SOME MODELS FOR THE ANALYSIS OF STRESSES IN A TUBULAR FUEL PIN FOR HIGH TEMPERATURE REACTORS BY FINITE ELEMENTS

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SUMMARY

In order to study the postulated time dependent stress problem, two different finite element computer code, a two-dimensional body of revolution BREL, and a three-dimensional TRISTAN code have been developed in the Nuclear Power Section of the Imperial College of Science and Technology. Recent experience, gained during 1972 has shown, that the 3-D programme can only be usefully used at extreme expense of time and money, and on large computers only. It also appears, that the "isoparametric" hexahedral elements used might have to be replaced by "subparametric" ("supergeometric") elements, which possess a displacement field distribution of a higher order than the element shape. This seems to be useful for structures with thermal and similar (irradiation induced) load.

The 2-D axi-symmetric code, on the other hand, has proven its high applicability for the determination of time dependent stresses in tubular fuel elements for high temperature gas-cooled reactors. Several cases of discontinuity and interaction (fuel-cladding) problems have been solved with this code. The authors describe various optional models for irradiation induced dimensional changes and creep of graphite, used with the 2-D code. Results are included with particular reference to Wigner shrinkage strain build-up during the reactor operation.

Previously published results have been based on the assumption that the shrinkage strain of nuclear graphite at a point within the structure depends on the total neutron dose accumulated at that point and on its current temperature. As a more realistic alternative, the strain might be assumed to be dependent on the entire temperature and dose history. The Wigner strain is then to be calculated by summing the increments. Early experience indicated that the choice of models did not greatly affect the calculated stresses, probably because the temperature variation at the point has normally been very moderate. However, recently it has been shown that unexpectedly high stresses may be computed after large instant power changes. To eliminate this anomaly, the build-up procedure has to be used. In the event of ramp changes, the new model may be used for the decreasing situation, and the old model might be preferable for increasing temperatures (due to the effects of the accelerated mobility of defects within the material).

1. Introduction

In the development of the High Temperature Gas-Cooled Reactor (HTR), attention in the United Kingdom has until recently been focussed on a form of core structures consisting of prismatic graphite blocks having multiple channels in which the fuel pins are located and along which the coolant flows. The fuel pins consist of coated fuel particles, bonded to form fuel compacts and placed inside graphite sleeves. At the First International Conference on Structural Mechanics in Reactor Technology, Berlin 1971, Head and Kinkead [1] described several geometric forms of the fuel pin, which have at various times been studied by the O.E.C.D. Dragon Project, and reviewed the methods of analysis used in the studies. The purpose of this paper is to review some recent analytical and experimental results and to describe developments of the computing methods.

2. Computer Codes

2.1 1-Dimensional Code (HASSAN)

HASSAN is a 1-dimensional code, developed especially for simultaneous thermal and structural analysis of tubular fuel pins, in which there exists the possibility of mechanical contact between the fuel compacts and the inner graphite sleeve. It has been reported by Nehrig [2] that tensile stresses resulting from mechanical interaction have caused cracking of fuel compacts in the Peach Bottom reactor, although the fuel particle coatings were not damaged.

Details of HASSAN have been described elsewhere by the present authors [3, 4]. Results obtained using this code, and included in the same references, indicate that cracking is not likely to occur in the tubular fuel pins studied by the Dragon Project. This conclusion is partially confirmed by the experimental results of Everett, Graham and Ridealgh [5]. Additional full-life irradiation tests now being undertaken (ref. [6]) should provide further confidence in the structural integrity of this type of fuel pin.

2.2 2-Dimensional Code (BREL)

BREL is a 2-dimensional finite element code, developed specifically for analysis of graphite core components with axial symmetry. Details of the code have been described elsewhere by the present authors [7]. This code has recently been used to compare the predicted stresses in a tubular fuel pin using different models for the evaluation of the irradiation induced dimensional changes. Results are included in this paper.

The element stiffnesses are calculated from

$$\underline{K}^e = \underline{B}^T \underline{D} \underline{B} V_e \tag{1}$$

and forces from

$$\{F_n^e\} = - \underline{B}^T \; \underline{D} \; \{\bar{\varepsilon}_n\} \; V_e \qquad (2)$$

where $V_e = 2 \, \Delta \, (d/2 + (\pi \, \bar{r} - d/2) \, p)$ (3)

and $p = 0$ for plane strain

$p = 1$ for the body of revolution case.

2.3 3-Dimensional Code (TRISTAN)

TRISTAN is a 3-dimensional finite element code, using isoparametric hexahedron elements with numerical integration to evaluate the element stiffness and the forces due to the combined thermal, irradiation and creep strains. The resulting system of equations is solved for displacements using the frontal method proposed by Irons [8]. A more detailed description of the code is given elsewhere by the authors [9].

The element stiffnesses are computed from

$$\underline{K}^e = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} \underline{Q}^T \, (\underline{H}^T \, \underline{D} \, \underline{H}) \, \underline{Q} \, |\underline{J}| \, d\xi \, d\eta \, d\zeta \qquad (4)$$

and forces from

$$\{F_n^e\} = - \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} Q^T \, (\underline{H}^T \, \underline{D} \, \underline{H}) \, (\underline{H}^{-1} \, \{\varepsilon_n^*\}) \, |\underline{J}| \, d\xi \, d\eta \, d\zeta \qquad (5)$$

Using Gaussian quadrature the nodal forces for the case of an isotropic material are found to be

$$F_{ji} = - \sum_{k=1}^{27} W_k \left(N_{i,j} \left(\varepsilon_v^n \lambda + \varepsilon_{jj}^n G \right) + G \sum_{h=1}^{3} N_{i,h} \, \varepsilon_{hj}^n \right)_k |\underline{J}|_k \qquad (6)$$

where $j = 1, 3$ and $i = 1, n$ ($n = 20$ for quadratic element and 32 for cubic element).

3. Modelling of Material Properties

The codes described in Section 2 permit the user to take account of anisotropy of graphite properties between the extrusion, or pressing, direction and the transverse direction. Most of the studies made by the authors have related to fuel pins manufactured by near-isotropic Gilsocarbon based graphites. For this material, it is reasonable to use an isotropic form of elasticity matrix. This material exhibits a greater degree of anistropy of some other properties, which is taken into account. The material behaviour is therefore represented in the following manner:

Elasticity matrix $\underline{D} = \lceil \underline{S} \; \underline{T} \rfloor$ (7)

where $$\underline{S} = [\lambda + 2G\,\delta_{ij}] \quad (i,\ j=1,\ 3) \tag{8}$$

$$\underline{T} = [G\,\delta_{ij}] \quad (i,\ j=4,\ 6) \tag{9}$$

Thermal strains $$\{\varepsilon^{th}\} = \{\alpha_{ij}\}\ T \quad (i,\ j=1,\ 3) \tag{10}$$

Irradiation (Wigner) strains

Two models have been used, one in which the Wigner strain at a point is assumed to depend on the neutron dose at the point and the current temperature only,

$$\{\varepsilon^{W}\} = \{\omega_{ij}\} \quad (i,\ j=1,\ 3) \tag{11}$$

where the elements of ω_{ij} are determined by direct interpolation of experimental data.

In the alternate model, the elements ω_{ij} are assumed to depend on the entire dose/temperature history and are obtained by summation of incremental strains over all dose intervals

$$\omega_{ij}(d_e) = \omega_{ij}(d_e - \Delta d_e) + \Delta\omega_{ij} \tag{12}$$

where the $\Delta\omega_{ij}$ depend on the neutron dose increment and current temperature.

Creep strains $$\{\varepsilon^{C}\,(d_e)\} = \{\varepsilon^{C}\,(d_e - \Delta d_e)\} + \dot{\underline{c}}\ \{\tilde{\sigma}\}\ \Delta d_e \tag{13}$$

where $$\underline{c} = \lceil \underline{a}\ \underline{b} \rfloor \tag{14}$$

$$\underline{a} = [(1+\mu)\,\delta_{ij} - \mu]\ C \quad (i,\ j=1,\ 3) \tag{15}$$

$$\underline{b} = [2\,(1+\mu)\,\delta_{ij}]\ C \quad (i,\ j=4,\ 6) \tag{16}$$

and where $\mu = 0.5$ in case of isochoric creep.

4. Results, Discussion and Conclusions

Fig. 1 shows the radial and axial displacements at the end closure of a tubular fuel pin with a prescribed temperature distribution. These results were obtained using TRISTAN. By virtue of the axial symmetry of the pin, there are no tangential displacements. Better results would be obtained from the 3-dimensional code by using more elements. Fig. 2 shows two possible alternate idealisations using respectively 93 and 184 quadratic elements, which the authors consider would yield satisfactory results, but at high cost. Fig. 3 illustrates the assessment of computing cost. Using the values of NKNVA and NUNKVA, the numbers of known and unknown variables respectively, the size of the largest stiffness matrix formed during movement of the front is

$$\text{LARGEST} = (\text{NUNKVA} + \text{NKNVA})\ (\text{NUNKVA} + \text{NKNVA} + 1)\ /\ 2 \tag{17}$$

and the computing cost is

$$\text{COST} = ((C+P/3)(1+ M/16) + (R+L+5B)/200)\ /\ 100 \tag{18}$$

For the 184 element idealisation, the estimated computing cost is £1700/step, but would be less for further time increments in a case where the stiffness may be assumed to remain constant or proportional to changes of a single parameter, Young's modulus, during irradiation.

Fig. 4 gives results from BREL, and shows contours of temperature and principal in-plane stress for initial and irradiated conditions. Fig. 5 shows the time dependent variation of stress on two significant elements, where the initial thermal stresses are greatest. The figure also shows the variable temperature history, including ramp and step changes, and the interaction pressure between the compacts and the inner sleeve. Fig. 6 indicates the corresponding variation of Wigner strain, calculated using the build-up procedure described in Section 3. As would be expected, the stress history is considerably affected by the choice of model, especially with instant step changes in temperature. This conclusion reverses the conclusion drawn by the authors several years ago, that the choice of model did not greatly influence the calculated stresses. The earlier work, however, related to components subjected to smaller variations of temperatures. The authors would recommend the use of the incremental procedure in a situation where large temperature changes occur, although it may be realistic to use the direct interpolation procedure in a situation of ramp-increasing temperature where vacancy mobility is increasing.

Finally, Fig. 7 shows a parametric study of graphite creep modelling. The effects of two different values of Poisson's ratio ($\mu = 0.5$ and $\mu = \nu$) on the computed stress are illustrated. The results indicate little disagreement between the two assumptions.

5. Notation

$\underline{a}$	...	creep compliance submatrix
$\underline{b}$	...	creep compliance submatrix
$\underline{B}$	...	shape matrix of finite elements
B	...	number of punched cards
$\underline{c}$	...	creep compliance matrix
C	...	creep compliance constant
C	...	central processor time (secs)
d	...	dose
d	...	thickness of triangular elements
$\underline{D}$	...	elasticity matrix
F	...	force
G	...	modulus of rigidity
$\underline{H}$	...	auxiliary matrix
$\underline{J}$	...	Jacobian matrix
$\underline{K}$	...	stiffness matrix of finite elements

L	number of printed lines
M	memory in units of 1K (1000 decimal) words
n	number of nodal points per finite element
N	shape functions
p	case indicator
P	peripheral processor time (secs)
$\underline{Q}$	modified shape matrix of finite elements
r	radius
R	number of cards read
$\underline{S}$	elasticity submatrix
$\underline{T}$	elasticity submatrix
T	temperature
V	volume
W	weight factor
α	C.T.E. (coefficient of thermal expansion)
Δ	area of triangular finite elements
Δ	difference, increment
δ	Kronecker's function
ϵ	strain
λ	Lamé constant
μ	Poisson's ratio for creep
ν	Poisson's ratio in elasticity
σ	stress
$\xi\ \eta\ \zeta$	local coordinates
ω	Wigner strain components

Indices:

c	creep
e	element, equivalent
i	nodal numbers
ij	directions
kh	auxiliary indices
n	non-elastic
th	thermal
v	volumetric
w	Wigner

Symbols:

T	transpose
,	derivative
$\bar{x}$	average distribution in finite elements
*	lower order distribution in finite elements
$\tilde{\bar{x}}$	mean value in dose interval
$\lvert\ \rvert$	determinant
$[\ ]$	rectangular matrix
$\lceil\ \rfloor$	diagonal matrix
$\{\ \}$	column matrix, vector
$\underline{X}$	matrix

6. References

[1] HEAD, J.L., KINKEAD, A.N., "Graphite fuel element structures for high-temperature gas-cooled reactors", Nucl. Eng. Design 18, pp. 155 - 144 (1972).

[2] NEHRIG, D.A., Conf. Graphite structures for nuclear reactors, London 1972 (ref. Nucl. Eng. Int. 17, p. 413)

[3] ALUJEVIC, A., HEAD, J.L., UHRY, J.C., HASSAN - a computer programme for thermal and structural analysis of prismatic HTR fuel elements, Dragon Project Report, D.P.R. 826 (1972).

[4] ALUJEVIC, A., HEAD, J.L., "Heat transfer and stress-strain analysis of tubular fuel elements", Atomkernenergie 20, 4 (1972).

[5] EVERETT, M.R., GRAHAM, L.W., RIDEALGH, F., "Studies on the behaviour of graphite structures irradiated in the Dragon reactor", Conf. Graphite structures for nuclear reactors, Proc. I. Mech. E. P-30, pp. 321-345, London (1972).

[6] Dragon Project, Private communication (1972).

[7] ALUJEVIC, A., HEAD, J.L., "Fuel element stresses at discontinuities and interactions by finite elements in two dimensions", Atomkernenergie 21, 2 (1973).

[8] IRONS, B.M., "A frontal solution program for finite element analysis", Int. J. Numer. Meth. Eng. 2, pp. 5-32 (1970).

[9] ALUJEVIC, A., HEAD, J.L., "Application of three-dimensional finite elements for analysis of irradiation-induced stresses in nuclear reactor fuel elements"; In: WHITEMAN, J.R. (Ed.), The mathematics of finite elements and applications, pp. 415-426, Academic Press, London (1973).

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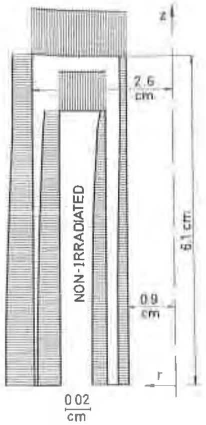

Fig 1 RADIAL AND AXIAL DISPLACEMENTS
[u, w ; hoop displ. v≐10^{-6}cm]
IN TUBULAR FUEL ELEMENT END CAP
[3-D TRISTAN code results]

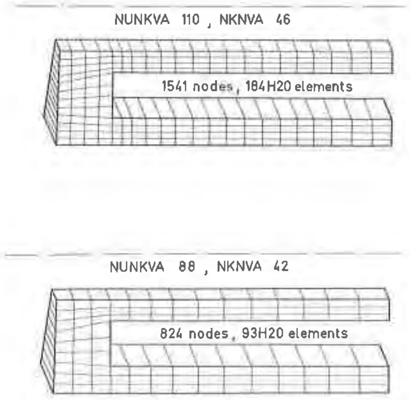

Fig 2 IDEALISATIONS OF STRUCTURES
(3-D hexahedron elem's)

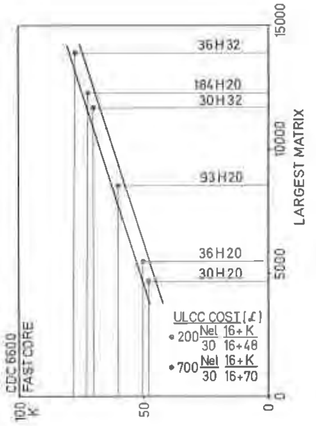

Fig 3 REQUIRED COMPUTER
MEMORY AND COST
(3-D TRISTAN)

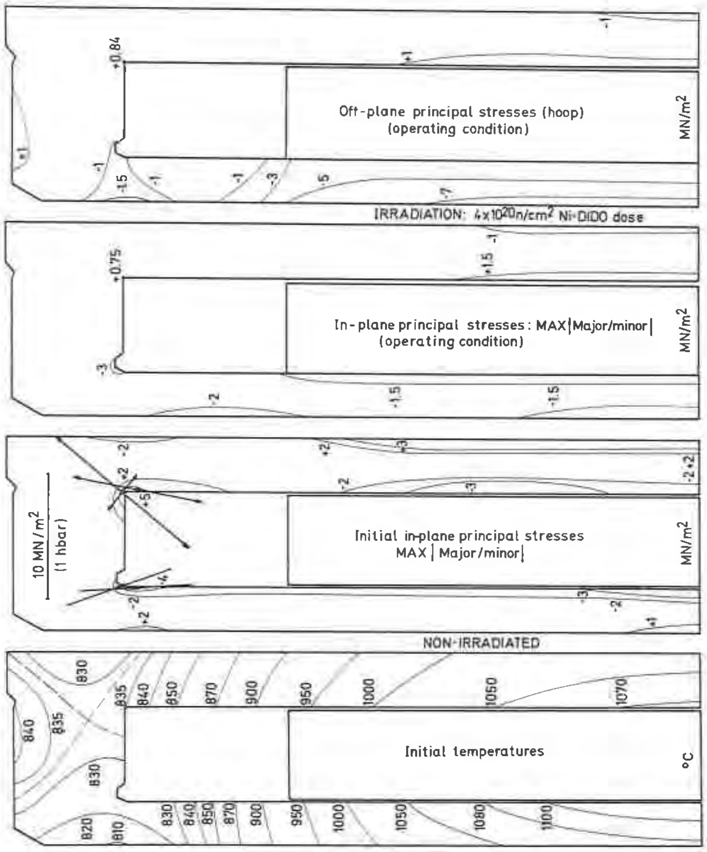

Fig 4 ISOTHERMS AND ISOBARS IN TUBULAR FUEL ELEMENT END CAP
[2-D BREL code results]

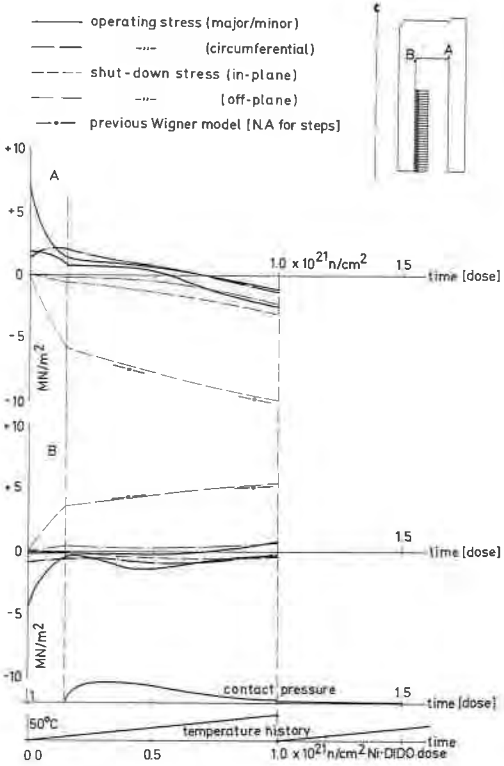

Fig 5 VARIATION OF STRESSES WITH IRRADIATION
[2-D BREL code results]

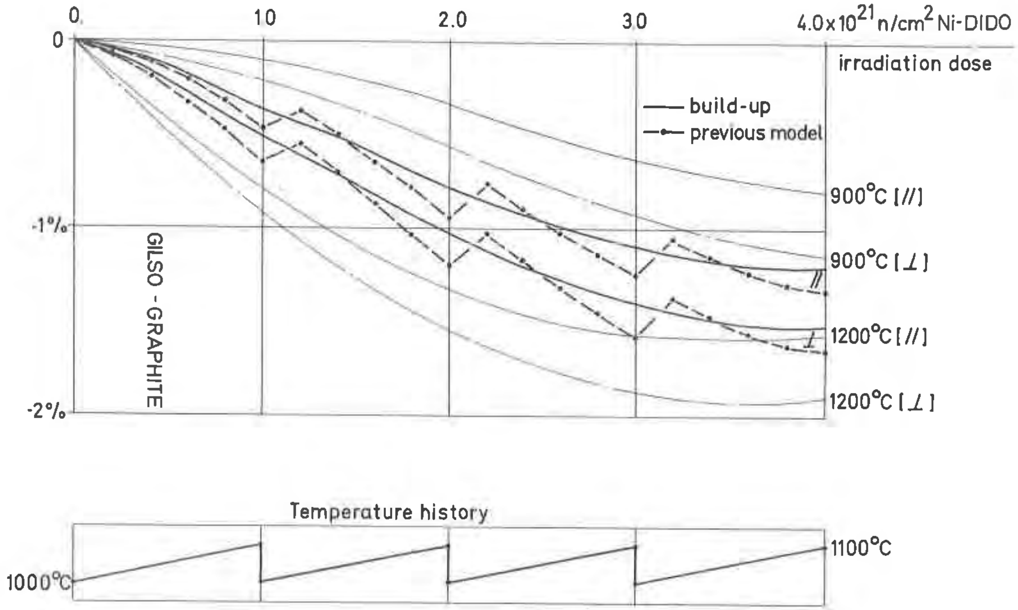

Fig.6 WIGNER SHRINKAGE STRAIN BUILD-UP

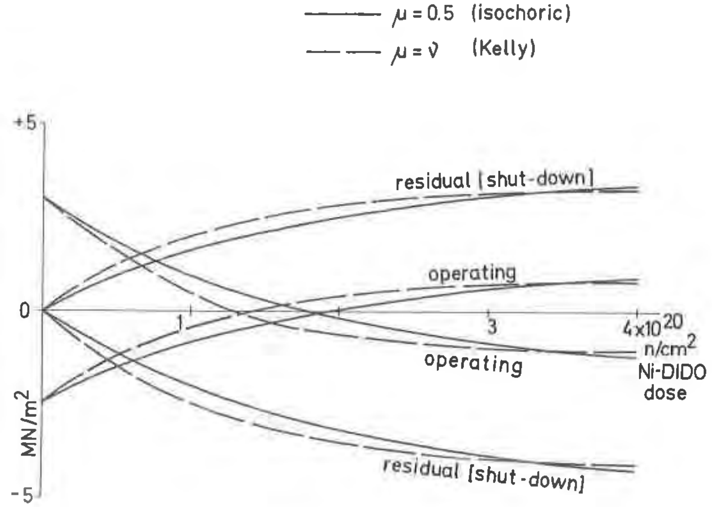

Fig.7 CREEP STRAIN MODEL EFFECT ON STRESS HISTORY