LIMIT LOAD ANALYSIS OF THE LATERAL SUPPORT FRAMEWORK FOR AN ICE CONDENSER SYSTEM

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SUMMARY

The ice condenser is a safeguard system for a nuclear power plant. Its function is to cool the steam released within the containment in the event of a design basis accident, thereby decreasing the pressure acting on the containment structure. The system extends around a 300 degree arc between the inner and outer-containments, and is approximately 60 feet in height.

Structurally, the ice condenser system consists of slender ice baskets, 48 feet in length, vertically extending through tiered lattice frames to a lower support structure which provides both vertical and horizontal constraint at the base of the baskets. The lattice frames provide lateral support to the ice baskets at six foot vertical intervals, and are the structures of principle interest to this paper. To facilitate installation maintenance, the ice baskets are not attached to the lattice frames but in fact, are separated from them by a slight gap.

As a major support structure, limit load analysis of a representative lattice frame was conducted to supplement tests and other analyses with a view to provide a better understanding of the behaviour and strength characteristics of the ice condenser system when subjected to its primary functional loads. The primary loads experienced by the lattice frame are inplane forces transmitted by the ice baskets as a result of a seismic event and / or a design basis accident. Corresponding design loads are determined from elastic nonlinear seismic time-history analyses and from dynamic pressure transient analyses.

In the paper, limit analysis of the lattice framework is conducted for three principle load conditions: radial, tangential, and radial + tangential. Depending on the load pattern, the analysis is formulated as a linear programming problem based on either the static or kinematic theorem of plastic collapse. In view of the size and complexity of the lattice frame, decomposition theory is employed to reduce the various problems to the smallest size possible. Multiple stress resultants affecting plasticity are considered, and corresponding piecewise linear yield surfaces are generated also using mathematical programming techniques. A general computer programme is employed for all analyses. Given the nature of the loads and the structure properties, the programme generates yield surfaces and formulates and solves the limit analysis in one operation.

The critical load pattern is identified as having the smallest limit load level. With a view to improve the accuracy of the solution, the limit analysis for this load case is repeated several times for increasingly refined yield surface approximations. The final analysis results concerning limit load(s) and plastic collapse mode(s) entirely confirmed the previously obtained results from experimental tests and elastic analysis. The limit load levels for all load cases were found to be in excess of the minimum qualification loads established by applying load factors to the primary functional design loads, thereby conclusively demonstrating the structural integrity of the lattice frame support system.

1. Introduction

Many engineering structures may experience extreme loads (accidental or otherwise) that cause one or more structural components to behave plastically, and it is imperative that the limit (plastic) strength of these components preclude (castastrophic) failure under any such loading conditions. The design of such structures, therefore, must be complemented by reliable experimental and/or analytical evidence as to the limit strength of some or all of their component parts. This is particularly true for a nuclear power plant.

The present investigation concerns the (analytical) limit strength of a major support framework in a nuclear power plant ice condenser system (Fig. 1). The ice condenser is a safeguard system in that its function is to cool the steam released into the ice condenser compartment in the event of a design basis accident, thereby decreasing the pressure acting on the containment structure. The annular area encompassed by the crane wall and the containment wall (Fig. 1) has inner and outer radii of approximately 45 and 60 feet, respectively. The 15°F insulated ice condenser compartment is approximately 60 feet in height and extends around a 300 degree arc and is located in the upper half of the containment structure.

Structurally, the system consists of 12 inch diameter performated ice baskets 48 feet in length that vertically extend through an eight-tier lattice frame assembly to a lower support structure. The frame assembly is vertically supported by columns and laterally supported by wall panels connected to the crane wall. The ice baskets are vertically and laterally constrained by the lower support, and laterally supported at six foot vertical intervals by the tiered frames. To facilitate installation, maintenance, and periodic weighing of the ice, the baskets are freestanding with a 1/2 inch tolerance gap at each lattice aperature (Fig. 6).

Each tier of the frame assembly consists of 72 lattice-frame modules lying in the horizontal plane over the 300 degree arc of the ice condenser compartment. Adjacent modules (hereafter termed 'lattice frame') share common vertical columns. Separate analytical studies determined that it was possible to uncouple a typical lattice frame for the purposes of representative investigation. This frame model (Fig. 2) is the structure of principle interest to the present study.

The primary loads experienced by the lattice frame are dynamic forces transmitted by the ice baskets as a result of a seismic event and/or a design basis accident. From (separate) elastic nonlinear time-history and dynamic pressure transient analyses, sets of equivalent static forces were established as the corresponding design loads. Then, as a supplement to tests and other analyses, limit load analysis of the lattice frame was conducted for these loads with a view to provide a better understanding of the strength of the ice condenser system when subjected to extreme loading conditions. The details of this analysis are the principle concern of the present study.

2. Mathematical Bases for Computer Analysis

For small deformations and elastic-perfectly plastic behaviour the limit strength of skeletal frameworks is characterized by that loading level at which sufficient plastic zones have formed as to cause the whole structure, or any part of it, to fail in a collapse mechanism mode. For a single stress resultant-type affecting plasticity (e.g., moment), the limit load analysis is precisely a linear programming (LP) problem, Charnes [1]; while for multiple stress resultants (e.g., moment, axial force, etc.) the analysis is theoretically a nonlinear programming (NLP) problem. The latter is generally the case for most engineering structures. Unfortunately, current (1975) NLP solution algorithms are inadequate for the economical analysis of practical-sized structures (e.g., the lattice frame considered herein). Such is not the case for LP algorithms, and, therefore, it is desireable to piecewise linearize the yield criterion governing the multiple stress resultants so as to reduce the analysis to a LP problem, Hodge, [2].

Consider a general member cross-section j (Fig. 3) for which as many as six (normalized) stress resultants $\tilde{\underline{Q}}^j = [Q_{1j}, Q_{2j},\ldots,Q_{6j}]$ may affect plasticity (i.e., biaxial moments and shears, torsion and axial force). The stress state at any continuum point i on the cross-section may include as many as three (normalized) stress components $\tilde{\underline{\sigma}}^i = [\tau_{xi},\tau_{yi},\sigma_{zi}]$, and the (known) Von Mises yield criterion is taken to govern rigid-plastic behaviour at each such point. The yield criterion defining the rigid-plastic yield surface for the entire cross-section is generally a nonlinear function of $\underline{Q}^j$. Except for a few special cases, the actual yield surfaces for most cross-sections are not known in the literature. Albeit, by dividing any given cross-section into an element mesh (Fig. 3) and employing simple numerical integration, a (lower-bound) stress point, $\underline{Q}^j_y$, on the actual yield surface is found by formulating the LP problem, Baset [3,4]

$$\alpha_y = \max \alpha \;||\; \tilde{\underline{n}}\underline{\sigma} - \underline{v} \leq \underline{0} \;, \quad \tilde{\underline{c}}\underline{\sigma} - \alpha\underline{R} = \underline{0} \;, \quad \alpha \geq 0 \tag{1}$$

and solving to give

$$\underline{Q}^j_y = \alpha_y\underline{R} \tag{2}$$

where: $\underline{n}$ is a normality matrix of the (known) unit normals to the hyperplanes of a preselected piecewise linear (PWL) Von Mises yield surface (e.g., Fig. 4) adopted to govern rigid-plastic behaviour of each of the elements i; $\underline{\sigma}$ is the vector of stress components $\underline{\sigma}^i$ extending over all elements; the unity vector $\underline{v}$ defines the (normalized) simple-tension yield stress for the material; the (known) entries of the compatibility matrix $\underline{c}$ are element strains (corresponding to $\underline{\sigma}^i$) due to unit deformations (corresponding to $\underline{Q}^j$) imposed individually on the cross-section; the (known) entries of the vector $\underline{R}$ are preselected to define a particular ray-direction from the coordinate origin of the stress space (e.g., $\tilde{\underline{R}} = [1,1,\ldots,1]$ defines the bisector axes of the positive orthant of the stress space); and α is a proportional scale factor on the stress resultants along the predetermined ray-direction. The solution to eg. (1) corresponds to the maximum amplification, α_y, of the ray-vector $\underline{R}$ for which the PWL Von Mises yield criterion (first constraint set) and equilibrium of stress resultants and components (second constraint set) is everywhere satisfied over the cross-section.

Since the (normalized) yield surface is symmetrical, eq. (2) defines a generic yield point for all orthants of the stress space through sign-permutation of the entries in $\underline{R}$ (e.g., for $\tilde{\underline{R}} = [-1, -1,\ldots,-1]$, $\underline{Q}^j_y$ is the stress point on the yield surface in the negative orthant). Other generic yield points are found by solving eq. (1) for different $\underline{R}$ (e.g., $\tilde{\underline{R}} = [1, .5, .5,\ldots,.5]$), until the desired level of approximation to the actual yield surface is reached. The final sets of yield-point vertices taken together constitute the matrix $\underline{Y}^j$ defining the PWL yield surface for member cross-section j (i.e., the column entries of $\underline{Y}^j$ are the $\underline{Q}^j_y$).

Any stress point $\underset{\sim}{Q}^j$ on the PWL yield surface may be expressed as a convex linear combination of the yield-point vertices $\underset{\sim}{Y}^j$, Zavelani-Rossi [5], i.e.,

$$\underset{\sim}{Q}^j = \underset{\sim}{Y}^j \underset{\sim}{B}^j \tag{3}$$

where the non-negative combination-factors sum to unity; i.e., $\tilde{\underset{\sim}{V}}\underset{\sim}{B}^j = 1$. Therefore, subject to eq. (3), the yield criterion for the cross-section is given by the single constraint

$$\tilde{\underset{\sim}{V}}\underset{\sim}{B}^j \leq 1 \tag{4}$$

This way of representing the yield criterion (i.e., as opposed to one constraint equation for each hyperplane) significantly improves the efficiency of the LP solution algorithm (which is very dependent on the number of constraints). The device - equivalent to applying LP Decomposition Theory - may also be employed to formulate eq. (2).

Having the (adopted) PWL yield surfaces for all members, the limit load analysis of the entire structure becomes a LP problem. Based on the Static Theorem, the limit analysis involves finding the maximum multiplier λ on the applied loads $\underset{\sim}{F}$ for which there exists a stress resultant field simultaneously satisfying the yield criteria and equilibrium conditions over the entire structure; i.e., from eqs. (3) and (4), to find, Baset [3,4],

$$\lambda_c = \max \lambda \;\|\; \tilde{\underset{\sim}{V}}\underset{\sim}{B} - \underset{\sim}{K} \leq 0,\; \tilde{\underset{\sim}{C}}\underset{\sim}{Y}\underset{\sim}{B} - \lambda\underset{\sim}{F} = \underset{\sim}{0},\; \underset{\sim}{B} \geq \underset{\sim}{0},\; \lambda \geq 0 \tag{5}$$

where: $\tilde{\underset{\sim}{V}}$ is a multi-diagonal unity matrix; $\underset{\sim}{B}$ is the vector of $\underset{\sim}{B}^j$ over all members; vector $\underset{\sim}{K}$ defines the (normalized) principle plastic capacities for the members; compatibility matrix $\underset{\sim}{C}$ relates member deformations (corresponding to $\underset{\sim}{Q}^j$) to independent nodal displacements (corresponding to $\underset{\sim}{W}$) for the structure; $\underset{\sim}{Y}$ is the matrix of $\underset{\sim}{Y}^j$ over all members (note that $\underset{\sim}{Y}\underset{\sim}{B}$ defines the stress resultant field for the structure). Depending on the different load patterns to be considered for the structure, eq. (3) is solved for different $\underset{\sim}{F}$ to find corresponding collapse load factors λ_c.

It is noted that eqs. (2) and (3) are both based on the Static Theorem, and that, alternatively, they both may be reformulated on the basis of the Kinematic Theorem. As such, with a view to computational efficiency, a choice of formulation such as to have the fewest number of constraints is afforded (e.g., the Kinematic formulation is more effecient for the limit analysis of planar frameworks, while the Static formulation is best for space frameworks).

A general computer programme (being) developed at the University of Waterloo incorporates all of the foregoing features. For input data concerning applied loading, structure geometry, member properties and desired level of approximation to actual yield surfaces, the programme generates PWL yield surfaces and performs the limit load analysis in one (sequential) LP operation. The programme: is based on both the Static and Kinematic Theorems; utilizes Decomposition Theory; provides for bounded solutions; and employs a general LP code, IBM [6], permitting the analysis of large and complex skeletal structures (e.g., over 4000 constraint conditions for any number of variables).

3. Limit Load Analysis of Lattice Frame

The lattice frame model is shown in Fig. 2. The wall panels at the crane wall were assumed to provide full end-fixity in the plane of the frame. The columns, however, were assumed to provide no lateral support and, as such, the (idealized) model is equivalent to a planar frame with fully-fixed (cantilever) supports at its base (Fig. 5). Strut members are not connected at their points of diagonal crossing, while the remaining member joints are welded connections typical of, so-called, rigid frames. The steel grade, common for all strut, rib and bracket members, has simple-tension yield stress σ_y. The response of the vertical ice baskets to a seismic event and/or a design basis accident is such as to transmit in-plane loads to the supporting lattice framework.

Concentrated (i.e., point) tangential and/or radial forces are transmitted to the framework through the connecting collar and spacer-plate guides for the vertical ice baskets (Fig. 6). For each ice-basket aperature, the tangential forces t act at the mid-points of rib members while the radial forces r act at the third-points of the strut (or bracket) members. The corresponding equivalent static loads were specified to act over three sections of the framework as fixed fractions of a common load value F (Fig. 5). It is noted that: common point loads prevail for each of the three frame sections (e.g., t_1 and r_1 are common for section 1); load magnitudes increase with distance from the crane wall (e.g., $t_1<t_2<t_3$); all tangential loads are considered to act in the same direction at any given time; and all radial loads are considered to act outwards from the crane wall.

Three load cases were determined to be of concern; tangential (i.e., t loads alone), radial (i.e., r loads alone) and combined tangential + radial (i.e., t and r loads together). For each load case, the limit strength of the framework is dictated by the maximum load value $F_c = \lambda_c F$ that can be sustained prior to failure in a collapse mechanism mode. The critical load case corresponds to the smallest F_c value.

In view of the planar nature of the loads, axial force and uniaxial moment and shear are the stress resultant-types experienced by the members. Shear forces were assumed to have (comparatively) negligible influence on plastic behaviour since the struts, ribs and brackets are all of shallow rectangular cross-section. As such, moment M and axial force N were taken as the principal stress resultant-types affecting plasticity. The corresponding principal plastic capacities of the members, Mp and Np, are given in Fig. 5.

The two normalized stress resultants for each member cross-section were defined as $Q_1 = M/Mp$ and $Q_2 = N/Np$. Since the corresponding stress state at any continuum point on the cross-section is specified by normalized axial stress $\sigma_z = \sigma/\sigma_y$ alone (where σ = actual stress), note that the normalized Von Mises yield criterion reduces to $-1 \leq \sigma_z \leq 1$. As it happens, the case of a solid rectangular cross-section for which moment and axial force alone affect plasticity is one of the few for which the actual nonlinear yield surface is known (i.e., curved locus - Fig. 7). Albeit, solving eq. (1) for two ray-directions R_1 and R_2 gives corresponding yield-point vertices lying exactly on the actual surface as shown in Fig. 7. In actual fact, two different PWL yield surfaces were generated for subsequent limit load analysis of the entire framework. From Fig. 7, the 8-side polyhedron defined by vertices 1 to 8 (i.e., broken lines) was initially adopted to govern plastic behaviour of the critical cross-sections for the limit analyses concerning all three load cases. Then, the solution for the critical load case was refined by adopting the 12-side polyhedron A to L (i.e., solid lines) and repeating the limit analysis.

The framework is 153 times statically indeterminate, and has 331 critical sections at which (concentrated) plasticity may occur (i.e., all points of support connection, member connection and load application). Therefore, for two prevailing stress resultant-types (i.e., moment and axial force), there are 2 x 331-151 = 511 independent equilibrium conditions of concern to the limit analysis. From eq. (5), limit analysis based on the Static Theorem involves solving a LP problem having 331 + 511 = 842 constraint conditions (i.e., yield + equilibrium). However, the same analysis based on the Kinematic Theorem involves only 663 constraints (i.e., 1 external work + 2 x 331 = 662 kinematic compatibility conditions) and, hence, is more efficient, Baset [3]. As such, the computer programme employed the kinematic formulation (not detailed herein) to conduct the limit analysis for all three load cases.

Based on the 8-side PWL yield surface 1-8 (Fig. 7), the limit load F for each of the three load cases was found to be as given in Table I (i.e., in terms of yield stress σ_y). The corresponding plastic collapse modes are shown in Fig. 8. From Table I, the critical loading for the framework is noted to be the tangential load case (i.e., $0.497\ \sigma_y <<< 3.852\ \sigma_y$). In fact, the tangential loads completely dominated the combined load case. Based on the 12-side PWL yield surface A-L (Fig. 7), the limit analysis was repeated for the tangential load case to find an improved (conservative) limit load $F_c = 0.504\ \sigma_y$, Grierson [7]. The corresponding collapse mode was again found to be the panel-sway mechanisms shown in Fig. 8(a). In view of the slight change in the value of F_c, further yield surface refinements were not considered necessary.

4. Conclusions

The analytically derived limit load(s) and plastic collapse mode(s) confirmed the results and conclusions of prototype tests and other analyses concerning the strength and behaviour characteristics of the lattice frame under extreme load conditions. The limit load levels for all load cases were found to be in excess of the qualification loads established by applying load factors to the primary functional design loads, thereby confirming the structural integrity of the lattice frame support system.

Finally, the rational adaption of PWL yield criteria to govern plastic behaviour at critical cross-sections permits efficient plastic analysis of large and complex frameworks. Using an IBM 360/75 computer (Waterloo), the execution time was less than 2 decimal minutes for each of the analyses reported herein.

5. Acknowledgements

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6. References

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TABLE I : Limit Load F_c

Loading Case	PWL Yield Surface	
	8-side	12-side
Tangential	$0.497\sigma_y$	$0.505\sigma_y$
Radial	$3.852\sigma_y$	-
Combined	$0.497\sigma_y$	$0.505\sigma_y$

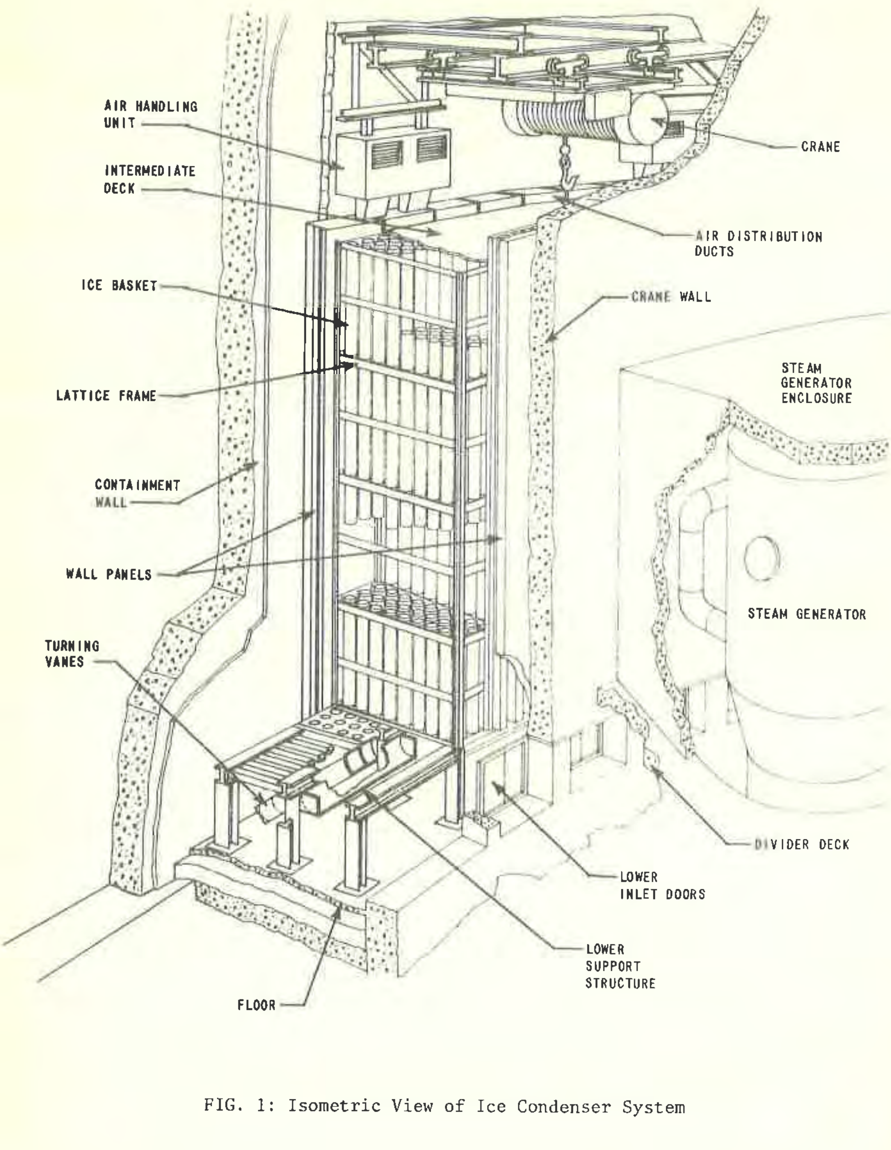

FIG. 1: Isometric View of Ice Condenser System

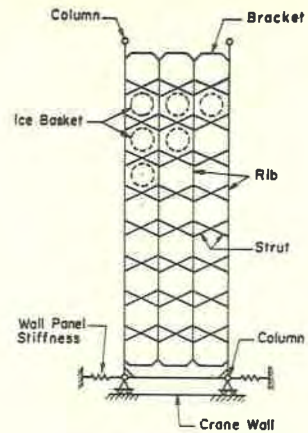

FIG. 2: Lattice Frame Model

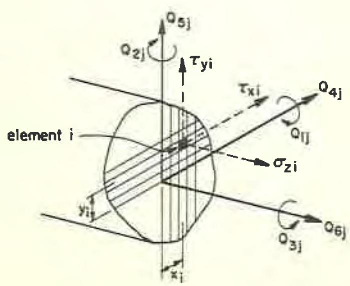

FIG. 3: Stress Resultants and Stress Components for General Member Cross-Section j

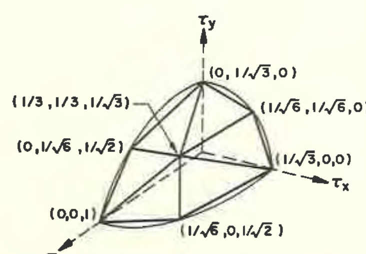

FIG. 4: PWL Von Mises Yield Surface (positive orthant)

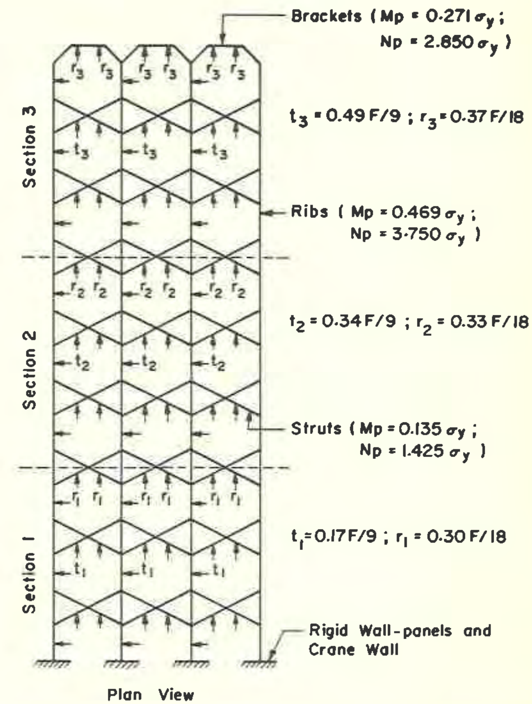

FIG. 5: Idealized Frame and Loading

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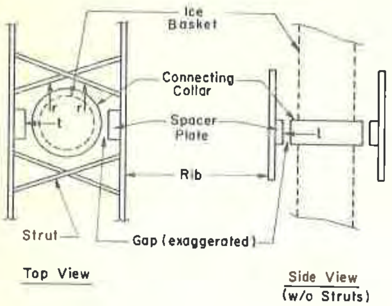

FIG. 6: Tangential (t) and Radial (r) Forces Exerted by Ice Basket

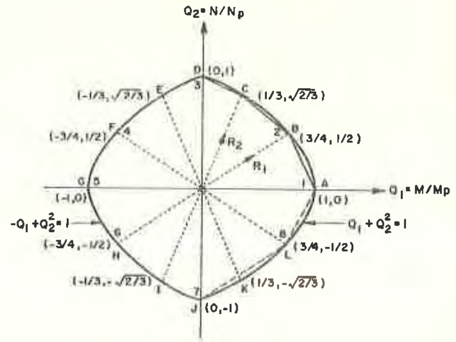

FIG. 7: PWL Yield Surface(s) for Rectangular Section Subjected to Moment (M) and Axial Force (N)

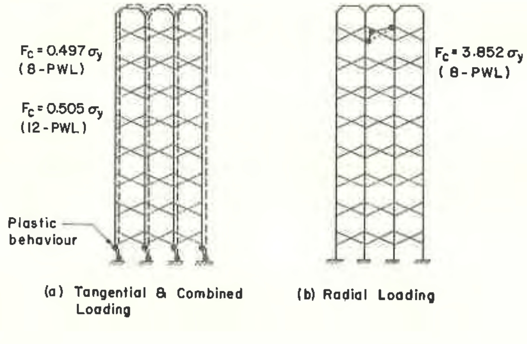

FIG. 8: Plastic Collapse Modes