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Vibrations and instabilities acting in a pipe flowing fluid

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ABSTRACT

This paper analyses the dynamics' and stability's mechanisms of flexible pipes containing flowing fluid for flow velocity constant. We already study the effect of a steady flow and show that hydroelastic instabilities are possible if the flow velocity is sufficiently high and analyse the response of a tube in time and frequency domain. Agreement between theoretical and experimental complex frequencies, displacements-time, displacements amplitudes and phases profiles diagrams is sufficient to validate a model.

1. INTRODUCTION

Instability analysis in elastic systems subject to non-conservative forces is presently required in several technological areas, such as oil prospection, nuclear engineering, aeronautic engineering, etc, wich demand high performance of the used components.

Tubular circuit networks flowing water under pressure are present in big industrial instalations and nuclear power plants; and display and important role nowadays. Instabilities or pressure fluctuations may induce significant transients in these circuits. For this reason, structure analysis becomes necessary to understand several failure scene particularly the elastic-fluid efect associated to the flow.

Previous studies, Thompson [11], Thimoshenko and Gere [10], De Morais and Pedroso [4], based on static analysis of the problem have been proved limited as to a broader comprehension of the phenomenon requiring a dynamic analysis of the problem.

Several authors, Gregory and Païdoussis [3], Païdoussis [7], Plaut and Huseyin [9], Nemat-Nasser *et al* [6], and Païdoussis and Issid [8] have presented complex eigenvalues formulation analysis in dynamic stability problems regarding elastic pipes flowing fluid under certain boundary conditions (either clampled-free or pinned-pinned). However, these analysis are based in analytic solutions just for some simplified boundary conditions.

This paper analyses the dynamics' and stability's mechanisms of flexible pipes containing flowing fluid, taking into acount that the flow velocity is entirely constant.

The finite differences procedure discretizes the model, determining therefore the mass, damping and rigidity matrices. By solving the eigenvalues problem, we obtain the vibration frequencies of the structural system. The structural and fluid damping leads the system to a second degree generalized eigenvalue problem, which was reduced to the form $\underset{\sim}{A}\vec{z}=\lambda\vec{z}$ by means of a coordinate transformation (Abo-Hamd & Utku [1]).

We compare the values obtained through a formulation developed with the classic results of buckling Euler's column, where the jet action mechanics on the pipe extremity provoke similar effects.

2. POSING THE PROBLEM

Given a core reactor with its primary circuit, represented by a cavity-pipe system. imagine a pressurized system (p_o) and a flow (U) provoked by a pipe burst, we could ask ourselves if the flow would not originate instability in the tubular circuit.

For this kind of problem there is no static solution, as it has been demonstrated in previous studies [4].

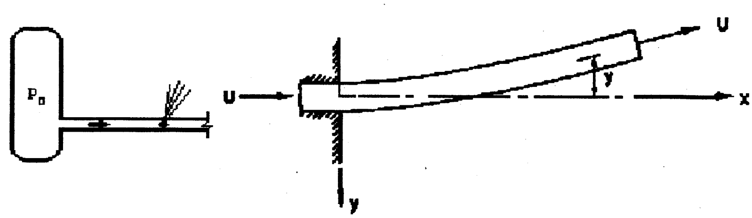

Figure 2.1 - Scheme of the problem posed and model

3. THEORETICAL DEVELOPMENT

3.1. Vibration in Pipes Flowing Fluid

Given a fluid element and a pipe to introduce the necessary basis for the analysis.

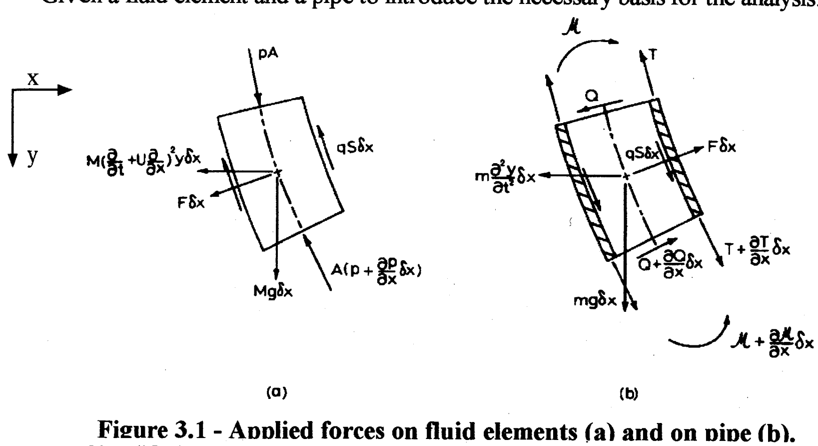

Figure 3.1 - Applied forces on fluid elements (a) and on pipe (b).

I - Simplified Hypothesis: (a) incompressible Fluid ($\omega D/c \ll 1$); (b) the pressures are measured according to atmospheric pressure (p_{atm}); (c) small displacement: ds=dx, reports to the equilibrium position.

II - Equilibrium of all Forces Acting on the System

In one of our previous studies [5], we established a general formulation of the equilibrium equation for an elastic pipe of length L with constant transversal section A, and flexion rigidity EI, flowing an incompressible fluid of density ρ, being U an out jet velocity, subject to small lateral motions $y(x,t)$. Figure (3.1) indicates respectively: the forces acting on a fluid element and on the pipe.

Consider the forces presents in the fluid-pipe system: Fds is a normal reaction, the force between the fluid and the pipe on the length ds; Q is a shear force; M, the bending moment; T, the axial force along the pipe; qds is the friction force between the pipe and the fluid; ρAds the fluid element mass, and $M\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2 yds$ and $m\frac{\partial^2 y}{\partial t^2}ds$ the accelerations of the fluid and the tube respectively.

The resultant forces in the x and y directions, on the fluid element and the pipe, after some algebric operations, simplifications and manipulations, and adding the expressions obtained for the fluid and the pipe in the y and x directions, becomes:

$$EI\frac{\partial^4 y}{\partial x^4}+\mu I\frac{\partial^5 y}{\partial x^4 \partial t}+M\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2 y+\frac{\partial}{\partial x}\left[(pA-T)\frac{\partial y}{\partial x}\right]+m\frac{\partial^2 y}{\partial t^2}=0 \qquad \textbf{(1)}$$

$$\frac{\partial}{\partial x}(T-pA)+(M+m)g=0 \qquad \textbf{(2)}$$

Integrating Eq.(2) from x to L, we obtain,

$$(T-pA)_L-(T-pA)_x+(M+m)(L-x)g=0 \qquad \textbf{(3)}$$

If P_L and T_L are the pressure (the system's pressurization) and the axial force at the pipe's extremity (L), therefore,

$$(T-pA)_{\text{in section } x}=T_L-p_L A+(M+m)(L-x)g=0 \qquad \textbf{(4)}$$

Introducing (4) into (1), we obtain,

$$\begin{aligned} &EI\frac{\partial^4 y}{\partial x^4}+\mu I\frac{\partial^5 y}{\partial x^4 \partial t}+\left[MU^2-(M+m)(L-x)g+p_L S-T_L\right]\frac{\partial^2 y}{\partial x^2} \\ &+2MU\frac{\partial^2 y}{\partial x \partial t}+(M+m)g\frac{\partial y}{\partial x}+(M+m)\frac{\partial^2 y}{\partial t^2}=0 \end{aligned} \qquad \textbf{(5)}$$

The Eq.(5) represent the dynamic equilibrium equation of the fluid-structure system for the small lateral motions of the pipe, and can be solved from the boundary conditions for each case in particular.

III - Non-Dimensional General Equation

In solving this kind of problem, it is convenient to represent the fundamental equations which rule the phenomenon in terms of non-dimensional variables.

Thus, considering the non-dimensional variables $\xi=x/L$; $\eta=y/L$, and after some algebraic manipulations the following equation may be obtained:

$$\upsilon\frac{\partial^5 \eta}{\partial \xi^4 \partial \tau}+\frac{\partial^4 \eta}{\partial \xi^4}+\left[u^2-\gamma(1+\xi)+\Pi-\Gamma\right]\frac{\partial^2 \eta}{\partial \xi^2}+2\beta^{1/2}u\frac{\partial^2 \eta}{\partial \xi \partial \tau}+\gamma\frac{\partial \eta}{\partial \xi}+\frac{\partial^2 \eta}{\partial \tau^2}=0 \qquad \textbf{(6)}$$

in which,

$$\begin{cases} \xi = \frac{x}{L};\ \eta = \frac{y}{L};\ \tau = \left(\frac{EI}{M+m}\right)^{1/2} \frac{t}{L^2};\ \nu = \left(\frac{I}{E(M+m)}\right)^{1/2} \frac{\mu}{L^2} \\ u = \left(\frac{M}{EI}\right)^{1/2} UL;\ \beta = \frac{M}{M+m};\ \gamma = \left(\frac{M+m}{EI}\right) L^2 g;\ \Gamma = \frac{T_o L^2}{EI};\ \Pi = \frac{p_o A L^2}{EI} \end{cases} \quad \textbf{(7)}$$

being, β - the mass relation $0<\beta<1$; γ - the pipe's gravity function and flexional properties; Π - the pressurization effect (P_o); Γ - the axial force effect; υ - the pipe's internal damping effect; u - the flow's non-dimensional velocity.

The non-dimensional frequency Ω is related to a circular frequency ω by:

$$\Omega = \left(M + {}^{m}\!/_{EI}\right)^{1/2} \omega L^2 \quad \textbf{(8)}$$

3.2. Numerical Solution of the Pipe Flowing Fluid

We discretize the non-dimensional equation for the movement of pipe flowing fluid (Eq.(6)), by using the finite differences procedure, which results in the Eq.(9).

$$\begin{aligned} &\frac{\nu}{\Delta\xi^4}\left\{\dot{\eta}_{i+2} - 4\dot{\eta}_{i+1} + 6\dot{\eta}_i - 4\dot{\eta}_{i-1} + \dot{\eta}_{i-2}\right\} + \frac{1}{\Delta\xi^4}\left\{\eta_{i+2} - 4\eta_{i+1} + 6\eta_i - 4\eta_{i-1} + \eta_{i-2}\right\} \\ &+\left[u^2 - \gamma(1-\xi) + \Pi - \Gamma\right]\frac{1}{\Delta\xi^2}\left\{\eta_{i+1} - 2\eta_i + \eta_{i-1}\right\} + \frac{\gamma}{2\Delta\xi}\left\{\eta_{i+1} - \eta_{i-1}\right\} + \frac{2\beta^{1/2}u}{2\Delta\xi}\left\{\dot{\eta}_{i+1} - \dot{\eta}_{i-1}\right\} + \ddot{\eta}_i = 0 \end{aligned} \quad \textbf{(9)}$$

Reorganizing Eq.(9), grouping the terms of displacement, velocity and acceleration, under the form of finite differences operators (δ), we obtain:

$$\vec{\ddot{\eta}}_i + \left\{\frac{\nu}{\Delta\xi^4}\delta_\xi^4 + \frac{2\beta^{1/2}u}{2\Delta\xi}\delta_\xi^2\right\}\vec{\dot{\eta}}_i + \left\{\left[u^2 - \gamma(1-\xi) + \Pi - \Gamma\right]\frac{1}{\Delta\xi^2}\delta_\xi^2 + \frac{1}{\Delta\xi^4}\delta_\xi^4 + \frac{\gamma}{2\Delta\xi}\delta_\xi\right\}\vec{\eta} = 0 \quad \textbf{(10)}$$

in which δ_ξ is a central finite differences operator of i order in relation to the space (ξ) applied to a differential variable which characterizes the cell composing the finite differences discretization.

Presenting Eq.(9) under matrix form, we have the discretized equation for the motion of a pipe flowing fluid.

$$\vec{\ddot{\eta}} + [C]\vec{\dot{\eta}} + [K]\vec{\eta} = \vec{0} \quad \textbf{(11)}$$

in which,

- $\vec{\eta}^T = \{\eta_1\ \eta_2\ \eta_3 \ldots \eta_n\}$ - corresponds to the displacement vector of the discretized nodes of the structure,
- $[K] = \left\{\left[u^2 - \gamma(1-\xi) + \Pi - \Gamma\right]\frac{1}{\Delta\xi^2}\delta_\xi^2 + \frac{1}{\Delta\xi^4}\delta_\xi^4 + \frac{\gamma}{2\Delta\xi}\delta_\xi\right\}_{nxn}$ - is the equivalent discretized rigidity matrix,

- $[C]=\left\{\frac{\nu}{\Delta\xi^4}\delta_\xi^4+\frac{2\beta^{1/2}u}{2\Delta\xi}\delta_\xi^2\right\}$ - is the equivalent descritized damping matrix.

3.2.1. Analysis in Time Domain - Direct Integration using the Newmark Method (MID-MNw)

The Newmark integration scheme in the time is adopted for a constant medium acceleration. The equilibrium equation (Eq.(11)) in the time t+Δt can be expressed as follows:

$$\underset{\sim}{M}\ {}^{t+\Delta t}\ddot{\vec{U}}+\underset{\sim}{C}\ {}^{t+\Delta t}\dot{\vec{U}}+\underset{\sim}{K}\ {}^{t+\Delta t}\vec{U}={}^{t+\Delta t}\vec{R} \tag{12}$$

Having obtained the relations ${}^{t+\Delta t}\dot{U}$ and ${}^{t+\Delta t}\ddot{U}$ in terms of ${}^{t+\Delta t}U$ and of the displacement, velocity and acceleration in time t, and substituing these relations in Eq.(12), the following equation result,

$$\left[\underset{\sim}{M}\frac{1}{\alpha\Delta t^2}+\underset{\sim}{C}\frac{\delta}{\alpha\Delta t}+\underset{\sim}{K}\right]{}^{t+\Delta t}U=$$

$$={}^{t+\Delta t}\vec{R}+\underset{\sim}{M}\left[\frac{1}{\alpha\Delta t^2}{}^{t}U+\frac{{}^{t}\dot{U}}{\alpha\Delta t}+\left(\frac{1}{2\alpha}-1\right){}^{t}\ddot{U}\right]+\underset{\sim}{C}\left[\frac{\delta}{\alpha\Delta t}{}^{t}U+\left(\frac{\delta}{\alpha}-1\right){}^{t}\dot{U}+\frac{\Delta t}{2}\left(\frac{\delta}{\alpha}-2\right){}^{t}\ddot{U}\right] \tag{13}$$

i.e.,

$$\underset{\sim}{\hat{K}}\ {}^{t+\Delta t}\vec{U}={}^{t+\Delta t}\vec{\hat{R}} \tag{14}$$

3.2.2. Analysis in Frequency Domain - Solution of Damped Modes

Suppose now that there is a solution for the equation of a pipe flowing fluid of the kind:

$$\eta=N_0e^{i\Omega\tau} \tag{15}$$

which substituted in the equation of motion (Eq.(11)), results in:

$$\left\{(-\Omega^2)+(i\Omega)\underset{\sim}{C}+\underset{\sim}{K}\right\}\vec{N_0}=\vec{0} \tag{16}$$

In order to solve the eigenvalues generalized problems presented above, we make the following variable transformation:

$$\xi=\dot{\eta}\ \text{e}\ \dot{\xi}=\ddot{\eta} \tag{17}$$

Applying this transformation into Eq.(11) and considering the indentity $[I]\vec{\xi}=\vec{\dot{\eta}}$, we have:

$$\begin{bmatrix}\underset{\sim}{0} & \underset{\sim}{I}\\ -\underset{\sim}{K} & -\underset{\sim}{C}\end{bmatrix}\begin{bmatrix}\eta\\ \xi\end{bmatrix}=\begin{bmatrix}\dot{\eta}\\ \dot{\xi}\end{bmatrix}\quad\therefore\ \underset{\sim}{A}\vec{z}=\vec{\dot{z}} \tag{18}$$

which is the matrix equation of the motion of a pipe flowing fluid. Admiting for $\vec{z}$ a solution similar to Eq.(15), we have:

$$\vec{z}=\begin{bmatrix}\eta\\ \xi\end{bmatrix}=\begin{bmatrix}N_0\\ \Xi_0\end{bmatrix}e^{i\Omega\tau},\quad \text{e}\ \vec{\dot{z}}=\begin{bmatrix}\dot{\eta}\\ \dot{\xi}\end{bmatrix}=\begin{bmatrix}N_0\\ \Xi_0\end{bmatrix}(i\Omega)e^{i\Omega\tau} \tag{19}$$

Substituting Eq.(19) in the equation of motion of the pipe, Eq.(18), we have:

$$\underset{\sim}{A}\vec{z}=\lambda\vec{z} \tag{20}$$

With these transformations, we end up in a typical eigenvalue problem, with complex frequencies ([A] is an asymmetrical matrix). To solve the problem, we recommend the reduction of matrix A into Hessemberge's superior form and the use of the QR-Method in order to determine the eingenvalues (Abo-Hamd e Utku [1]).

4. COMPUTATIONAL ASPECTS

The INSTUBO (Instability in Pipes) code was developed in FORTRAN language. It uses the finite differences method to discretize tubulation (Eq.(20)), and solves the problem of vibrations, in the frequency domain by resolving eigenvalues problems for asymmetric sparse matrices, getting solutions in complex frequency form [12]. The code also solves the problem in the time domain, through the central finite differences procedure, with the Newmark Method [2].

5. PRESENTATION AND DISCUSSION OF THE RESULTS

In order to validate the program, several tests were performed in a straight iron tubulation, containg a dense fluid (water). The velocity range (u = 0-15.75) was such that it reached velocities higher than critical velocities (U_{cr}), point in which the tubulation loses its stability.

The characteristic of the tubulation are: A_{iron} = 112.9 cm^2, $A_{internaly}$ = 781.3 cm^2, L_{tube} =5.00m, E =210GPa, I = $0.497 \cdot 10^{-4}$ m^4, γ_{water} = 1000kgf/m^3 e γ_{iron} = 7800kgf/m^3, theoretical dimensionless frequencies (Ω) (with u=0.00): Ω_1 = 3.51, Ω_2 = 22.03, Ω_3 = 61.69, Ω_4 = 120.90.

Figure 5.1 shows the dimensionless complex frequencies (Ω) trajectory as an continuous function of u, according to Argand diagram for the case of a cantilever pipe in equilibrium with mass function β=0.200, and dissipation effect υ=0.000.

In this figure, we adequate the scale of the real and imaginary axes, in order to facilite the comparison with the results of Païdoussis [7], given in Figure 5.2.

Comparing Figures 5.1 e 5.2, we notice that are a good accordance between them, and the trajectories of the frequencies obtained numerically (Figure 5.1) reproduce satisfactorily the results of the literature (Fig.5.2): (a) The numerical trajectory of the frequencies for the first mode crosses the imaginary axis at u≅6.38, whereas in literature u=7.00 (ε=-8.86%); (b) The second mode of vibration, along with the other even modes (4°, 6°,...), determines the critical velocity of the dynamic instability. When crossing the real axis, the imaginary term, that is, the damping of the system becomes negative, and the motions increase continuously until the collapse of the structure.

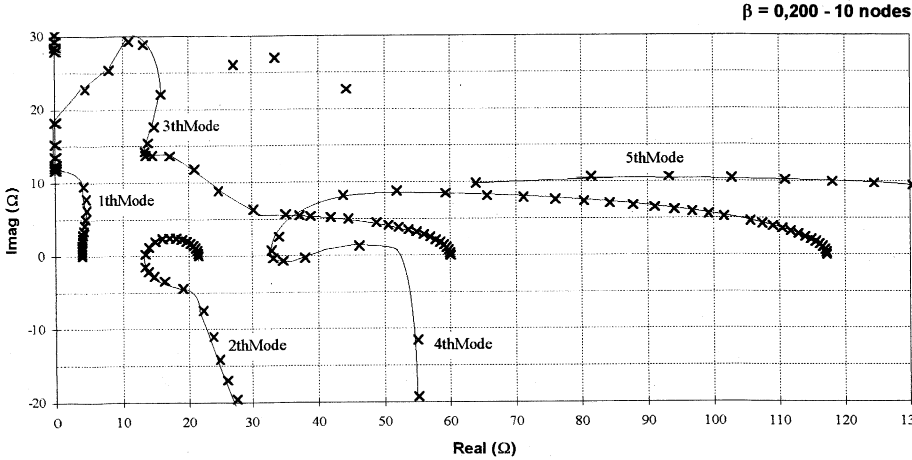

Figure 5.1 - Non-dimensional complex frequencies (Ω) evolution in function of the non-dimensional velocity (u)

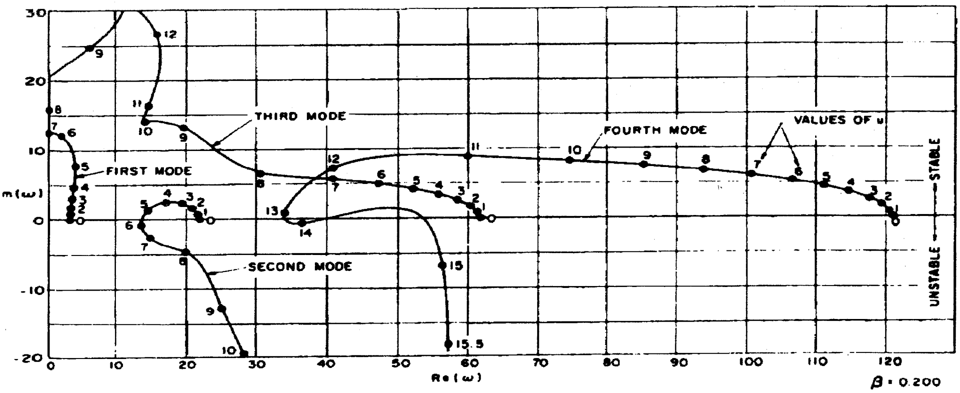

Figura 5.2 - Theoretical results - (Païdoussis [7])

The approximate value of the critical velocity (Eq.(7)), may be calculated as $U_{cr} = 24.82$m/s (u=5.70), which can be a flowing velocity attainable in tubular circuits in either industrial or nuclear power plants.

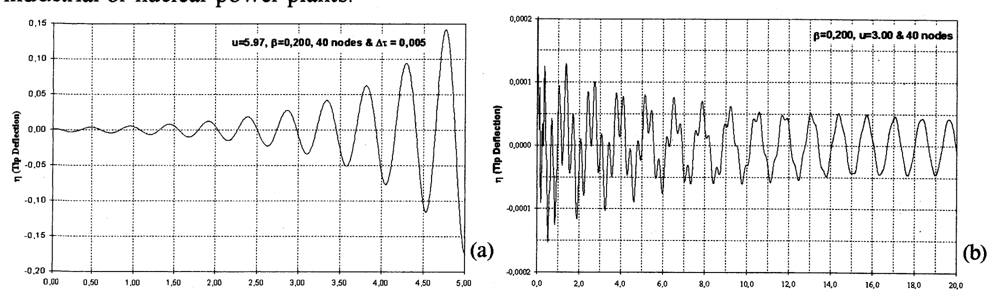

Figure 5.3 - Non-dimensional time deflexion record for two examples of flowing velocity

In Figure 5.3, we present the history of motion for the extemity of the pipe obtained through the direct integration scheme. We used 40 nodes to discretize the tube, considering the time step Δτ=0.005. In Figure 5.3a the flowing velocity is superior to the critical velocity, i.e., u=5.97 and β=0.200.

Given a small perturbation on the pipe, we notice that the oscillactory instability occurs according to the second mode of vibration. Thus, the displacement amplitudes increase continuously, leading the structure to collapse. However, this shall not be the real behaviour of the system. Indeed, if we add the non-linear terms to the equation of motion, the displacement amplitudes shall tend to a limit cycle, which in this case will be stable. However, from a practical point of view, the existence of a stable limit cycle shall not alter the velocity of utilization of the system once the displacement amplitudes within the limit cycle might cause damage to the structure.

In Figure 5.3b, the flow velocity u=3.00. Thus, once the effects of the initial perturbation have stopped, the pipe acquires a damping oscillatory motion with preponderance of the first mode of vibration.

6. CONCLUDING COMMENTS

In view of the results obtained in these studies, we can conclude that:

i) the instability problem in pipes flowing fluid can appear in a band value of current use flowing velocity, which determines the use of project checking and preventive methods.
ii) when $U=U_{cr}$, the system is at the instability limit for a given frequency (normally the lowest, with ($\omega_n=0$)), and U_{cr} associated to the pipe's jet force, correspondent to Euler's charge (statically determinated).
iii) instability is associated with the asymmetric even modes of the pipe, which are represented like a lashing such as, for instance, as flexible hose when a flow runs through it.
iv) in a non-conservative system, complex frequencies appear and determine the zone of instability for critical velocity bands. When one of the frequencies curves crosses the imaginary axis to penetrate the positive real plan, the system is unstable, a fact observed in several simulations.
v) in this study on hydroelastic instabilities, we can observe the following aspects (for a cantilever pipe): (a) the flow increases or diminishes the natural frequencies of oscillations according to the modes and the values of u and β; (b) the flow damping the free movements of the pipe (for small non-dimensional flow velocity u).

7. ACKNOWLEGDEMENTS

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