranged from 0 to 8, divided into four categories from high to low. Individuals are considered to be at the same level if they fall within the same category.

Table A.6: Model Fit (ECA).

	Distribution of Prediction		Observed
	Mean	Median	Value
Actions			
$\sum_{i \in I} D_i / n$	0.628	0.610	0.610
Interaction between Action and Links			
$\sum_{i \neq j} (g_{ij} * D_i * D_j)/n$	0.796	0.470	0.461
$\sum_{i \neq j} [g_{ij} * (1-D_i) * (1-D_j)]/n$	0.516	0.117	0.155
Network			
Avg degree	0.961	0.942	0.970
Min degree	0.492	0.000	0.000
Max degree	6.159	4.844	5.295
Density	0.056	0.005	0.005
$\sum_{i>j>k} g_{ij} * g_{ik} * (1-g_{jk})/n$	4.357	0.460	0.497
$\sum_{i>j>k} g_{ij} * g_{jk} * g_{ik}/n$	5.585	0.024	0.066
Network Formation Patterns			
Homophily Index	0.515	0.492	0.015
Coleman's Inbreeding Homophily Index	-0.287	-0.292	-0.098
Freeman Segregation Index	0.574	0.599	0.418

Note. $\frac{\sum_{i \in I} D_i}{n}$ represents the average participation rate in an action-network configuration. The term $\frac{\sum_{i \neq j} g_{ij} \cdot D_i \cdot D_j}{n}$ represents the average number of occurrences where both an individual and her peer participate in the ECA program. Similarly, $\frac{\sum_{i \neq j} g_{ij} \cdot (1-D_i) \cdot (1-D_j)}{n}$ represents the average number of occurrences where both the individual and her peer do not participate in the ECA program. For network structure, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot (1-g_{jk})}{n}$ represents the average occurrence where an individual is a mutual friend of two unconnected individuals in the action-network configuration. Lastly, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot g_{jk}}{n}$ represents the average number of link triangles (or clustering among three individuals) in the action-network configuration.

Table A.7: Alternative Model – Counterfactual Analysis.

Assignment	Baseline		Predicted (School Effect Off)		
Ratio	% Participation	Effect	% Participation	Effect	Multiplier
Counterfactual Analysis: Scenario 1 - Mandatory Assignment					
0%	12.9	-	-	-	-
5%	17.26	4.36	17.94	5.04	1.16
10%	21.61	8.71	23.69	10.79	1.24
20%	30.32	17.42	38.74	25.84	1.48
30%	39.03	26.13	51.39	38.49	1.47
40%	47.74	34.84	64.61	51.71	1.48
50%	56.45	43.55	91.11	78.21	1.80
Counterfactual Analysis: Scenario 2 - Fixed Amount Incentive Boost					
0%	12.9	-	-	-	-
5%	15.08	2.18	15.11	2.21	1.01
10%	17.26	4.36	17.72	4.82	1.11
20%	21.61	8.71	23.87	10.97	1.26
30%	25.97	13.07	30.58	17.68	1.35
40%	30.32	17.42	36.84	23.94	1.37
50%	34.68	21.78	48.23	35.33	1.62
Counterfactual Analysis: Scenario 3 - Fixed Amount Incentive Boost with Target Effort					
0%	12.9	-	-	-	-
5%	15.11	2.21	15.64	2.74	1.24
10%	17.69	4.79	18.82	5.92	1.24
20%	22.96	10.06	26.95	14.05	1.40
30%	27.15	14.25	35.35	22.45	1.58
40%	32.94	20.04	45.88	32.98	1.65
50%	38.17	25.27	56.06	43.16	1.71

Note. The first column lists the assignment ratio, which is the proportion of the population receiving assignments. The second and third columns display the baseline participation rate post-assignment and the change in participation rate, respectively, if peer effects are not considered. The fourth to sixth columns show the predicted participation rate, the predicted change in post-assignment participation rate considering dynamic networks with school effect switched off, and the multiplier ratio between the predicted participation rate change and the baseline participation rate change.

Table A.8: Model without Global Peer Effect – Treatment Utility Parameter Estimates.

	(1) Action[a] (Treatment Participation)		(2) Network Formation[b]
Log HH Income	0.033	Base Links	1.898***
	(0.036)		(0.434)
Parent HSD/B.A	0.090*	Same Sex	0.707***
	(0.064)		(0.050)
White	0.070***	Same Grade	0.810***
	(0.018)		(0.190)
Age	-0.025***	Same Race	0.244***
	(0.006)		(0.095)
Male	0.049**	Same SE	0.138***
	(0.022)		(0.060)
Self Efficacy	0.043***	Same RA	0.076
	(0.014)		(0.043)
Risk Aversion	0.008*	Friendship Cost	-0.142***
	(0.004)		(0.030)
English GPA	0.033***	Clustering	0.709***
	(0.016)		(0.182)
Math GPA	0.059***		
	(0.016)		
Part. Peer Effect	0.037**		
	(0.009)		
NonPart. Peer Effect	0.002		
	(0.003)		
Base PYD Utility	0.122***		
	(0.030)		

Note. The posterior sample consists of 100,000 simulations, with the initial 20% discarded as burn-in.This table reports the posterior mean of the simulations, with asymptotic standard errors in parentheses. Each simulated confidence interval (90%, 95%, and 99%) is checked to determine whether it contains zero. Confidence intervals that do not include zero are indicated by */**/*** for the confidence intervals 90%, 95%, and 99%, respectively.

[a] Column (1) results are presented as marginal probabilities, expressed in percentage points.

[b] Column (2) results are presented as relative marginal probabilities, expressed as percentages.

Table A.9: Model Fit – Alternative Model.

	Distribution of Prediction		Observed
	Mean	Median	Value
Actions			
$\sum_{i \in I} D_i / n$	0.102	0.100	0.127
Interaction between Action and Links			
$\sum_{i \neq j} (g_{ij} * D_i * D_j)/n$	0.052	0.009	0.068
$\sum_{i \neq j} [g_{ij} * (1-D_i) * (1-D_j)]/n$	0.755	0.754	0.755
Network			
Avg degree	1.245	0.961	0.970
Min degree	0.276	0.000	0.000
Max degree	4.935	4.710	5.295
Density	0.007	0.005	0.005
$\sum_{i>j>k} g_{ij} * g_{ik} * (1-g_{jk})/n$	0.558	0.460	0.497
$\sum_{i>j>k} g_{ij} * g_{jk} * g_{ik}/n$	7.675	0.022	0.066
Network Formation Patterns			
Homophily Index	0.016	0.009	0.093
Coleman's Inbreeding Homophily Index	-0.097	-0.095	-0.098
Freeman Segregation Index	0.433	0.441	0.418

Note. $\frac{\sum_{i \in I} D_i}{n}$ represents the average participation rate in an action-network configuration. The term $\frac{\sum_{i \neq j} g_{ij} \cdot D_i \cdot D_j}{n}$ represents the average number of occurrences where both an individual and her peer participate in the PYD program. Similarly, $\frac{\sum_{i \neq j} g_{ij} \cdot (1-D_i) \cdot (1-D_j)}{n}$ represents the average number of occurrences where both the individual and her peer do not participate in the PYD program. For network structure, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot (1-g_{jk})}{n}$ represents the average occurrence where an individual is a mutual friend of two unconnected individuals in the action-network configuration. Lastly, $\frac{\sum_{i>j>k} g_{ij} \cdot g_{ik} \cdot g_{jk}}{n}$ represents the average number of link triangles (or clustering among three individuals) in the action-network configuration.

APPENDIX

B

APPENDIX FOR CHAPTER 2

B.1 Standard Deferred Acceptance Algorithm

In this subsection, we detail student proposing DA as defined in Gale and Shapley (1962) and Abdulkadiroğlu and Sönmez (2003).

Step 1: Each student, i, applies to their first choice school, possibly $\emptyset$. Each school, s, tentatively accepts the highest priority applicants up to capacities, the remaining applicants are declined.

For each $k > 1$:

Step k:

Each student who was rejected in the previous step proposes to her next choice. Each school then evaluates both their previous tentative accepts and new proposals from students and tentatively accepts the highest priority applicants up to capacities, the remaining applicants are declined.

The algorithm ends when no new rejections/proposals occur.

B.2 Incremental Deferred Acceptance Algorithm

Parameters: a base assignment μ, student preferences P, assignment capacities q^a, and magnet priorities $\succ$.

Initialize: We truncate each student's preferences immediately above her base school. Formally, for each student i, P_i' is defined by: (i) for every schools s and s', $s\ P_i'\ s'$ if and only if $s\ P_i\ s'$; and (ii) for every school s such that $\mu(i)\ R_i\ s$, $\emptyset\ P_i'\ s$. For each school s, define $q_s^1 := q_s^a$.
Round k **(**$k \geq 1$**):** Let $\nu^k = DA(q^k)$. For each school s, define $\Delta_s^{k+1} := |\{i | i \in \mu(s) \text{ and } \nu^k(i) \neq \emptyset\}|$. Note that Δ_s^{k+1} is the number of base students assigned to a different school at step k.[1] For each school s, set $q_s^{k+1} := q_s^a + \Delta_s^{k+1}$.
Output: Let n be the first round where $q^n = q^{n+1}$. The assignment made by Incremental DA is $\nu^n = DA(q^n)$.

B.3 Waitlists

To illustrate the potential issues caused by the presence of endogenous capacity, we will provide two examples below.

Example 3. *There are five magnet schools, $S = \{a, b, c, d, e\}$, each with one seat, and four students, $I = \{i_1, i_2, i_3, i_4\}$. Note that student i_1 has school a as her base school, and i_2, i_3, i_4, have school e as their base school. The student priority for each school $s \in S$ and the preferences of each student $i \in I$ are as follows:*

Priority					Preference			
$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$	P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
i_3	i_2	i_1	i_4	i_1	b	d	d	d
i_2	i_1		i_2		c	a	a	
			i_3		d	b		

Under DA, the unique outcome ν is

$$\begin{pmatrix} \underline{i_1} & \underline{i_2} & \underline{i_3} & \underline{i_4} \\ c & b & a & d \end{pmatrix}$$

In Wake County, students who do not receive a seat at their first-choice magnet school will be placed in an applicant pool/waitlist for that school. The table below displays the waitlist for each school in the WCPSS style, based on example 1.

[1] As a reminder, we truncated preferences above a student's base school. Therefore, if $\nu^k(i) \neq \emptyset$, then i was assigned to a school different than her base school.

Waitlist				
$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
	i_1		i_2	
			i_3	

It's important to note that even if school a has one additional seat available due to student i_1 accepting a seat at another school and not attending her base school, there are no students on the waitlist to fill the vacancy. This may not be an issue if there are no other students who would prefer school a over their current assignment. However, under matching ν, there exists a student-school pair (i_2, a) such that $a \; P_{i_2} \; \nu(i_2) = b$ and $|\nu(a)| < q_a^a + \tilde{q}_a = 2$, which implies a "waste".

Consider another matching ν'

$$\begin{pmatrix} \underline{i_1} & \underline{i_2} & \underline{i_3} & \underline{i_4} \\ b & a & a & d \end{pmatrix}$$

Given that ν' is fair and results in a better outcome for both i_1 and i_2, it suggests that there is room for improvement in the standard DA mechanism where endogenous capacity exists.

Example 4. *There are five magnet schools, $S = \{a, b, c, d, e\}$, each with one seat, and five students, $I = \{i_1, i_2, i_3, i_4, i_5\}$. Note that student i_1 has school a as her base school, and i_2, i_3, i_4, have school e as their base school. The student priority for each school $s \in S$ and the preferences of each student $i \in I$ are as follows:*

Priority					Preference				
$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$	P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
i_5	i_2	i_1	i_4	i_1	b	d	a	d	a
i_2	i_3		i_2		c	a	b		
i_3	i_1					b	c		

Under standard DA, the resulting outcome ν and waitlist in WCPSS style for each school are

$$\begin{pmatrix} \underline{i_1} & \underline{i_2} & \underline{i_3} & \underline{i_4} & \underline{i_5} \\ c & b & \emptyset & d & a \end{pmatrix}$$

Waitlist				
$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$
i_3	i_1		i_2	

As student i_1 was accepted by school c, this would result in school a having one seat released. The empty seat would then be filled by student i_3.

Therefore, the final outcome would be

$$\begin{pmatrix} \underline{i_1} & \underline{i_2} & \underline{i_3} & \underline{i_4} & \underline{i_5} \\ c & b & a & d & a \end{pmatrix}$$

However, this outcome is not fair since there's a student-school pair (i_2, a) such that $a P_{i_2} \nu(i_2) = b$ and $i_2 \succ_a i_3$. To put it differently, when school capacity changes, the admission threshold changes as well. As a result, schools may have already rejected students whom they would have otherwise preferred to accept, especially those who passed the new admission threshold at a later time.

Note that merely modifying the waitlist system is insufficient to address the challenges posed by endogenous capacity. This is because the manner in which the waitlist allocates schools shares similarities with the school proposal DA mechanism, a known approach that lacks both strategyproofness and optimality for students.

B.4 Proofs

Here we prove a number of properties about Decremental DA. We repeat the definition of Decremental DA for convenience.

Decremental DA

Parameters: a base assignment μ, student preferences P, assignment capacities q^a, and magnet priorities $\succ$.

Initialize: We truncate each student's preferences immediately above her base school. Formally, for each student i, P_i' is defined by: (i) for every schools s and s', $s\ P_i'\ s'$ if and only if $s\ P_i\ s'$; and (ii) for every school s such that $\mu(i)\ R_i\ s, \emptyset\ P_i'\ s$. For each school s, define $\Delta_s^1 := |\mu(s)|$ and $q_s^1 := q_s^a + \Delta_s^1$.

Round k **(**$k \geq 1$**):** Let $\nu^k = DA(q^k)$. For each school s, define $\Delta_s^{k+1} := |\{i \in \mu(s) | \nu^k(i) \neq \emptyset\}|$. Note that Δ_s^{k+1} is the number of base students assigned to a different school at step k.[2] For each school s, set $q_s^{k+1} := q_s^a + \Delta_s^{k+1}$.

Output: Let n be the first round where $q^n = q^{n+1}$. The assignment made by Decremental DA is $\nu^k = DA(q^k)$.

Notation: We will refer to Δ^k, q^k, and ν^k as defined in the algorithm above. We let ν denote the outcome of Decremental DA (i.e. ν^n defined on the final capacities). We will denote the

[2] As a reminder, we truncated preferences above a student's base school. Therefore, if $\nu^k(i) \neq \emptyset$, then i was assigned to a school different than her base school.

final assignment induced by Decremental DA as δ. Specifically, for each student i, $\delta(i) = \nu(i)$ if $\nu(i) \neq \emptyset$ and $\delta(i) = \mu(i)$ otherwise.

There is an equivalent way of writing capacities in each round that will sometimes be convenient. As

$$\begin{aligned} q_s^k &= q_s^a + \Delta_s^k \\ &= q_s^a + |\{i \in \mu(s) | \nu^{k-1}(i) \neq \emptyset\}| \\ &= q_s^a + |\mu(s)| - |\left\{j \in \mu(s) | \nu^{k-1}(j) = \emptyset\right\}|, \end{aligned}$$

an equivalent way to write the capacities is:

$$q_s^k = q_s^a + |\mu(s)| - |\left\{j \in \mu(s) | \nu^{k-1}(j) = \emptyset\right\}| \tag{B.1}$$

First, we establish that the algorithm is well defined by showing that Δ^k is decreasing in k. As it is bounded below by $\bar{0}$, this implies that for some k, $\Delta^k = \Delta^{k+1}$; therefore, the algorithm must terminate.

Lemma 3. *In the definition of Decremental DA, Δ^k is decreasing in k.*

Proof. We prove by induction on k that for every school s, Δ_s^k is decreasing. For the base case, $\Delta_s^1 = |\mu(s)|$ and by definition, $\Delta_s^2 := |\{i | i \in \mu(s) \text{ and } \nu^1(i) \neq \emptyset\}|$. Therefore, trivially, $\Delta_s^1 \geq \Delta_s^2$. For the induction step, suppose for every school s, $\Delta_s^{k-1} \geq \Delta_s^k$. Equivalently, $q_s^{k-1} \geq q_s^k$. Therefore, $\left\{i \in \mu(s) | DA(q^{k-1})(i) = \emptyset\right\} \subseteq \left\{i \in \mu(s) | DA(q^k)(i) = \emptyset\right\}$ since additional capacity makes each student weakly better off (see Lemma 1 on Page 61). Therefore,

$$\begin{aligned} \Delta_s^{k+1} &= |\{i \in \mu(s) | DA(q^k)(i) \neq \emptyset\}| \\ &\leq |\{i \in \mu(s) | DA(q^{k-1})(i) \neq \emptyset\}| \\ &= \Delta_s^k \end{aligned}$$

□

The following are straightforward corollaries of Lemmas 1 and 3.

Lemma 4. *For any student i, if there exists a k where $\nu^k(i) = \emptyset$, then $\delta(i) = \mu(i)$.*

Proof. A student's assignment gets weakly worse in each subsequent round. Therefore, if $\nu^k(i) = \emptyset$, then for every $k' > k$, $\nu^{k'}(i) = \emptyset$. In particular, $\nu(i) = \emptyset$. By construction, a student who does not receive a magnet assignment is assigned to her base school. □